Earth's temperature, the greenhouse effect, and the global warming

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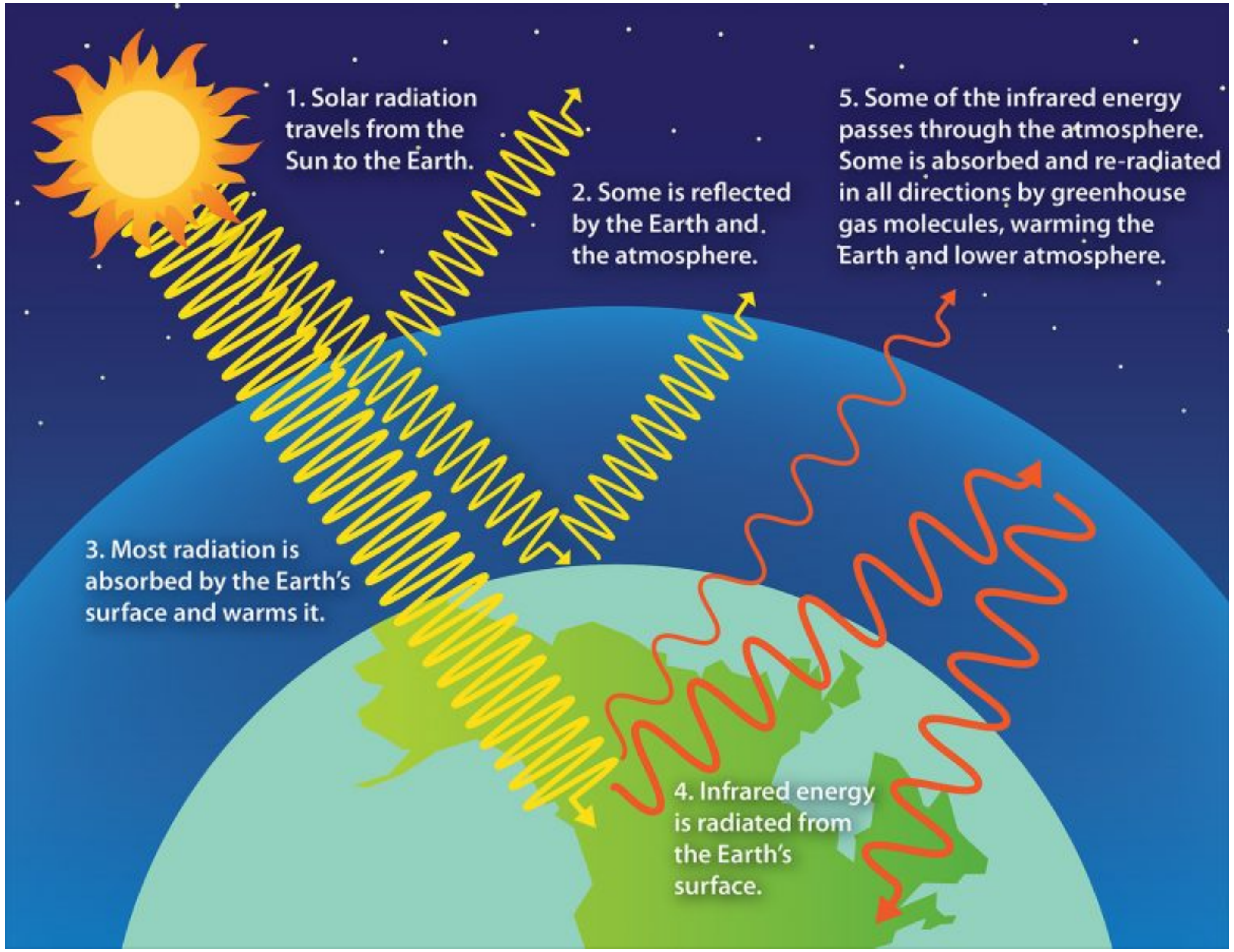
The greenhouse effect

- The **greenhouse gases** in the atmosphere (carbon dioxide, methane, water vapor, and other gases) first **absorb** the **Earth's infrared radiation** and then **re-radiate this absorbed energy**, heating the surrounding air and the ground below it.

- Greenhouse gases comprise a very small proportion of the Earth's dry atmosphere, **Carbon dioxide** is just 0.04 percent by volume.

- The most powerful greenhouse gas is **water vapor**, but we cannot control the concentration of water vapor in the atmosphere, while we can control that of carbon dioxide.

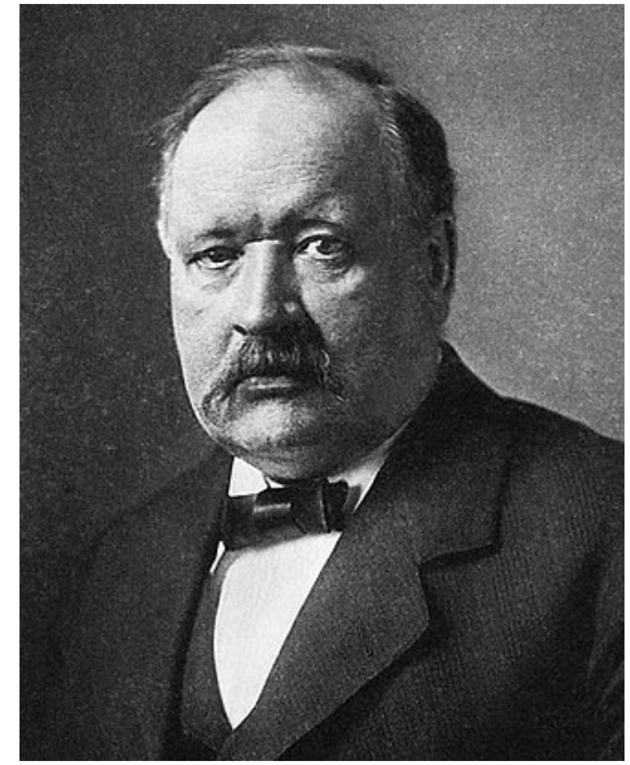
- The greenhouse effect is essential for life on Earth. Without the greenhouse effect, the surface temperature would barely exceed –18°C.



History

- Arrhenius was the first to use the principles of **physical chemistry** to estimate the extent to which **increases in atmospheric carbon dioxide** are **responsible for the Earth's rising surface temperature**.

- It was his colleague, meteorologist Nils Ekholm who, in 1901, was the first to use the word **greenhouse** in describing the atmosphere's absorption and re-radiation of heat.



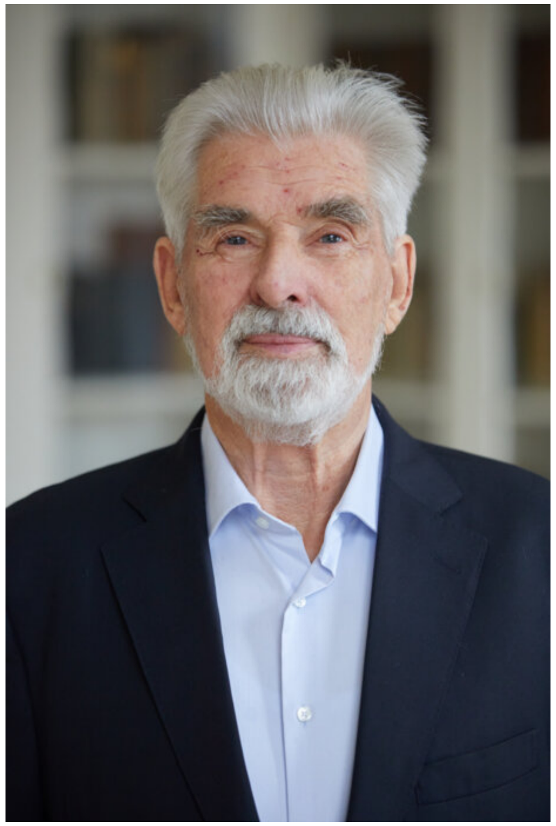
Svante Arrhenius (1859-1927)

Nils Gustaf Ekholm (1848-1923)

Physics Nobel prize 2021

The Nobel Prize in Physics 2021 was awarded "for groundbreaking contributions to our understanding of "**complex physical systems**" with one half jointly to **Syukuro Manabe** and **Klaus Hasselmann** "**for the physical modeling of Earth's climate, quantifying variability and reliably predicting global warming**"

Syukuro Manabe (1931-)



Klaus Hasselmann (1931-)

Climate models

- **Syukuro Manabe,** in the 1960s, was the first person to explore the interaction between **radiation balance** and the **vertical transport of air masses** due to **convection,** as well as **the latent heat of water vapor**. His work laid the foundation for the development of climate models.

