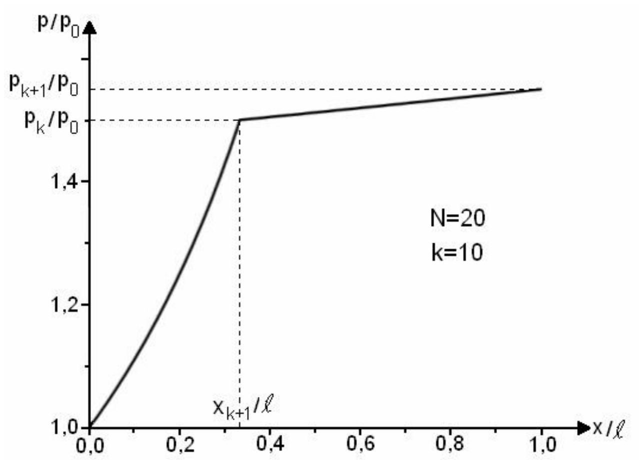
Marking scheme
Points are given for any correct solution

Problem II
Bicycle pump

Nr. item	*Task No. 1*		Point
1.a.	For:		0.80p
	expression of number of moles of air, initially found in the tire at atmospheric pressure p_0 and at T_0 $\nu_{initial} = \frac{p_0 \cdot V_r}{R \cdot T_0}$	0.20p	
	expression of number of moles of air pumped inside the tire at each pumping cycle. $\begin{cases} \nu_0 = \frac{p_0 \cdot V_p}{R \cdot T_0} \\ \nu_0 = \frac{p_0 \cdot V_r}{R \cdot T_0 \cdot N} \end{cases}$	0.20p	
	expression of the number of moles of air, ν_k that are within the tire after the student pumped air k times using the pump with the hand operated piston $\nu_k = \nu_{initial} + k \cdot \nu_0$	0.20p	
	$\nu_k = \frac{p_0 \cdot V_r}{R \cdot T_0} \cdot \left(1 + \frac{k}{N}\right)$	0.20p	
1.b.	For:		0.40p
	expression of the air pressure p_k within the tire, after the student pumped air k times $p_k \cdot V_r = \nu_k \cdot R \cdot T_0$	0.20p	
	$p_k = p_0 \cdot \frac{N + k}{N}$	0.20p	
1.c.	For:		0.40p
	isothermal transformation law applied to the air found in the bicycle pump $p_0 \cdot \frac{V_r}{N} = p_k \cdot S \cdot (\ell - x_{k+1})$	0.20p	
	$x_{k+1} = \ell \cdot \frac{k}{N + k}$	0.20p	

