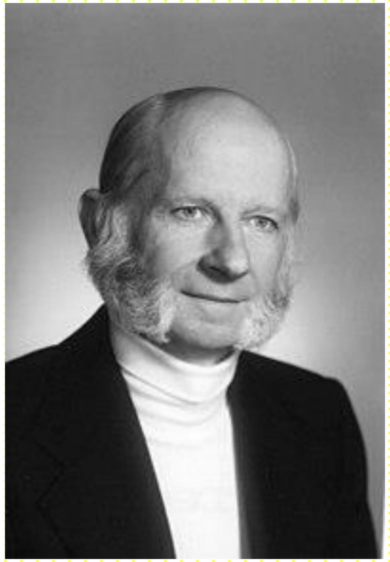
Trapping Ions and Cooling Atoms

IPHO 2024-IRAN

A. Paul Trap



The Nobel Prize in Physics 1989 was divided, ... the other half jointly to Hans G. Dehmelt and Wolfgang Paul "for the development of the ion trap technique". From nobelprize.org

How to trap an Ion?

Earnshaw's theorem

a collection of point charges cannot be maintained in a stable stationary equilibrium configuration solely by the electrostatic interaction of the charges.

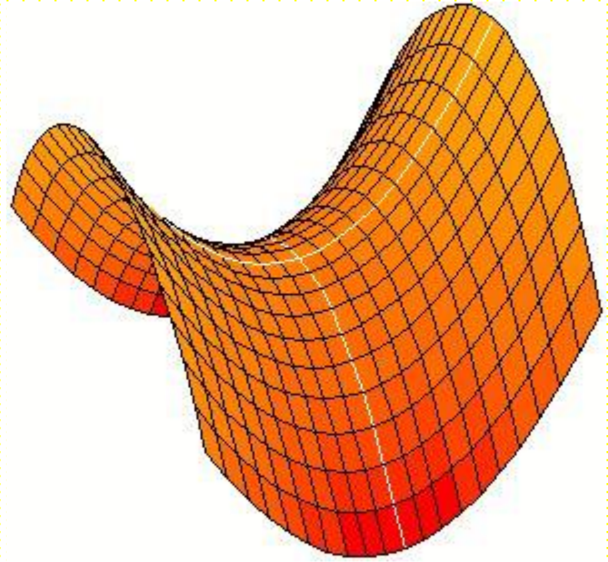
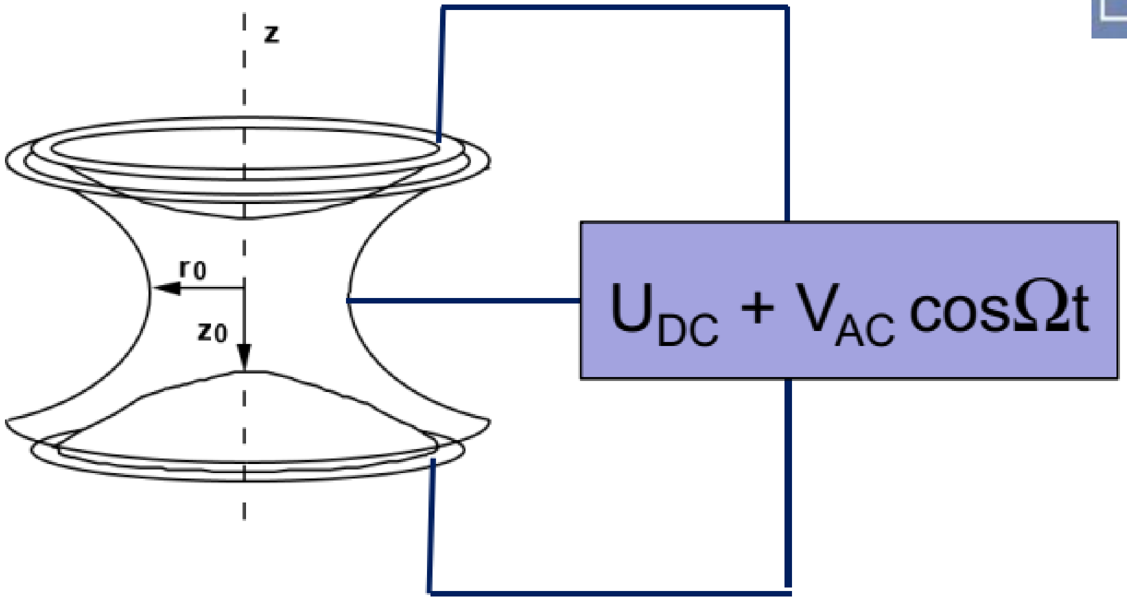
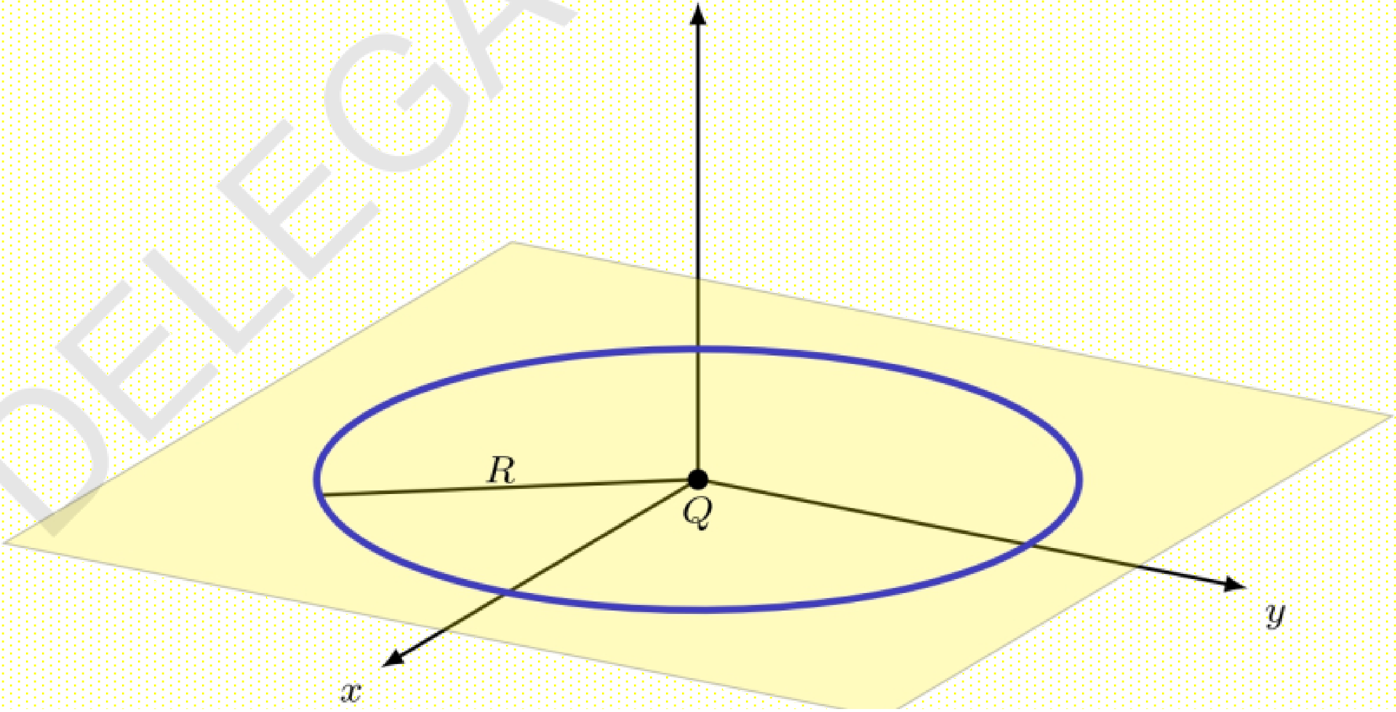