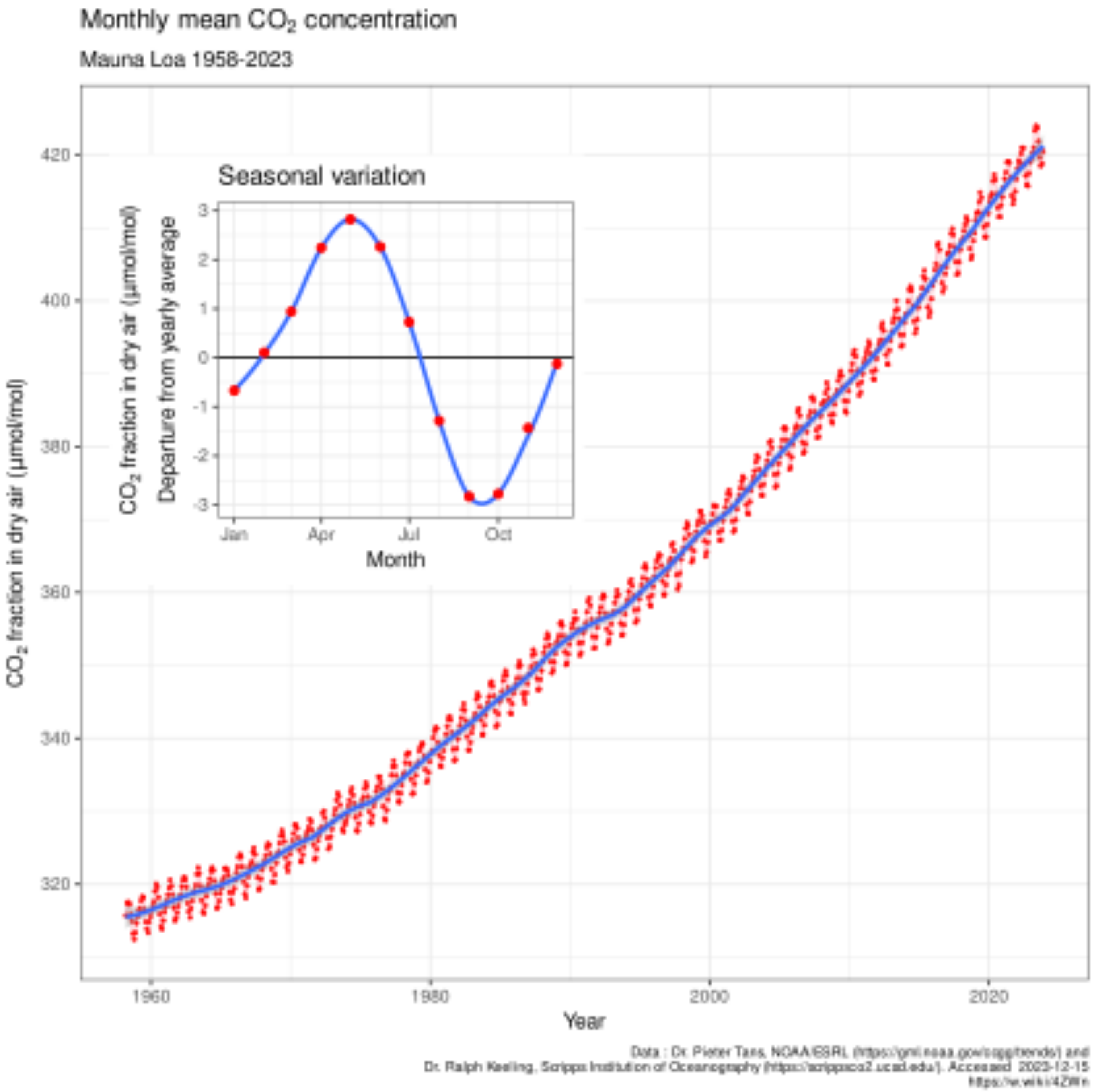
- **Hasselmann** also developed methods for identifying specific signals, and fingerprints, that both **natural phenomena** and **human activities** imprint in the climate. His methods have been used to prove that the **increased temperature in the atmosphere is due to human emissions of carbon dioxide**.

Keeling curve

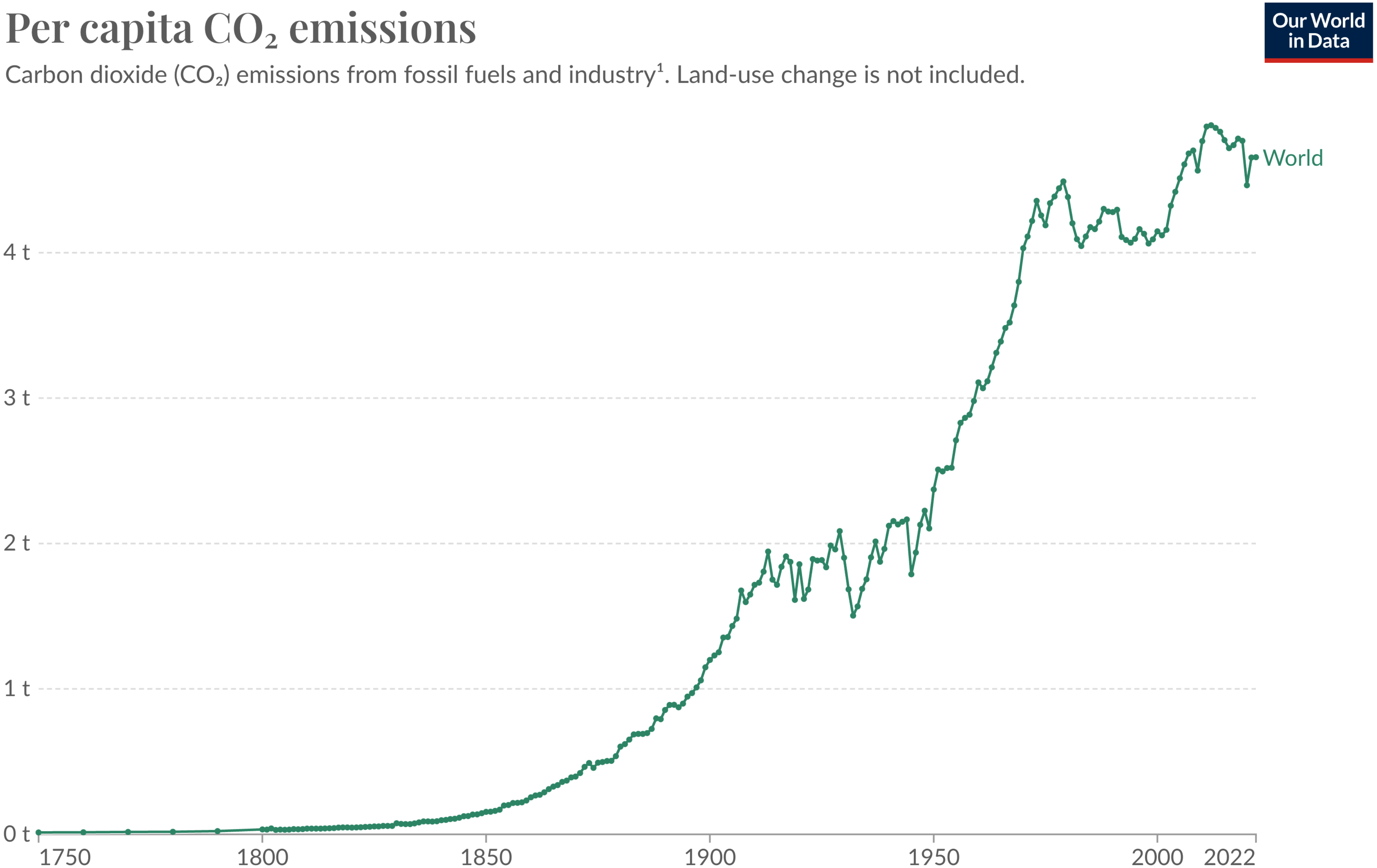
- **Charles David Keeling** was an American scientist whose recording of carbon dioxide at the **Mauna Loa Observatory** confirmed Arrhenius's proposition of the possibility of **anthropogenic** contribution to the greenhouse effect and global warming, by documenting the steadily rising carbon dioxide levels.