1.d.	For: $p_0 \cdot \frac{V_r}{N} = p(x) \cdot \frac{V_r}{N \cdot \ell} \cdot (\ell - x)$, for $0 \le x \le \frac{k \cdot \ell}{N+k}$ 0.20p $p(x) \cdot \left[V_r + (\ell - x) \cdot \frac{V_r}{N \cdot \ell} \right] = p_{k+1} \cdot V_r$, for $\frac{k \cdot \ell}{N+k} < x \le \ell$ 0.20p $p(x) = \begin{cases} \frac{p_0}{\left(1 - \frac{x}{\ell}\right)}, & 0 \le x \le \frac{k \cdot \ell}{N+k} \\ \frac{p_0 \cdot (N+k+1)}{(N+1) - \frac{x}{\ell}}, & \frac{k \cdot \ell}{N+k} < x \le \ell \end{cases}$ 0.20p	**0.60p**
1.e.	For: $p'(x) = \begin{cases} \frac{p_0/\ell}{\left(1 - \frac{x}{\ell}\right)^2}, & 0 \le x < \frac{k \cdot \ell}{N+k} \\ \frac{p_0 \cdot (N+k+1)/\ell}{\left[(N+1) - \frac{x}{\ell}\right]^2}, & \frac{k \cdot \ell}{N+k} < x \le \ell \end{cases}$ 0.20p *Observation: First-order derivative of the function* $p(x)$ *is positive over the entire range of definition* $\begin{cases} p_s'(x_k) = \frac{p_0/\ell}{\left(1 - \frac{k}{N+k}\right)^2} = \frac{p_0 \cdot (N+k)^2}{\ell \cdot N^2} \\ p_d'(x_k) = \frac{p_0 \cdot (N+k+1)/\ell}{\left[(N+1) - \frac{x}{\ell}\right]^2} = \frac{p_0 \cdot (N+k)^2}{\ell \cdot N^2 \cdot (N+k+1)} \end{cases}$ 0.20p *Observation: Left and right of the point where the valve opens, the pressure shows an angular point.* $p''(x) = \begin{cases} \frac{2p_0/\ell^2}{\left(1 - \frac{x}{\ell}\right)^3}, & 0 \le x < \frac{k \cdot \ell}{N+k} \\ \frac{2p_0 \cdot (N+k+1)/\ell^2}{\left[(N+1) - \frac{x}{\ell}\right]^3}, & \frac{k \cdot \ell}{N+k} < x \le \ell \end{cases}$ 0.20p *Observation: Second-order derivative is also positive on the whole range of definition.*	**0.80p**

Nr. item	Task No. 2	Points
	 *Observation: Graphic representation refers to the 11-th pumping cycle for N = 20* — 0.20p	
2.a.	For: $p_{k_0-1} < n \cdot p_0 \leq p_{k_0}$ — 0.20p	**1.20p**
	$p_{k_0} = p_0 \cdot \frac{N+k_0}{N}$ — 0.20p	
	$k_0 \geq N \cdot (n-1)$, k_0 number of pumping cycle is a natural number — 0.20p	
	$k_0 = N \cdot (n-1)$, if $N \cdot (n-1)$ is a natural number or $k_0 = [N \cdot (n-1)] + 1$, if $N \cdot (n-1)$ is not a natural number — 0.60p	
2.b.	For: find the work that the student needs to do on the ν_1 mole of air, evolving isothermally from pressure p_o to pressure p_{k-1} $L_{K,I} = \nu_1 \cdot R \cdot T_0 \cdot \ln \frac{p_{k-1}}{p_0}$ — 0.20p	**1.20p**
	$L_{K,I} = \frac{p_0 \cdot V_r}{N} \cdot \ln \frac{N+k-1}{N}$ — 0.20p	
	expression of the work that the student needs to do on the ν_k mole of air, evolving isothermally from pressure p_{k-1} to pressure p_k $L_{K,II} = \nu_k \cdot R \cdot T_0 \cdot \ln \frac{p_k}{p_{k-1}}$ — 0.20p	