Photo from wikipedia

Paul Trap: Using Dynamical Stability



Simplification of the question:



$$\lambda = \lambda_0$$

$$\lambda = \lambda_0 + u \cos \Omega t$$

A-1	a) In cartesian coordinates (x, y, z), obtain the electric field in the vicinity of the ring's center to the first order in x/R, y/R, and z/R. b) Find the frequency of small oscillations around the center of the ring in the directions for which a stable equilibrium exists.	1.5 pt

tromagnetism

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The ideas needed to solve:

1) Writing Coulomb's Law and very simple integrating.
2) Using suitable approximations and expansions.
3) Using Gauss's Law.

$$\vec{E}(x, y, x) \simeq \frac{\lambda}{4\epsilon_0 R^2}(-x, -y, 2z)$$

Finding the Eq. of Motion in Dynamical case:

$$\lambda = \lambda_0 + u\cos\Omega t \longrightarrow \ddot{z} = (+k^2 + a\Omega^2 cos\Omega t)z$$

A-2	Write a and k in terms of the known parameters.	0.6 pt

Syllabus: Mechanics, Oscillations and waves

The idea needed to solve:

Writing Newton's Second Law

$$F_z = Q\,E_z = \left(\frac{Q\lambda_0}{2\epsilon_0 R^2 m} + \frac{Qu}{2\epsilon_0 R^2 m}\cos\Omega t\right)$$

$$\frac{Q\lambda_0}{2\epsilon_0 R^2 m} \rightarrow k^2 \qquad \frac{Qu}{2\epsilon_0 R^2 m} \rightarrow a\Omega^2$$