- The Keeling Curve measures the progressive buildup of carbon dioxide in the atmosphere.

Charles David Keeling (1928-2005)



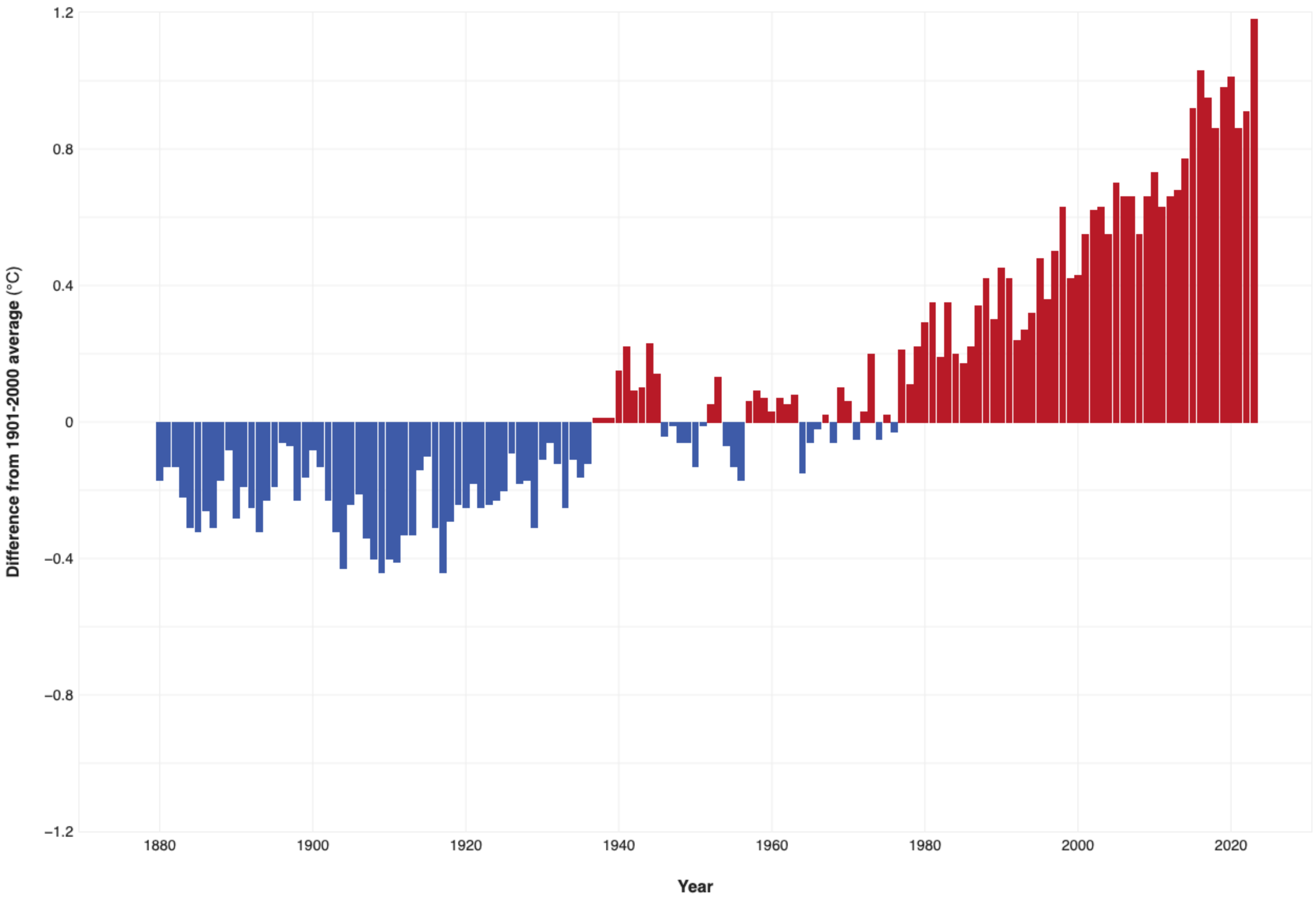
Fossil fuels—including coal, oil, and natural gas—have been powering economies for over 150 years, and currently supply about 80 percent of the world's energy.



Data source: Global Carbon Budget (2023); Population based on various sources (2023)
OurWorldInData.org/co2-and-greenhouse-gas-emissions | CC BY

1. Fossil emissions: Fossil emissions measure the quantity of carbon dioxide (CO_2) emitted from the burning of fossil fuels, and directly from industrial processes such as cement and steel production. Fossil CO_2 includes emissions from coal, oil, gas, flaring, cement, steel, and other industrial processes. Fossil emissions do not include land use change, deforestation, soils, or vegetation.

GLOBAL AVERAGE SURFACE TEMPERATURE



Reference of the question

1. Knox, Robert S. "Physical aspects of the greenhouse effect and global warming." *American Journal of Physics* 67, no. 12 (1999): 1227-1238.

Basic Concepts

Basic concepts

Black body Radiation

- A **black body** is an idealized physical body that **absorbs all incident electromagnetic radiation**, regardless of frequency or angle of incidence.

- The radiation emitted by a black body in **thermal equilibrium** with its **environment** is called **black-body radiation**.

- Black body **spectral radiance**: the **radiative power** per **unit area** per **unit wavelength** is given by

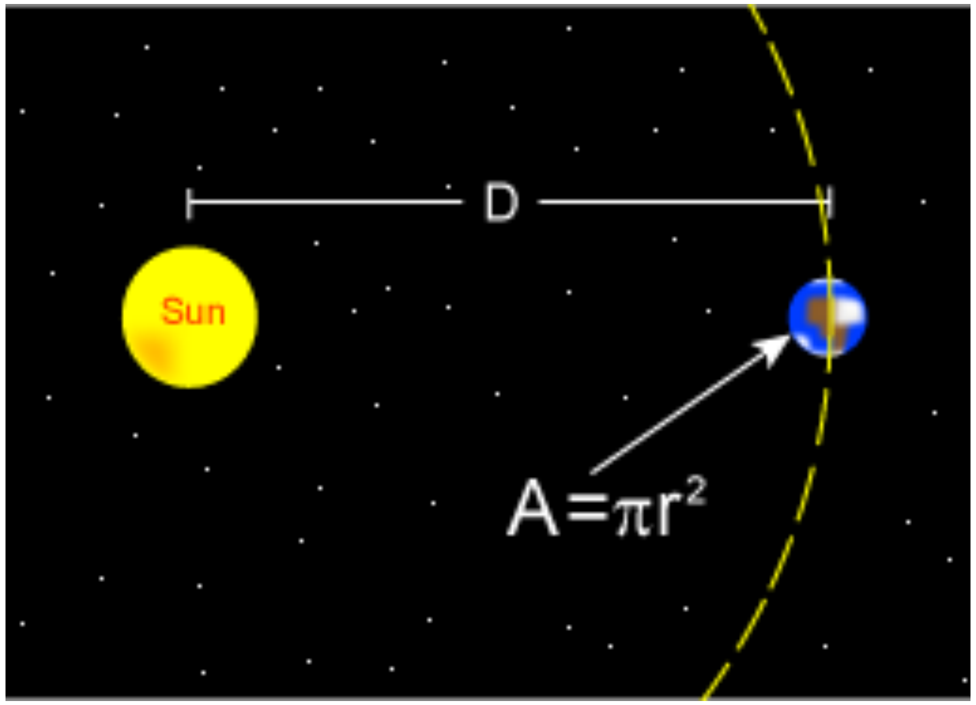
$$u(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(\frac{hc}{\lambda K_B T}) - 1}$$

- **Stefan-Boltzmann**'s law: **total radiative power per unit area** for a black body is given by $U(T) = \sigma T^4$

Solar constant

- The **solar constant** (S_0) is the amount of **solar radiative power per uint era** received by a given area **one astronomical unit** (the average Earth-Sun distance) away from the Sun.