Nr. item	Task No. 3	Points
	$L_{K,II} = \frac{p_0 \cdot V_r \cdot (N+k)}{N} \cdot \ln \frac{N+k}{N+k-1}$ 0.20p	
	expression of the total work that the student needs to do, during the k pumping cycle. $\begin{cases} L_{K,total} = L_{K,I} + L_{K,II} \\ L_{K,total} = \frac{p_0 \cdot V_r}{N} \cdot \ln \frac{N+k-1}{N} + \frac{p_0 \cdot V_r \cdot (N+k)}{N} \cdot \ln \frac{N+k}{N+k-1} \\ L_{K,total} = \frac{p_0 \cdot V_r}{N} \cdot \left[\ln \frac{N+k-1}{N} + (N+k) \cdot \ln \frac{N+k}{N+k-1} \right] \end{cases}$ 0.20p	
	expression of the total work that the student do from the moment he starts pumping till the moment when the pressure inside tire is $n \cdot p_0$ $L_{total} = \frac{p_0 \cdot V_r}{N} \cdot \sum_{k=1}^{N \cdot (n-1)} \left[\ln \frac{N+k-1}{N} + (N+k) \cdot \ln \frac{N+k}{N+k-1} \right]$ 0.20p	
3.a.	For: number of air moles within the tire after $k = 10$ pumping cycles $\nu_{10} = 0{,}43\, moli$ 0.20p	0.20p
3.b.	For: value of the air pressure within the tire after 10 pumping cycles $p_{10} = 1{,}52 \cdot 10^5 \frac{N}{m^2}$ 0.20p	0.20p
3.c. For	For: number of cycles needed for the air pressure within the tire to reach the value $n \cdot p_0$ $\begin{cases} k_0 = [20 \cdot (2{,}51 - 1)] + 1 \\ k_0 = 31 \end{cases}$ 0.20p	0.20p
3.d.	For: value of the work that the student has to do during the 10th pumping cycle $\begin{cases} L_{10,total} = \frac{\left(1{,}01 \cdot 10^5 \frac{N}{m^2}\right) \cdot \left(7{,}00 \cdot 10^{-3} m^3\right)}{20} \cdot \left[\ln \frac{29}{20} + 30 \cdot \ln \frac{30}{29} \right] \\ L_{10,total} \cong 49{,}1 J \end{cases}$ 0.20p	0.20p

Nr. item	*Task No. 4*	Points
4.a.	For: expression of partial pressure $p_{aer}(T_0)$ of air inside the tire $\begin{cases} p_{aer}(T_0) = p - p_{s,0} \\ p_{aer}(T_0) = 2{,}477 \cdot p_0 \end{cases}$ 0.20p	**0.80p**
	$\begin{cases} p_{aer}(T_1) = p_{aer}(T_0) \cdot \dfrac{T_1}{T_0} \\ p_{aer}(T_1) = 2{,}229 \cdot p_0 \end{cases}$ 0.20p	
	$\begin{cases} p_1 = p_{aer}(T_1) + p_{s,1} \\ p_1 = 2{,}235 \cdot p_0 \end{cases}$ 0.20p	
	The value of the pressure inside the tire in specified conditions of use $p_1 = 2{,}257 \times 10^5 N \cdot m^{-2}$ 0.20p	
4.b.	For : expression of the concentration n_e^* of nitrogen molecules in the room where the tire is kept $\begin{cases} n_e^* = \dfrac{p_0}{T_0} \cdot \dfrac{1}{k_B} \\ n_e^* = \dfrac{p_0}{T_0} \cdot \dfrac{N_A}{R} \end{cases}$ 0.20p	**1.20p**
	expression of the concentration n_i^* of nitrogen molecules in the tire $\begin{cases} n_i^* = 2{,}51 \cdot \dfrac{p_0}{T_0} \cdot \dfrac{N_A}{R} \\ n_i^* = 2{,}51 \cdot \dfrac{p_0}{T_0} \cdot \dfrac{1}{k_B} \end{cases}$ 0.20p	
	expression of the density of the flux of molecules N_e^* coming from all directions, from outside the tire $\begin{cases} N_e^* = \dfrac{1}{4} \cdot n_e^* \cdot \overline{v} \\ N_e^* = \dfrac{1}{4} \cdot \dfrac{p_0}{T_0} \cdot \dfrac{N_A}{R} \cdot \sqrt{\dfrac{8R \cdot T_0}{\pi \cdot \mu_{azot}}} \end{cases}$ 0.20p	

