Theoretical Problem No. 2 (10 points)

Compton scattering

An electron storage ring contains circulating electrons with high energy produced by an accelerator and rotating in an appropriate magnetic field. X - ray photons are directed so as to collide with electrons stored in storage ring. Phenomenon that occurs is known as Inverse Compton Scattering.

A photon of wavelength λ_i is scattered by a moving, free electron. As a result the electron stops and the resulting photon of wavelength λ_0 is scattered at an angle $\theta = 60°$ with respect to the direction of the incident photon; this photon is again scattered by a second free electron at rest. In this second scattering process a photon with wavelength of $\lambda_f = 1{,}25 \times 10^{-10} m$ emerges at an angle $\theta = 60°$ with respect to the direction of the photon of wavelength λ_0.

To characterize the photons and the electrons during the processes use the following notation:

	initial photon	photon – after the first scattering	final photon		first electron before collision	first electron after collision	second electron before collision	Second electron after collision
moment	$\vec{p}_i$	$\vec{p}_0$	$\vec{p}_f$	moment	$\vec{p}_{1e}$	0	0	$\vec{p}_{2e}$
energy	E_i	E_0	E_f	energy	E_{1e}	E_{0e}	E_{0e}	E_{2e}
wavelength	λ_i	λ_0	λ_f	speed	$\vec{v}_{1e}$	0	0	$\vec{v}_{2e}$

The following constants are known:

$h = 6{,}6 \times 10^{-34} J \cdot s$ - Planck's constant

$m_0 = 9{,}1 \times 10^{-31} kg$ - rest mass of the electron

$c = 3{,}0 \times 10^{8} m/s$ - speed of light in vacuum

Task No. 1 - First collision

1.a. Draw simple sketches marking the moments of the electron and photon before and after the first collision. Clearly specify the coordinate system used.

1.b. Express the energy and moment of the electron implied in the first collision as a function of the initial speed of the electron $\vec{v}_{1e}$ and its rest mass m_0.

1.c. Express the energy and wavelength of the photon after the first collision as a function of the wavelength of the initial photon λ_i, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

Task No. 2 - Second collision

2.a. Draw simple sketches marking moment of the electron and photon before and after the second collision. Clearly specify the coordinate system used.

2.b. Express the energy and wavelength of the photon after the second collision as a function of the wavelength of the photon before the collision λ_0, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

2.c. Express the kinetic energy $(T_2 = E_{2e} - E_{0e})$ and momentum p_{2e} of the electron after the second collision as a function of the photon wavelengths after the collision λ_f, m_0, c and h.

Task No. 3 - Quantitative description of processes

Using the values of the given physical constants and numerical values of λ_f and θ determine the expressions and numerical values for:

3.a. the De Broglie wavelength of the initial electron;

3.b. the energy and frequency of the initial photon;

3.c. the speed of the second electron after collision;

3.d. the variation of the photon wavelength after each collision process.

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ANSWER SHEET

Theoretical Problem No. 2 (10 points)

Compton scattering

Task No. 1 - First collision

1.a. Draw simple sketches marking the moments of the electron and photon before and after the first collision. Clearly specify the coordinate system used.

0,50p

1.b. Express the energy and moment of the electron implied in the first collision as a function of the initial speed of the electron $\vec{v}_{1e}$ and its rest mass m_0.

0,80p

1.c. Express the energy and wavelength of the photon after the first collision as a function of the wavelength of the initial photon λ_i, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

1,20p

Task No. 2 - Second collision

2.a. Draw simple sketches marking moment of the electron and photon before and after the second collision. Clearly specify the coordinate system used.

0,50p

2.b. Express the energy and wavelength of the photon after the second collision as a function of the wavelength of the photon before the collision λ_0, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

1,20p

2.c. Express the kinetic energy $(T_2 = E_{2e} - E_{0e})$ and momentum p_{2e} of the electron after the second collision as a function of the photon wavelengths after the collision λ_f, m_0, c and h.

0,80p

Task No. 3 - Quantitative description of processes

3.a. Determine the expression for the de Broglie wavelength of the initial electron		1,00p
Determine the numerical value for the de Broglie wavelength of the initial electron		1,00p
3.b. Determine the expressions for the energy and for the frequency of the initial photon		0,50p
Determine the numerical values for the energy and for the frequency of the initial photon		0,50p
3.c. Determine the expression for the speed of the second electron after collision		0,50p
Determine the numerical value for the speed of the second electron after collision		0,50p

3.d. Determine the expression for the variation of the photon wavelength after each collision process		0,50p
Determine the numerical value for the variation of the photon wavelength after each collision process		0,50p