- It is measured on a surface **perpendicular to the rays**, one astronomical unit (au) from the Sun.



Kirchhoff's law of thermal radiation

- For a body of any **arbitrary material** emitting and absorbing thermal electromagnetic radiation at every wavelength in **thermodynamic equilibrium**, the **ratio of its emissive power** to its **dimensionless coefficient of absorption** is equal to a **universal function** only of radiative **wavelength** and **temperature**, that is the **perfect black-body emissive power**.

- Hence, the emissive power of an arbitrary object with absorptivity ϵ at temperature T is given by:

$$U(T) = \epsilon\sigma T^4$$

Thermal Current balance

- In thermal equilibrium, the total thermal in-current and out-current should balance each other: $I_Q^{in} = I_Q^{out}$

- Moreover, all points on the surface are assumed to be at a constant temperature.

Part A

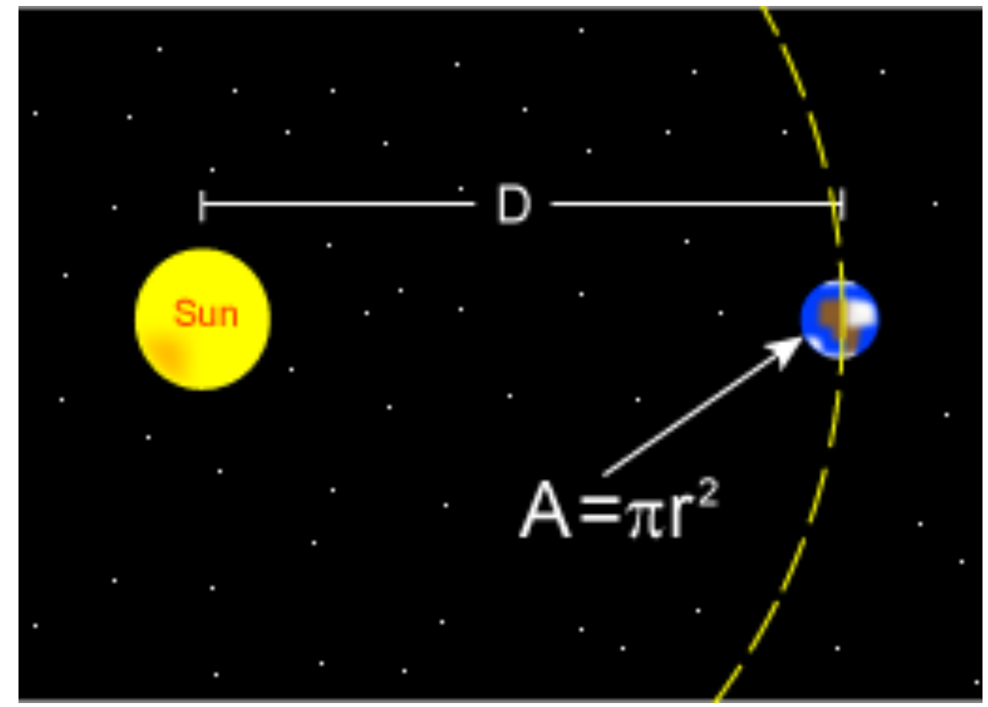
Assumptions

1. The Sun and the Earth as perfect black bodies.
2. Thermal Equilibrium for the Earth.
3. No atmosphere, no greenhouse effect (one-layer model).

Part A

A.1. The solar constant S_0.

(analytically and numerically)