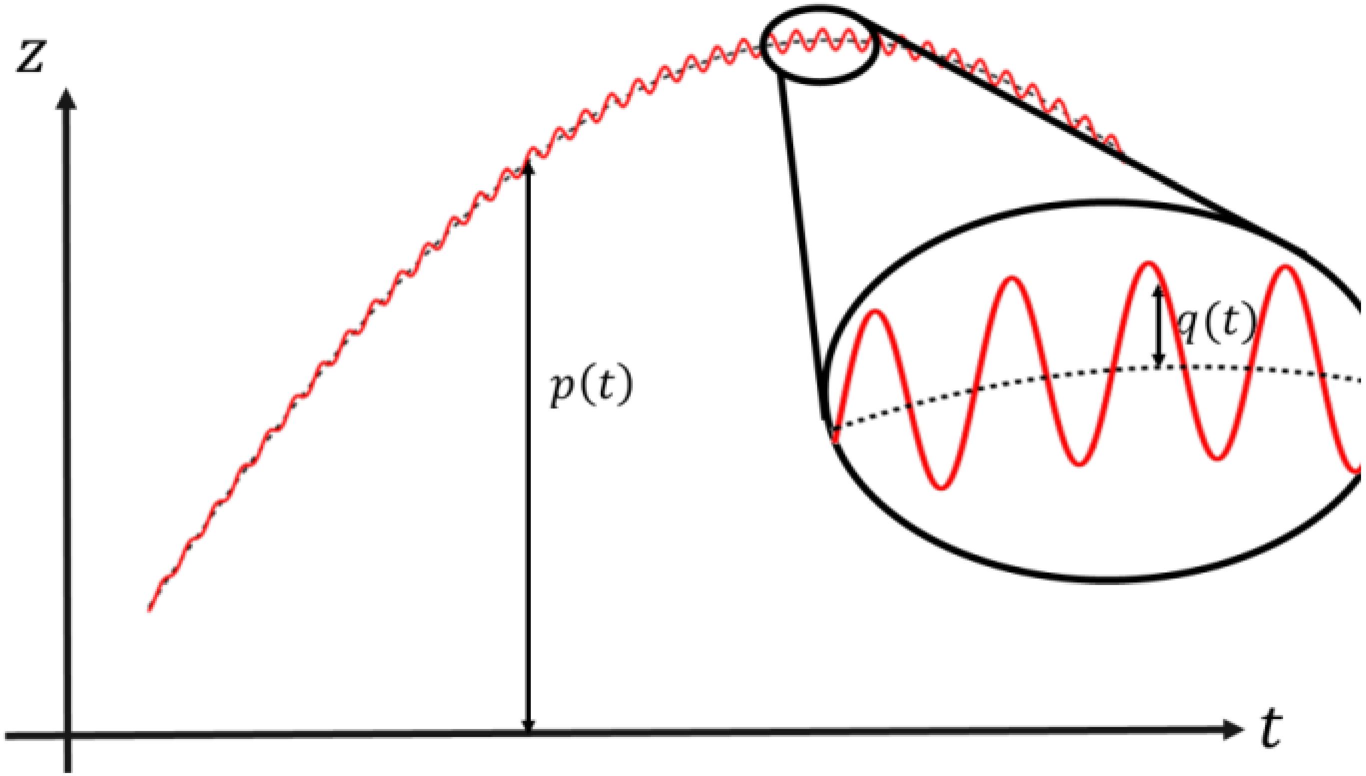
Mathieu Equation

Some simplifying assumptions:

$a \ll 1$

$\Omega \gg k$

$\Omega^2 a \gg k^2$

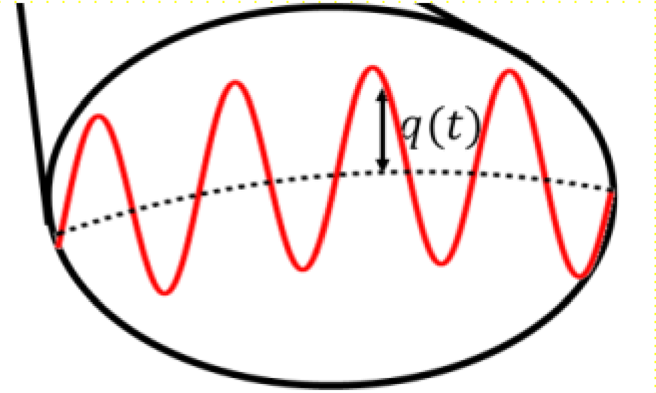


Slowly varying,
big amplitude

$$z(t) = p(t) + q(t)$$

Fast with small amplitude

The fast varying part:



A-3	a) Find the equation of motion for $q(t)$ in terms of a, k, Ω, and $p(t)$. b) Find the solution of this equation by considering appropriate initial conditions corresponding to the required properties of this function.	1.8 pt

Syllabus: Mechanics, Oscillations and waves

The ideas needed to solve:

p is nearly constant

$q \ll p$

$\Omega^2 a \gg k^2$

$$\cancel{\ddot{p}} + \ddot{q} = (\cancel{k^2} + a\Omega^2 \cos \Omega t)(p + \cancel{q})$$

$$\ddot{q} = a\Omega^2 \cos \Omega t \; p$$

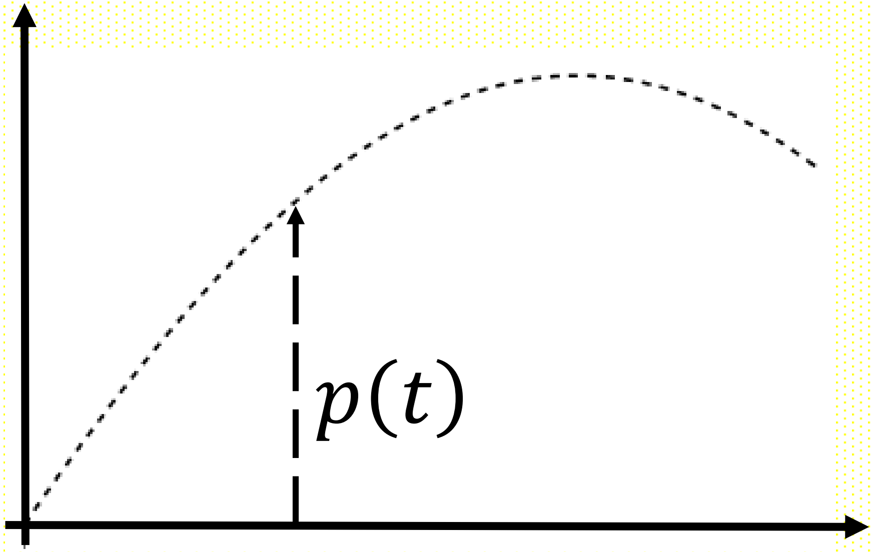
$$\ddot{q} = a\Omega^2 \cos \Omega t \; p$$

