

Marking scheme

Any other solution that leads to correct results will be scored accordingly.

Theoretical Problem nr. 3 - Water world

A.	Part A. Falling droplets	Points
A.1.	$p_{up} + p_{hydro} = p_{down}$ $\sigma = \frac{\rho \cdot g \cdot h \cdot (h^2 - \delta^2)}{2\delta} \quad \sigma \cong \frac{\rho \cdot g \cdot h^3}{2\delta}$	0.3p 0.5p
	$\sigma = 6.5 \times 10^{-2} N \cdot m^{-1}$	0.2p
B.	Part B. Stalagmometer	Points
B.1.	$G = F_{\sigma} \quad R = \sqrt[3]{\frac{3 \cdot d \cdot \sigma}{4 \cdot g \cdot \rho}}$	0.3p 0.5p
	$R = 7.9 \times 10^{-4} m$	0.2p
B.2.	$G + F_H = F_{\sigma} \quad \left\{ \begin{array}{l} R_H = \sqrt[3]{\frac{3}{4} \left(\frac{d \cdot \sigma}{\rho \cdot g} - \frac{d^2 \cdot H}{4} \right)} \\ R_H = R \cdot \sqrt[3]{1 - \frac{d \cdot H \cdot \rho \cdot g}{4\sigma}} \end{array} \right.$	0.3p 0.5p
	$R_H = 7.4 \times 10^{-4} m$	0.2p
C.	Part C. Electrically charged droplets	Points
C.1	<p>The variation of the volume of the conductive drop, if its radius increases with the infinitely small amount ΔR</p> $\Delta V = \frac{4\pi}{3} \cdot [(R + \Delta R)^3 - R^3] \cong 4\pi \cdot R^2 \cdot \Delta R$	0.3p 2.0p
	<p>Variation of the capacitance of the spherical capacitor represented by the drop</p> $\Delta C = 4\pi \cdot \epsilon_0 \cdot \Delta R$	0.3p
	<p>The variation of the electrostatic energy, accumulated in the drop at the constant potential</p> $\Delta W_{\epsilon} = 2\pi \cdot \epsilon_0 \cdot \phi^2 \cdot \Delta R$	0.4p
	<p>Mechanical work L_{ϵ} of electrostatic pressure p_{ϵ}, when increasing the volume with ΔV</p> $\begin{cases} L_{\epsilon} = p_{\epsilon} \cdot \Delta V \\ L_{\epsilon} = p_{\epsilon} \cdot 4\pi \cdot R^2 \cdot \Delta R \end{cases}$	0.4p
	$\Delta W_{\epsilon} = L_{\epsilon}$	0.3p
	$p_{\epsilon} = \frac{\epsilon_0 \cdot \phi^2}{2R^2}$	0.3p

C.2.	The expression of the pressure exerted towards the outside of the drop, just before the moment of its spraying $p_e = \frac{\epsilon_0 \cdot \phi_{max}^2}{2R^2}$	0.2p	1.0p
	The expression of the pressure exerted towards the inside of the drop $p_\sigma = \frac{2\sigma}{R}$	0.2p	
	$p_\sigma = p_e$	0.2p	
	$\phi_{max} = 2 \cdot \sqrt{\frac{\sigma \cdot R}{\epsilon_0}}$	0.2p	
	$\phi_{max} = 5.4 \times 10^3 \text{ V}$	0.2p	
C.3.	The pressure due to the surface tension in each of the n small drops $p_{\sigma r} = \frac{2\sigma}{r} \quad p_{\sigma R} = \frac{2\sigma}{R} \cdot \sqrt[3]{n}$	0.2p	1.0p
	The electrostatic potential of each small drop $\frac{R}{n} \cdot \phi_{max} = r \cdot \phi_\pi$	0.2p	
	$\phi_\pi = \frac{1}{\sqrt[3]{n^2}} \cdot 2 \cdot \sqrt{\frac{\sigma \cdot R}{\epsilon_0}} \quad \phi_\pi = 2 \cdot \sigma^{1/2} \cdot \epsilon_0^{-1/2} \cdot R^{1/2} \cdot n^{-2/3}$	0.2p	
	The outward pressure determined by the electric charges on each small drop $p_{e\pi} = \frac{\epsilon_0 \cdot \phi_\pi^2}{2r^2} \quad p_{e\pi} = \frac{2\sigma}{R} \cdot n^{-2/3}$	0.2p	
	Expression of the pressure leading to the spherical shape of the droplets resulting from the spray $p_\pi = \frac{2\sigma}{R} \cdot (n^{1/3} - n^{-2/3})$	0.2p	
	$p_\pi = 5.1 \times 10^2 \text{ N} \cdot \text{m}^{-2}$	0.2p	
D.	<i>Part D. Water in magnetic field</i>		<i>Points</i>
D.1.	$\Delta W = w_w - w_0 = \frac{B^2}{2\mu_0} \cdot \left(\frac{1}{\mu_r} - 1 \right)$	0.3p	0.3p
D.2.	$\Delta W = v \cdot \frac{B^2}{2\mu_0} \cdot \left(\frac{1}{\mu_r} - 1 \right)$	0.4p	1.5p
	$L = v \cdot (p_N - p_M)$	0.4p	
	$v \cdot (p_N - p_M) = v \cdot \frac{B^2}{2\mu_0} \cdot \left(\frac{1}{\mu_r} - 1 \right)$	0.5p	
	$p_N - p_M = \frac{B^2}{2\mu_0} \cdot \left(\frac{1}{\mu_r} - 1 \right)$	0.2p	
D.3.	$p_N = p_0 + \rho \cdot g \cdot \frac{L}{4} \cong p_0$	0.3p	1.5p
	$p_M = p_0 - \frac{B^2}{2\mu_0} \cdot \left(\frac{1}{\mu_r} - 1 \right)$	0.4p	
	$p_M = p_s \quad p_s = p_s(70^\circ\text{C})$	0.3p	
	$I = \sqrt{\frac{2\mu_0 \cdot (1 + \chi) \cdot (p_0 - p_s)}{K^2 \cdot \chi}}$	0.3p	
	$I = 2.7 \times 10^3 \text{ A}$	0.2p	

E.	Part E. Rising bubbles	Points
E.1.	$L_{bubble} = \pi \cdot R_{bubble}^2 \cdot h_0 \cdot \rho \cdot \frac{v_{bubble}^2}{2}$	0.4p 0.4p
E.2.	$F_{asc} = \frac{4\pi}{3} \cdot R_{bubble}^3 \cdot g \cdot \rho$ <p style="text-align: center;"><i>Note: the weight of the vapor is negligible</i></p> $F_{dis} = \pi \cdot R_{bubble}^2 \cdot \rho \cdot \frac{v_{bubble}^2}{2}$	0.2p 0.8p
	$\vec{F}_{asc} + \vec{F}_{dis} = 0 \quad v_{bubble} = \sqrt{\frac{8g \cdot R_{bubble}}{3}}$	0.2p
	$t_{up} = \frac{h_0}{\sqrt{\frac{8g \cdot R_{bubble}}{3}}}$	0.2p
	$t_{up} = 6.2 \times 10^{-1} \text{ s}$	0.2p
Total points		10p

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