$$S_0 = \sigma T_S^4 (\frac{R_S^2}{D^2}) = 1.35 \times 10^3 \ W/m^2$$

$$T_S = 5.77 \times 10^3 K$$

$$D = 1.50 \times 10^1 1m$$

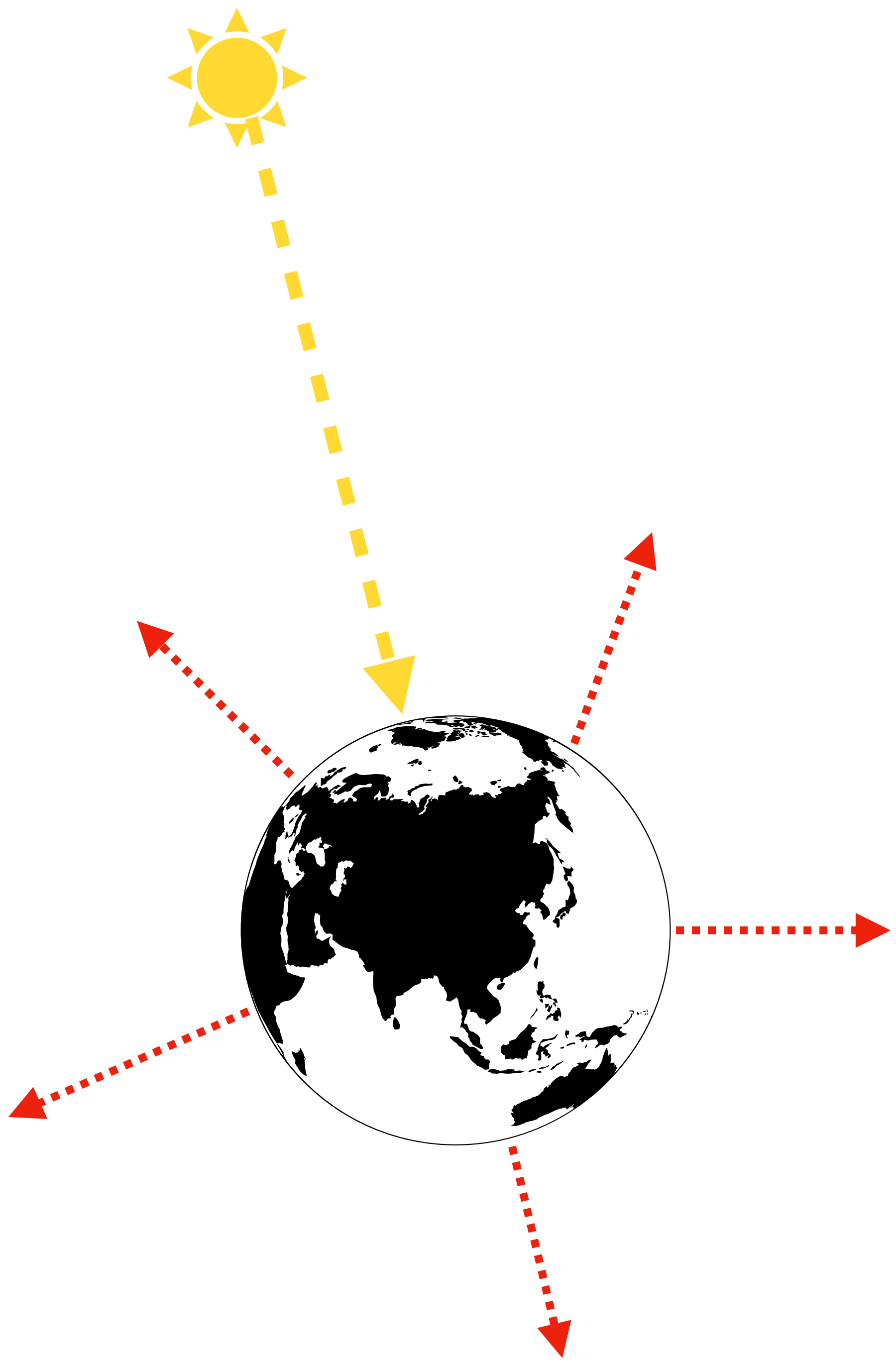
$$R_S = 6.96 \times 10^7 m$$

Part A

A.2. The Earth's temperature T_E

$$S_0 \pi R_E^2 = 4\pi R_E^2 \sigma T_E^4$$

$$T_E = (\frac{S_0}{4\sigma})^{1/4} = 278K$$



Wien's displacement law

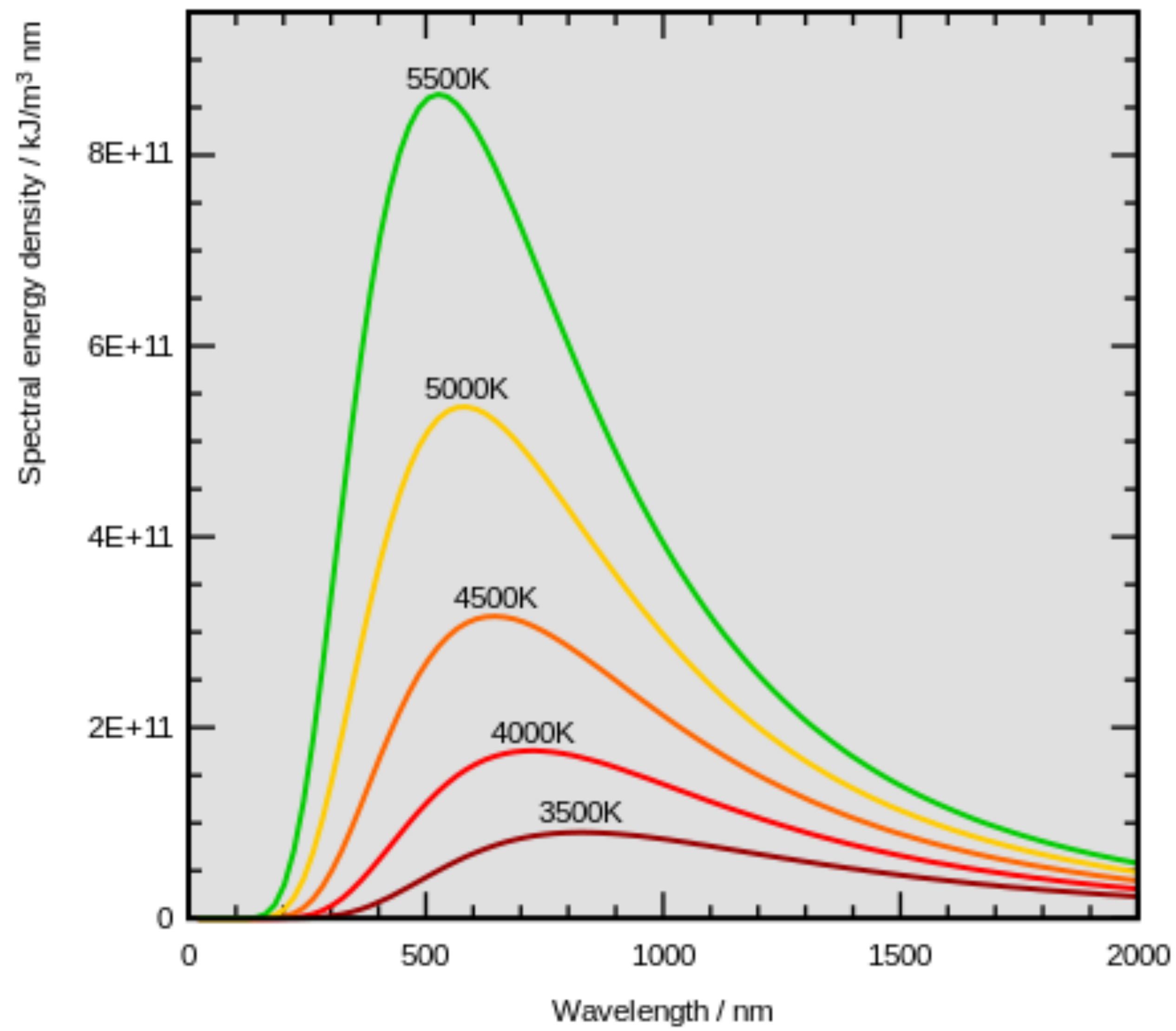
- The **spectral radiance** of blackbody radiation per unit wavelength, $u(\lambda, T)$, peaks at the wavelength, λ_{max}, which is inversely proportional to the temperature:

$$\lambda_{max} = \frac{b}{T}$$

$$b = \frac{hc}{x_m k_B}$$

where x_m satisfies the equation:

$$f(x) = 0$$



Part A

A.3. The function $f(x)$.

$$\frac{du(\lambda,T)}{d\lambda}=0\rightarrow x_m=\frac{hc}{\lambda_{max}K_BT}$$

$$f(x)=5(1-e^{-x_m})-x_m=0$$

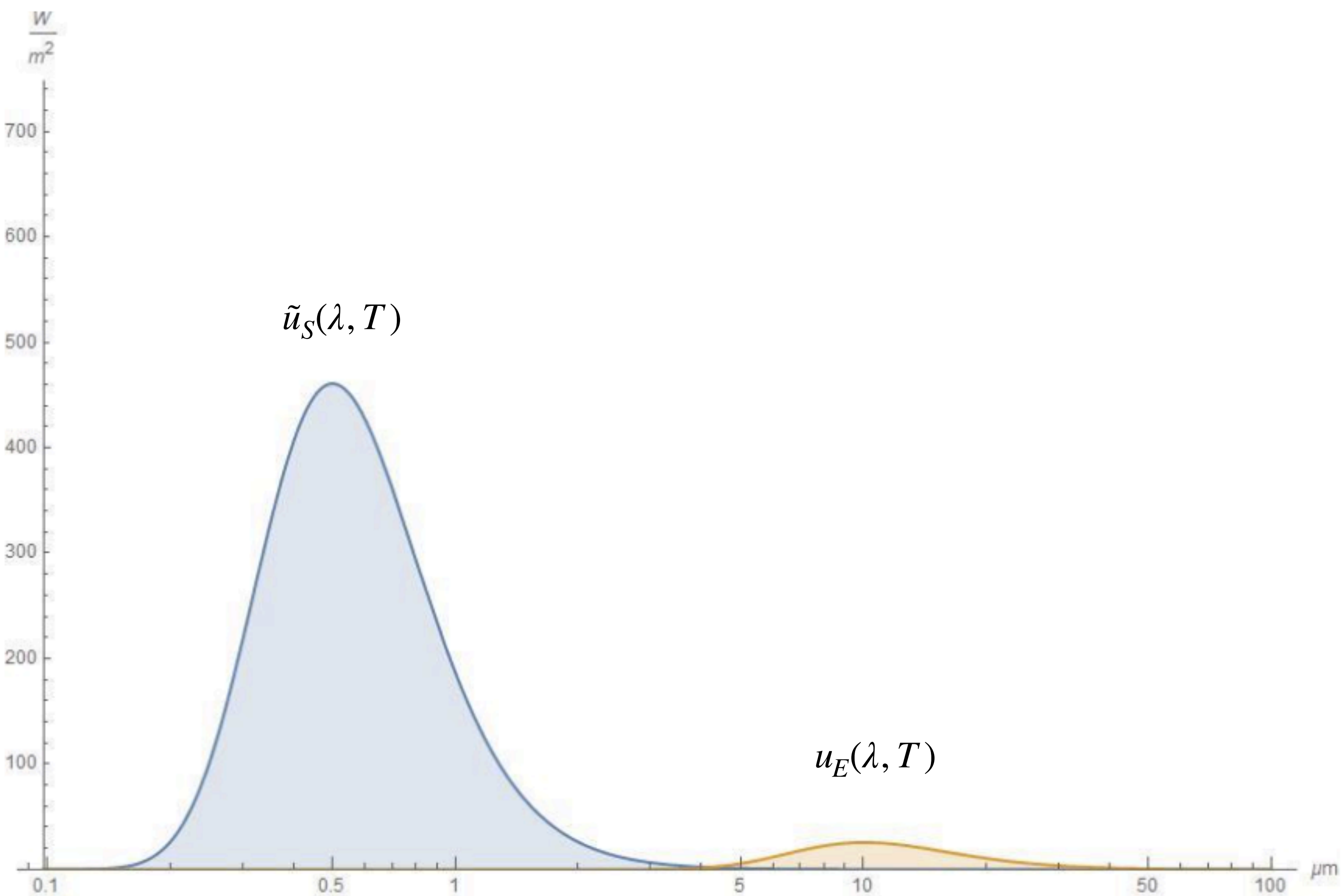
A.4. Numerical values of x_m and b

$$x_m=5(1-e^{-x_m})\rightarrow x_m=4.97$$

$$b=\frac{hc}{x_mK_B}=2.90\times10^3\ nm\,.K$$

A.5. λ_{max} for the Sun and the Earth.