p is nearly constant

$$q = -a\, p \cos \Omega t + c_1 t + c_2$$

q should remain small with zero mean

The slow varying part:



A-4	a) Consider the mean effect of the rapidly varying component and obtain an effective equation of motion for $p(t)$. b) Investigate the stability of the equilibrium point and find the condition for a stable equilibrium.	1.5 pt

Syllabus: Mechanics, Oscillations and waves

The ideas needed to solve:

1) Substituting the answer for $q(t)$ in the original Eq. of motion.

2) Using the mean values $\langle \cos \Omega t \rangle = 0$ and $\langle \cos^2 \Omega t \rangle = 1/2$

$$\ddot{p} = \left(k^2 p - \frac{a^2\Omega^2 p}{2}\right)$$

Stability Condition: $\Omega > \sqrt{2}\,\frac{k}{a}$

Numerical estimate of the frequency needed

Assume that $\lambda_0 = 8 \times 10^{-9}$ C/m and $R = 10$ cm. We would like to use this device to trap a singly ionized atom 100 times heavier than a hydrogen atom.

A-5	Calculate k. Assume $a = 0.04$ and estimate the smallest frequency required to stabilize the motion of this ion. Use the data given at the end of the question.	0.4 pt

$k = 2 \times 10^5$ rad/s $\qquad \Omega_{min} \simeq 7 \times 10^6$ rad/s

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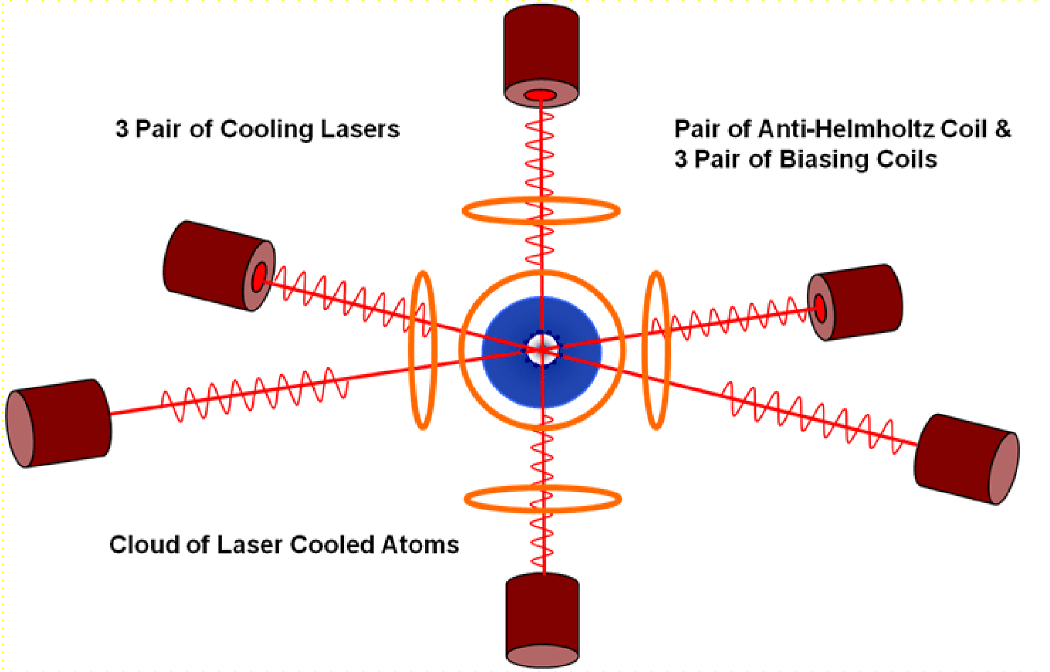
B. Cooling Atoms



