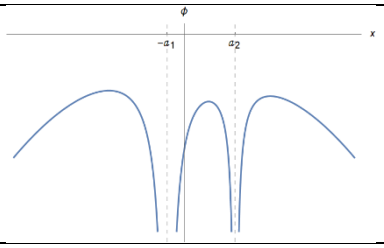
Marking Scheme Q3 (10 points)

Part A (5.0 pt)

A-1	$\Phi(x,y) = -\frac{GM_1}{\sqrt{\left(x+\frac{M_2}{(M_1+M_2)}a\right)^2+y^2}} - \frac{GM_2}{\sqrt{\left(x-\frac{M_1}{(M_1+M_2)}a\right)^2+y^2}} - \frac{1}{2}\frac{G(M_1+M_2)}{a^3}(x^2+y^2)$ [Gravitational part (0.5 pt)] [Centrifugal part (0.5 pt)]	1.0 pt
A-2	[Correct behavior at infinity (0.1 pt)] [Three maximums (0.3 pt)] [Two vertical asymptotes (0.3 pt)] 	0.7 pt
A-3	$\frac{x_0}{a} = 0.36$ [In case of obtaining correct equation but not solving it (0.2 pt)] [Obtaining the numerical result with one decimal figure (0.3 pt)]	0.5 pt
A-4	$\dot{a} = -2\beta a\left(\frac{1}{M_1}-\frac{1}{M_2}\right)$ (0.3 pt) $\dot{P} = -6\pi\sqrt{\frac{a^3}{GM}}\beta\left(\frac{1}{M_1}-\frac{1}{M_2}\right)$ (0.3 pt) [Only correct approach (conservation of momentum) (0.2 pt)]	0.6 pt
A-5	$T = \left(\frac{GM_1\beta}{8\pi\sigma r^3}\right)^{\frac{1}{4}}$ [Correct approach (Energy relation) (0.5 pt)] [Correct solution (0.5 pt)]	1.0 pt
A-6	$a = \left[\frac{P^2G(M_S+M_{NS})}{4\pi^2}\right]^{\frac{1}{3}}$ (0.3 pt) $T = \left(\frac{500\pi M_{NS}\beta}{\sigma P^2(M_S+M_{NS})}\right)^{\frac{1}{4}}$ (0.1 pt) $T = 9\times10^3$ K (0.1 pt) [If the final answer for T is correct the complete pt will be given]	0.5 pt
A-7	$E' = \frac{1}{2}\mu' v'^2 - \frac{GM_1'M_2}{a} < 0$ (0.2 pt) $v'_{max} = \sqrt{\frac{2G(M_1'+M_2)}{a}}$ (0.2 pt) $v' = v$ (0.2 pt) $M'_{1min} = \frac{M_1-M_2}{2}$ (0.1 pt)	0.7 pt

Part B (5.0 pt)

B-1	$g = -\frac{4\pi G \rho_c r}{3}$	0.2 pt
B-2	$h_1(\rho, r) = r^2 \rho^{\gamma-2}$ $h_2(r) = \frac{4\pi G r^2}{K\gamma}$ $[\vec{F} = -\frac{GM(\vec{r})\rho}{r^2} \mathrm{A}\, \Delta r - \Delta p A = 0$ (0.3 pt)]	0.6 pt
B-3	$r_0 = G^{-\frac{1}{2}} p_c^{\frac{1}{2}} \rho_c^{-1}$	0.4 pt
B-4	$A_1(u, x) = x^2 u^{\gamma-2}$ $A_2(x) = \frac{4\pi x^2}{\gamma}$ The answer would be correct up to a constant coefficient	0.3 pt
B-5	$f(x) = A \sin(\sqrt{2\pi}x) + B\cos(\sqrt{2\pi}x)$ (0.3 pt) $A = \frac{1}{\sqrt{2\pi}}$ (0.2 pt) & $B = 0$ (0.1 pt)	0.6 pt
B-6	$u'(0) = 0$ (0.1 pt) $\lim_{x\to 0} \frac{u'(x)}{x} = u''(0)$ (0.4 pt) $\gamma = -\frac{4\pi}{3u''(0)}$ (0.2 pt) $\gamma \sim 1.66$ (0.1 pt)	0.8 pt
B-7	$\tilde{\rho} \simeq \rho(1-3\epsilon)$ (0.6 pt) $[\tilde{\rho} = \rho(1+\epsilon)^{-3}$(0.4pt)] $\tilde{g} \simeq g(1-2\epsilon)$ (0.3 pt) $[\tilde{g} = g(1+\epsilon)^{-2}$(0.2pt)]	0.9 pt
B-8	$\ddot{\tilde{r}} = \tilde{g} - k\gamma\tilde{\rho}^{\gamma-2}\frac{\partial\tilde{\rho}}{\partial\tilde{r}}$	0.6 pt
B-9	$\ddot{\epsilon} = -\frac{4\pi G\rho_c}{3}(3\gamma-4)\epsilon$ (0.4 pt) $\gamma_{\min} = \frac{4}{3}$ (0.1 pt) $\omega = \sqrt{\frac{4\pi G\rho_c}{3}(3\gamma-4)}$ (0.1 pt)	0.6 pt