$$\lambda^s_{max}=\frac{b}{T_s}=5.03\times10^2 \qquad \lambda^E_{max}=\frac{b}{T_E}=1.04\times10^4$$



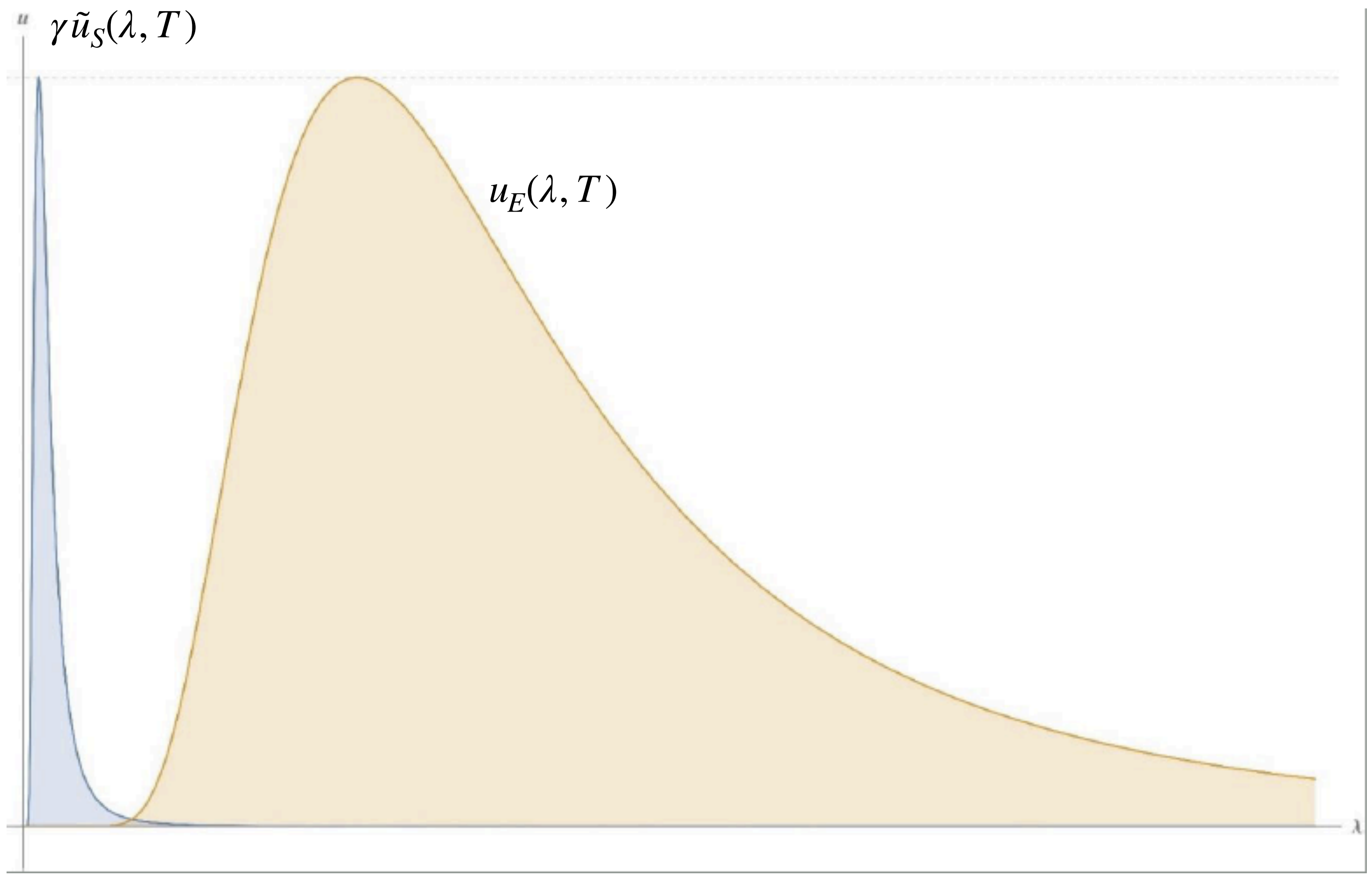
Part A

A.6. Determine γ

$$\tilde{u}_S(\lambda, T) = (\frac{R_s}{D})^2 u_S(\lambda, T)$$

$$\frac{\tilde{u}_S(\lambda^S_{max}, T)}{u_E(\lambda^E_{max}, T)} = (\frac{R_S}{D})^2 (\frac{T_S}{T_E})^5$$

$$\gamma = (\frac{\tilde{u}_S(\lambda^S_{max}, T)}{u_E(\lambda^E_{max}, T)})^{-1} = (\frac{D}{R_S})^2 (\frac{T_E}{T_S})^5 = 1.20 \times 10^{-2}$$



Part B

Assumptions

1. Two-layer model for Earth (Earth's surface and a thin layer of atmosphere a few kilometers above the Earth's surface).

2. The atmosphere layer is not a perfect black body with absorptivity ϵ in infrared wavelengths.

3. The absorption of the Sun's radiation in the atmosphere is neglected.

4. The absorptivity of Earth's surface in infrared wavelengths is equal to 1.

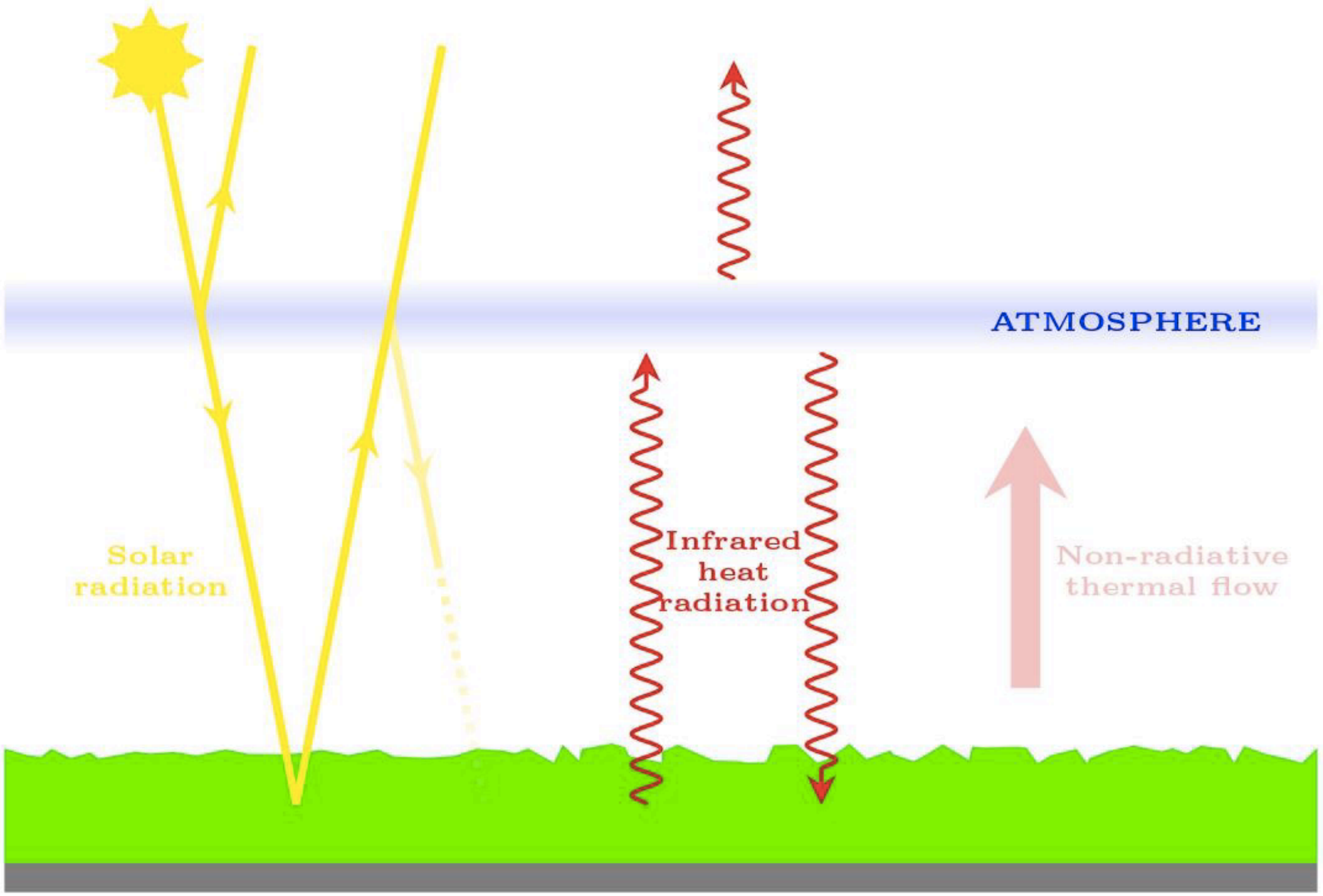
5. The atmosphere layer and Earth's surface reflect a fraction of the incident Sun's radiation spectrum in the ultraviolet to visible blue wavelengths with reflectivity r_A and r_E, respectively.