https://worldbuilding.stackexchange.com/

The same concept in IPhO 2009 Mexico, yet very different approach.

Laser Cooling

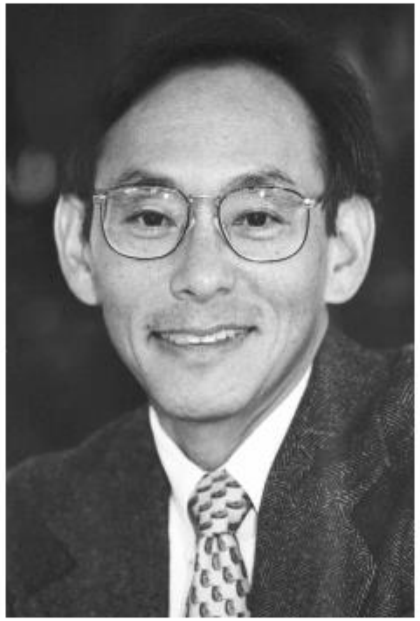


Photo from the Nobel Foundation archive.

Steven Chu

Prize share: 1/3

Photo from the Nobel Foundation archive.

Claude Cohen-Tannoudji

Prize share: 1/3

Photo from the Nobel Foundation archive.

William D. Phillips

Prize share: 1/3

The Nobel Prize in Physics 1997 was awarded jointly to Steven Chu, Claude Cohen-Tannoudji and William D. Phillips "for development of methods to cool and trap atoms with laser light" From nobelprize.org

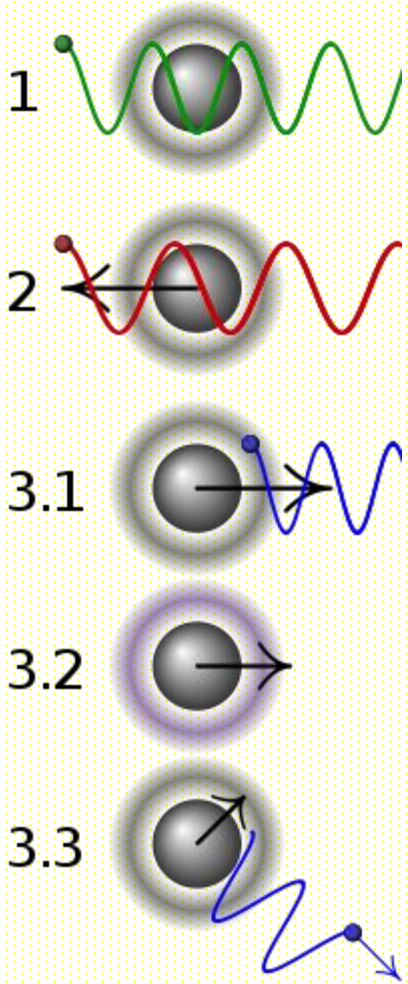


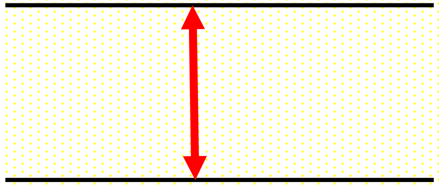
Photo from wikipedia

The atom absorbs mainly the blue-shifted lights due to Doppler effect.

