

Theoretical Problem No. 1 (10 points)

Fred & Barney's car

Fred and Barney built a car having as "wheels" two identical square prisms (figure 1). The wheels are perfectly suited to the road's shape (that is a periodical repetition of identical bumps), so that the center of mass of the car does not move vertically during the trip. The wheels never slip on the road. During the car's motion, the wheel's long edge always touches a „valley” of the road; road „heights” are reached by a line passing through the half of a rectangular face of the "wheels". The vehicle's movement starts in the tops of two road bumps. Initially, the horizontal translational velocity is \vec{v}_0 .



Figure 1

The total weight of the vehicle excluding its „wheels” is $M \cdot \vec{g}$, distributed equally on the two wheel axles. Further, assume that the only forces acting on car are gravity and normal force.

Task No. 1 - Kinetic energy of the car

1.a. Determine the expression of J - the moment of inertia of wheel reported at its own axles if its mass is m and the length of its square side is $2a$.

1.b. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels:

- i. when the car pass on the tops of the bumps;
- ii. when the car pass with each wheel through a „valley”.

Task No. 2 - Road's shape

In the figure 2 is shown the motion of the lower side of the square cross-section of the wheel on a bump (the curve passing through the points x_s, T, x_d).

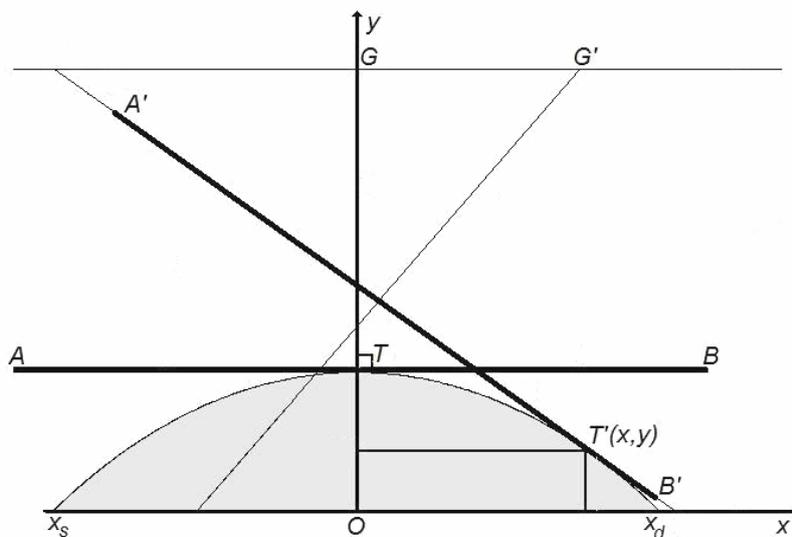


Figure 2

The road is a periodic repetition of such a bump.

The Ox axis of the coordinate system passes through valleys of the relief and the Oy axis passes through bump tip. AB is wheels side, T is the contact point between wheel and road and G is wheel axis position – in initial position. $A'B'$, T' , G' have the same meaning – but for a certain position. In cross-section the wheel axis is always on the same vertical line with the contact point between wheel and road.

2.a. Give an explanation for this finding.

2.b. Determine the analytical form $y = y(x)$ of cross-section of the road.

Hint: The derivative $y'(x)$ of the function $y = y(x)$ is the slope of the tangent to the graph of function in x . Consider known that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$. We recommend to use

the notation $ch(x) = \frac{e^x + e^{-x}}{2}$; $sh(x) = \frac{e^x - e^{-x}}{2}$ with property $ch^2(x) - sh^2(x) = 1$

2.c. Determine the expression of the length (on horizontal) of a bump of road.

2.d. Determine the expression of minimum distance between wheel axles Fred's car.

2.e. Decide whether a machine wheeled regular hexagonal prism conveniently chosen could run on the road whose form was determined in 2.b., without vertical displacement of the center of mass of the wheel.

Task No. 3 - Accident

Assuming that analytical expression of road's bumps is $y = k - h \cdot ch(x/a)$ where k and h are two constants.

3.a. Determine the domain of possible values of the horizontal velocity of the car, under the circumstances established in problem's statement.

3.b. Determines how the amount of heat that can be released by plastic collision of Fred's car with an obstacle depends on position of obstacle on road. Immediately after the collision the car stops.

3.c. Determine the expression of heat released by collision.

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ANSWER SHEET

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Task No. 1 - Kinetic energy of the car

1.a. Determine the expression of J - the moment of inertia of wheel reported at its own axles.

1,00p

1.b.i. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels, when the car passes on the tops of the bumps.

1,00p

1.b.ii. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels: when the car passes with each wheel through a „valley”.

1,00p

Task No. 2 - Road's shape

2.a. In cross-section the wheel axis is always on the same vertical line with the contact point between wheel and road. Give an explanation for this finding.

0,50p

2.b. Determine the analytical form $y = y(x)$ of cross-section of the road.

2,00p

2.c. Determine the expression of the length (on horizontal) of a bump of road.

0,50p

2.d. Determine the expression of minimum distance between wheel axles Fred's car.

0,50p

2.e. Decide whether a machine wheeled regular hexagonal prism conveniently chosen could run on the road whose form was determined in 2.b., without vertical displacement of the center of mass of the wheel. Give a short explanation.

0,50p

Task No. 3 - Accident

3.a. Determine the domain of possible values of the horizontal velocity of the car, under the circumstances established in problem's statement.

2,00p

3.b. Determines how the amount of heat that can be released by plastic collision of Fred's car with an obstacle depends on position of obstacle on road. Give a short explanation.

0,50p

3.c. Determine the expression of heat released by collision.

0,50p