6. Earth's surface and atmosphere are in thermal equilibrium at different temperatures T_E and T_A, respectively.

7. The non-radiative thermal current density from the Earth's surface to the atmosphere is only convective and is given by $J_{NR} = k(T_E - T_A)$.

Manabe's climate model (simplified)

Two-layer model



Part B

B.1. Assume that $\epsilon = 1$ and $r_E = 0$,
and Calculate T_E and T_A.

Thermal current balance equation for the Earth's surface:

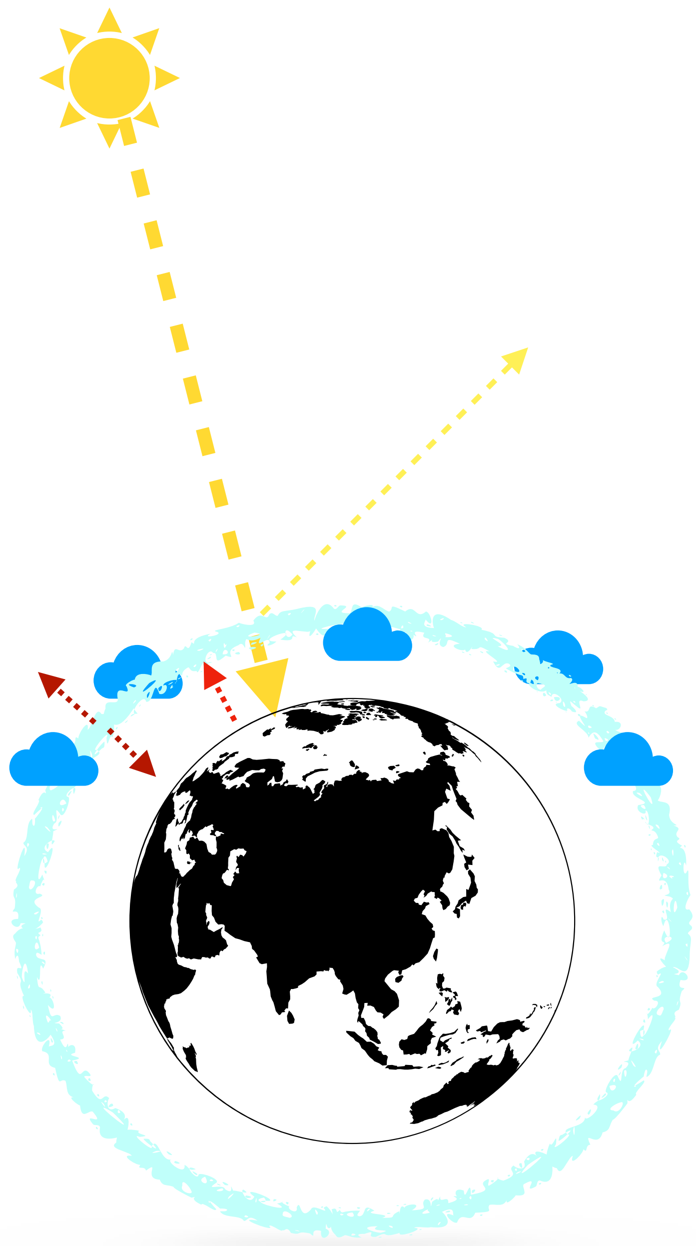
$$(\pi R_E^2)(1 - r_A)S_0 + (4\pi R_E^2)\sigma T_A^4 = (4\pi R_E^2)\sigma T_E^4$$

Thermal current balance equation for the the atmosphere layer:

$$(4\pi R_E^2)\sigma T_E^4 = 2(4\pi R_E^2)\sigma T_A^4$$

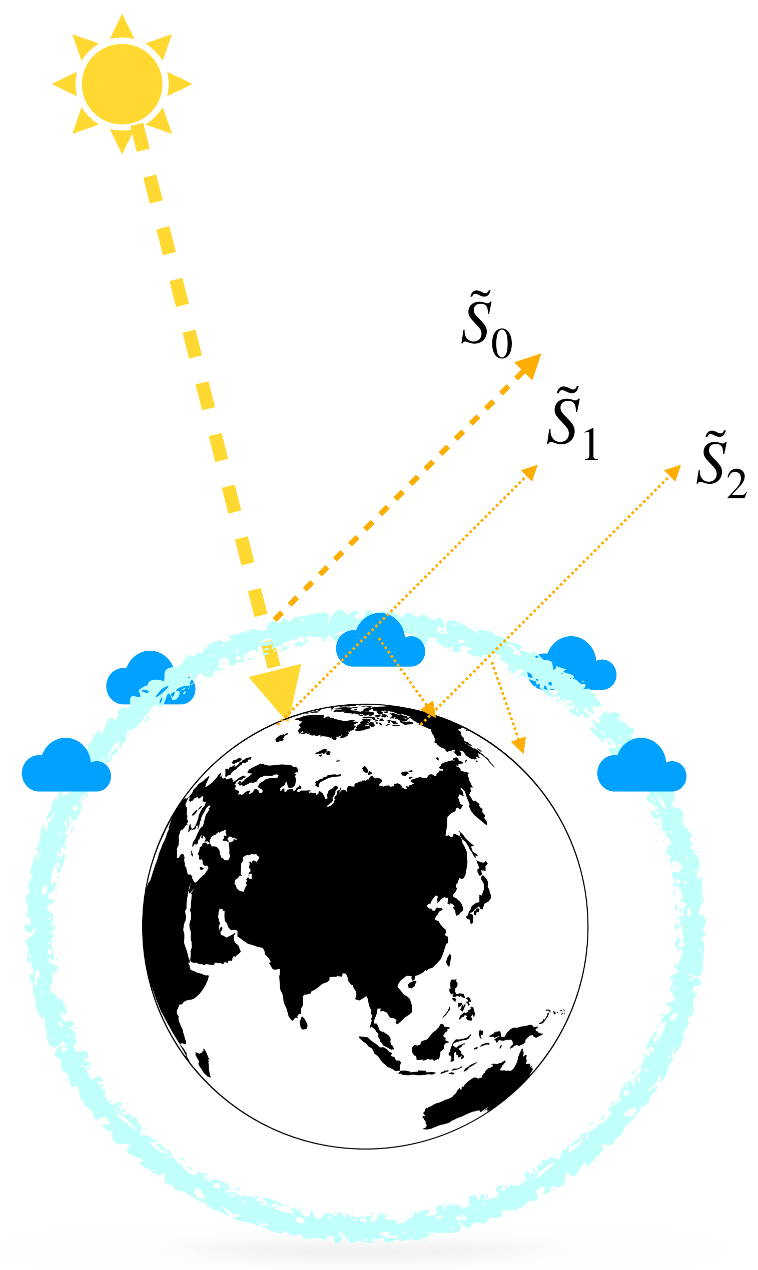
$$T_A = \left(\frac{(1 - r_A)\frac{S_0}{4}}{\sigma}\right)^{\frac{1}{4}} = 2.58 \times 10^2 K$$