The mechanism effectively reduces the energy of the atom

Finding the spectrum width knowing the lifetime of an atomic state

$$E_A = \hbar\omega_A$$



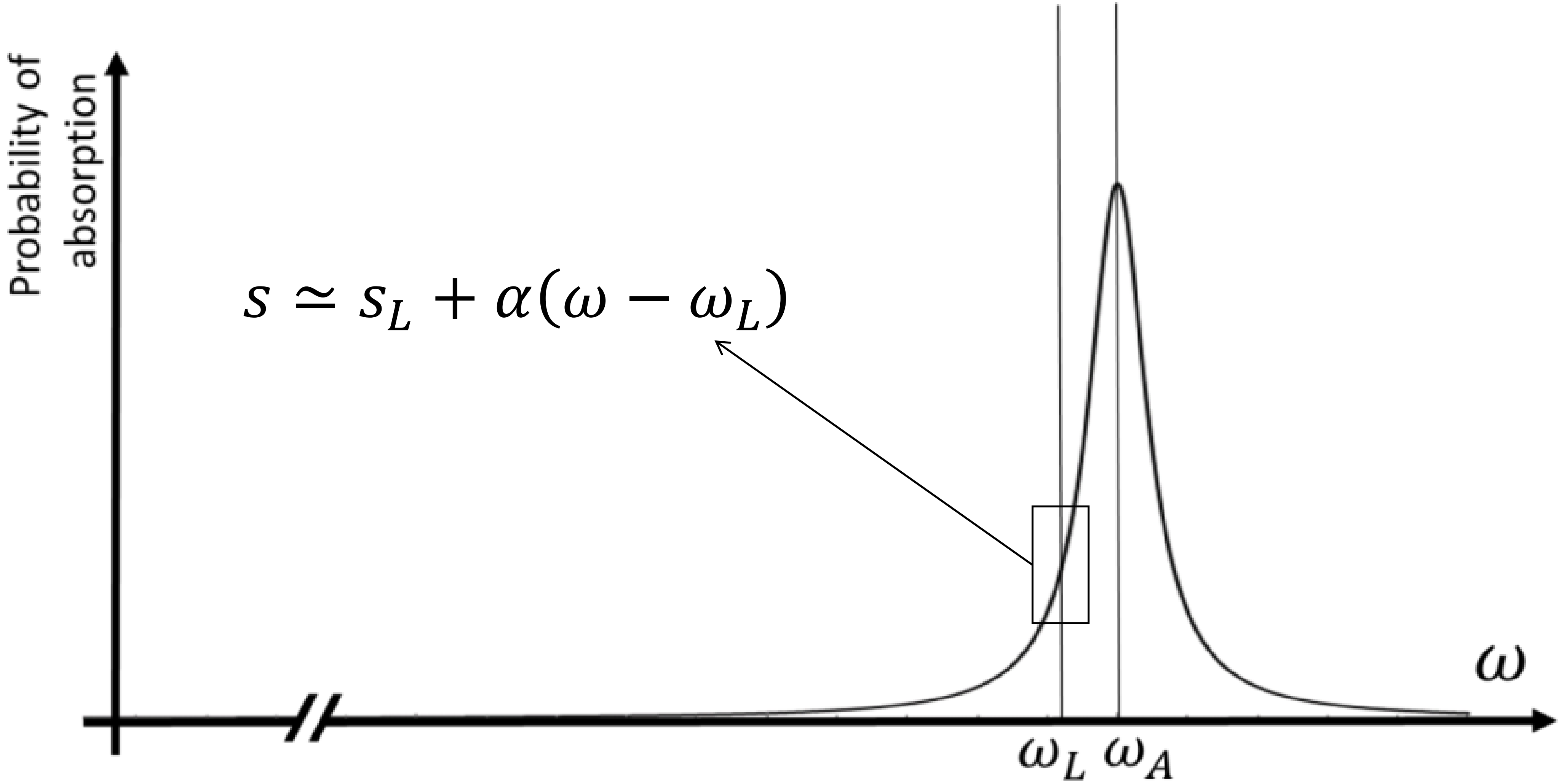
τ = Lifetime

B-1	Use the Heisenberg's uncertainty principle to find Γ.	0.5 pt

$$\Delta E \times \Delta t \simeq \hbar \longrightarrow \Gamma = \Delta\omega \simeq 1/\tau$$