	expression of the density of the flux of molecules N_i^* coming from all directions, from inside the tire $\begin{cases} N_i^* = 2{,}51 \cdot \frac{1}{4} \cdot n_i^* \cdot \bar{v} \\ N_i^* = 2{,}51 \cdot \frac{1}{4} \cdot \frac{p_0}{T_0} \cdot \frac{N_A}{R} \cdot \sqrt{\frac{8R \cdot T_0}{\pi \cdot \mu_{azot}}} \end{cases}$	0.20p
	expression of the number N_i of nitrogen molecules falling onto the surface of pinhole in unit of time coming from all directions, from inside the tire $N_i = 2{,}51 \cdot \frac{1}{4} \cdot \frac{p_0}{T_0} \cdot \frac{N_A}{R} \cdot S \cdot \sqrt{\frac{8R \cdot T_0}{\pi \cdot \mu_{azot}}}$ $N_i = 2{,}51 \cdot p_0 \cdot N_A \cdot S \cdot \sqrt{\frac{1}{2\pi \cdot \mu_{azot} \cdot R \cdot T_0}}$	0.20p
	expression of the number N_e of nitrogen molecules falling onto the surface of pinhole in unit of time coming from all directions, from outside the tire $N_e = \frac{1}{4} \cdot \frac{p_0}{T_0} \cdot \frac{N_A}{R} \cdot S \cdot \sqrt{\frac{8R \cdot T_0}{\pi \cdot \mu_{azot}}}$ $N_e = p_0 \cdot N_A \cdot S \cdot \sqrt{\frac{1}{2\pi \cdot \mu_{azot} \cdot R \cdot T_0}}$	0.20p
4.c.	For:	**1.80p**
	expression of the number of nitrogen molecules falling onto the surface S of the small orifice in time dt coming from outside the tire $\begin{cases} \aleph_e = S \cdot N_e^* \cdot dt \\ \aleph_e = \frac{1}{4} \cdot \frac{p_0}{T_0} \cdot \frac{N_A}{R} \cdot \bar{v} \cdot S \cdot dt \end{cases}$	0.20p
	expression of the number of nitrogen molecules falling onto the surface S of the small orifice in time dt coming from inside the tire depends on pressure $\begin{cases} \aleph_i = \frac{1}{4} \cdot n^*(p) \cdot \bar{v} \cdot S \cdot dt \\ \aleph_i = \frac{1}{4} \cdot \frac{p}{T_0} \cdot \frac{N_A}{R} \cdot \bar{v} \cdot S \cdot dt \end{cases}$	0.20p
	number of molecules inside the tire $\aleph = \frac{p \cdot V_r}{R \cdot T_0} \cdot N_A$	0.20p
	expression of the variation of numbers of molecules inside the tire $\begin{cases} d\aleph = \aleph(t+dt) - \aleph(t) \\ d\aleph = \frac{V_r \cdot N_A}{R \cdot T_0} \cdot (p(t+dt) - p(t)) \\ d\aleph = \frac{V_r \cdot N_A}{R \cdot T_0} \cdot dp \end{cases}$	0.20p
	$d\aleph = \aleph_e - \aleph_i$	0.20p

	$\frac{V_r \cdot N_A}{R \cdot T_0} \cdot dp = \frac{1}{4} \cdot \frac{p_0}{T_0} \cdot \frac{N_A}{R} \cdot \bar{v} \cdot S \cdot dt - \frac{1}{4} \cdot \frac{p}{T_0} \cdot \frac{N_A}{R} \cdot \bar{v} \cdot S \cdot dt$	0.20p	
	$\frac{dp}{(p - p_0)} = -\frac{1}{4} \cdot \frac{\bar{v} \cdot S}{V_r} \cdot dt$	0.20p	
	$\int_{2{,}51 \cdot p_0}^{1{,}10 \cdot p_0} \frac{dp}{(p - p_0)} = -\frac{1}{4} \cdot \frac{\bar{v} \cdot S}{V_r} \cdot \int_0^{t_f} dt$ $\quad t_f = \frac{4 \cdot V_r}{\bar{v} \cdot S} \cdot \ln 15{,}1$	0.20p	
	$\begin{cases} t_f = 1{,}59 \times 10^4 s \\ t_f \cong 4{,}4\, ore \end{cases}$	0.20p	
TOTAL Problem II			**10p**

Prof. dr. Delia DAVIDESCU
Conf. univ. dr. Adrian DAFINEI