Syllabus: Quantum Physics

The probability of absorption in the atom's reference frame



The changes to absorption rate due to the Doppler effect and the effective force on the atoms

B-2	a) Assume that the trapped atom is moving with a velocity, $v = v_x$ in the lab frame. In the frame of reference of the atom, calculate the collision rate of the photons, incident from each of the two directions, with the atoms (denoted by s_+ and s_-) and the rate of absorption of momentum in each direction (denoted by π_+ and π_-). b) Determine the effective force on the atom as a function of v, $k_L = \omega_L / c$, $\hbar$, and α, in the reference frame of the laboratory. Assume $s_L \ll \alpha\omega_L$,	1.7 pt

Syllabus: Mechanics, Relativity

In the reference frame of the atom:

$$\omega_+ = \omega_L \left(1 + \frac{v}{c}\right) \qquad s_+ = s_L + \alpha\omega_L \frac{v}{c}$$

$$\omega_- = \omega_L \left(1 - \frac{v}{c}\right) \qquad s_- = s_L - \alpha\omega_L \frac{v}{c}$$

The rate of momentum absorption:

$$\pi_+ = s_+ \times (-\hbar k_+)$$

$$\pi_- = s_- \times (+\hbar k_-)$$

$$\longrightarrow \quad \pi_+ + \pi_- = -2\hbar k_L \alpha \omega_L \frac{v}{c}\left(1 + \frac{s_L}{\alpha \omega_L}\right)$$

$$F = -(2\alpha\hbar k_L^2)v$$

The same force in the lab's frame up to the order v/c

Estimating the ultimate temperature: I

Finding the energy absorption rate of a completely stopped atom

B-3	Considering the momentum of the atom after such a process for the two possible outcomes, calculate the average power absorbed by the atom.	1.0 pt

Syllabus: Mechanics

Before absorbing the photon

$$p_0 = 0$$

After absorbing the photon:

$$p_1 = \hbar k_L$$

After emitting a photon:

$$p_{f1} = 0$$

$$p_{f2} = 2\hbar k_L$$

$$\langle E_f \rangle = \frac{1}{2} \times 0 + \frac{1}{2} \times \frac{(2\hbar k_L)^2}{2m} = \frac{\hbar^2 k_L^2}{m}$$

Input power $= \dfrac{\hbar^2 k_L^2}{m\tau}$

Estimating the ultimate temperature: II

Finding the energy dissipation rate of an atom and the steady state temperature

B-4	Consider the force calculated in Task B-2 and calculate the output power. Then, calculate the average value of v^2 at equilibrium. Using your knowledge of the kinetic theory of gases estimate the temperature of the atoms.	0.8 pt

Syllabus: Mechanics, Thermodynamics and statistical physics

Rate of energy loss:

$$P_{\text{out}} = F.v = -2\,\alpha\hbar k_L^2 v^2$$

Steady state condition:

$$P_{\text{out}} + P_{\text{in}} = 0 \longrightarrow \overline{v^2} = \frac{\hbar\Gamma}{2\alpha m}$$

$$T = \frac{\hbar\Gamma}{2\alpha k_B}$$

Numerical estimate of the ultimate temperature:

B-5	Estimate this temperature, for an atom 100 times heavier than a hydrogen atom. Assume that $\omega_{\mathrm{L}} = 2 \times 10^{16}$ rad/s, $\tau = 5 \times 10^{-9}$ s, and $\alpha = 4$.	0.4 pt

$$T = 2 \times 10^{-4} K$$

Thanks!