$$T_E = (2T_A^4)^{\frac{1}{4}} = 3.07 \times 10^2 K$$



Part B

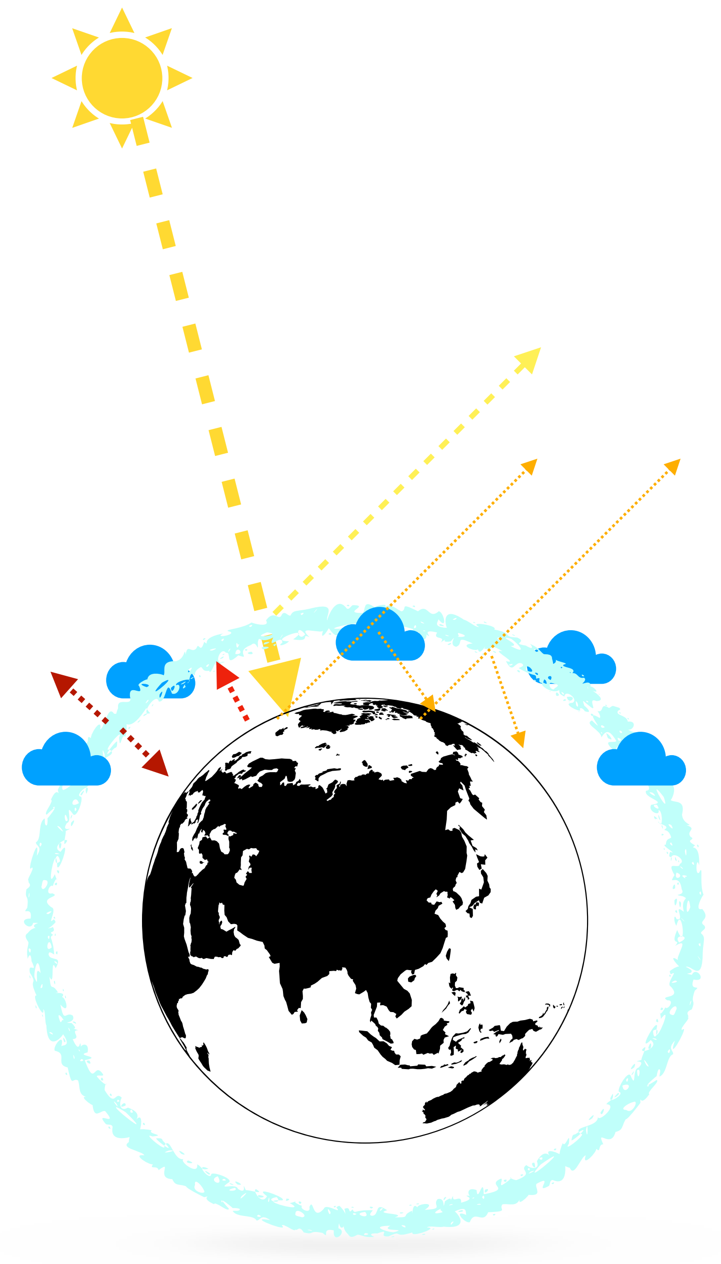
B.2. Determine the Albedo, α, in terms of r_E and r_A. Then calculate its numerical value assuming $r_E = 0.1$ and $r_A = 0.225$.



$$\tilde{S} = \tilde{S}_0 + \tilde{S}_1 + \tilde{S}_2 + \cdots$$

$$\alpha = \frac{\tilde{S}}{S_0} = r_A + \frac{(1-r_A)^2 r_E}{1 - r_A r_E} = 3.12 \times 10^{-1}$$

Part B



B.3. Express the Earth's temperature in terms of σ, α, S_0, and ϵ, and then using the given data and the calculated albedo, find the value of ϵ which leads to the current temperature of $T_E = 288\ K$ for the Earth.

Thermal current balance equation for the Earth's surface:

$$(4\pi R_E^2)\epsilon\sigma T_A^4 + (\pi R_E^2)(1-\alpha)S_0 = (4\pi R_E^2)\sigma T_E^4$$

Thermal current balance equation for the the atmosphere layer:

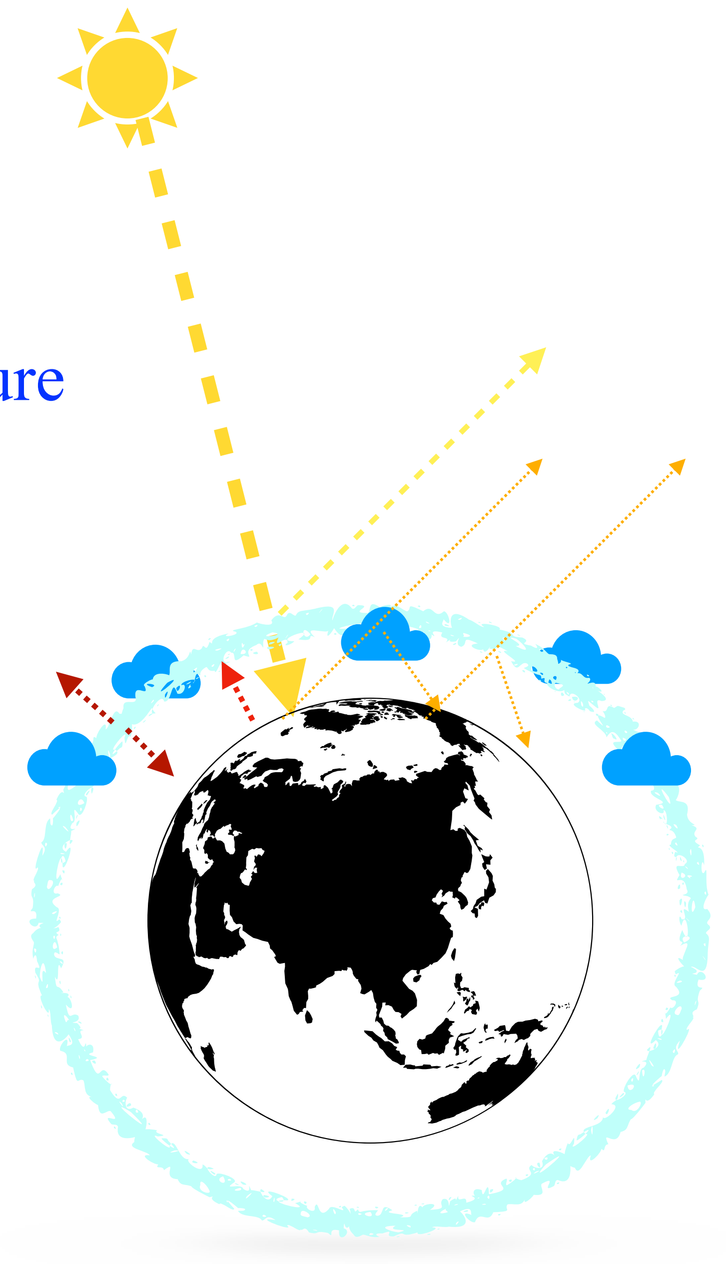
$$(4\pi R_E^2)\epsilon\sigma T_E^4 = 2(4\pi R_E^2)\epsilon\sigma T_A^4$$

$$T_E = \left[\frac{(1-\alpha)}{2\sigma(2-\epsilon)}S_0\right]^{\frac{1}{4}}$$

$$\epsilon = 0.808$$

Part B

B.4. Find $\frac{dT}{d\epsilon}$ and determine by how much the Earth's temperature increases if ϵ increases by one percent.

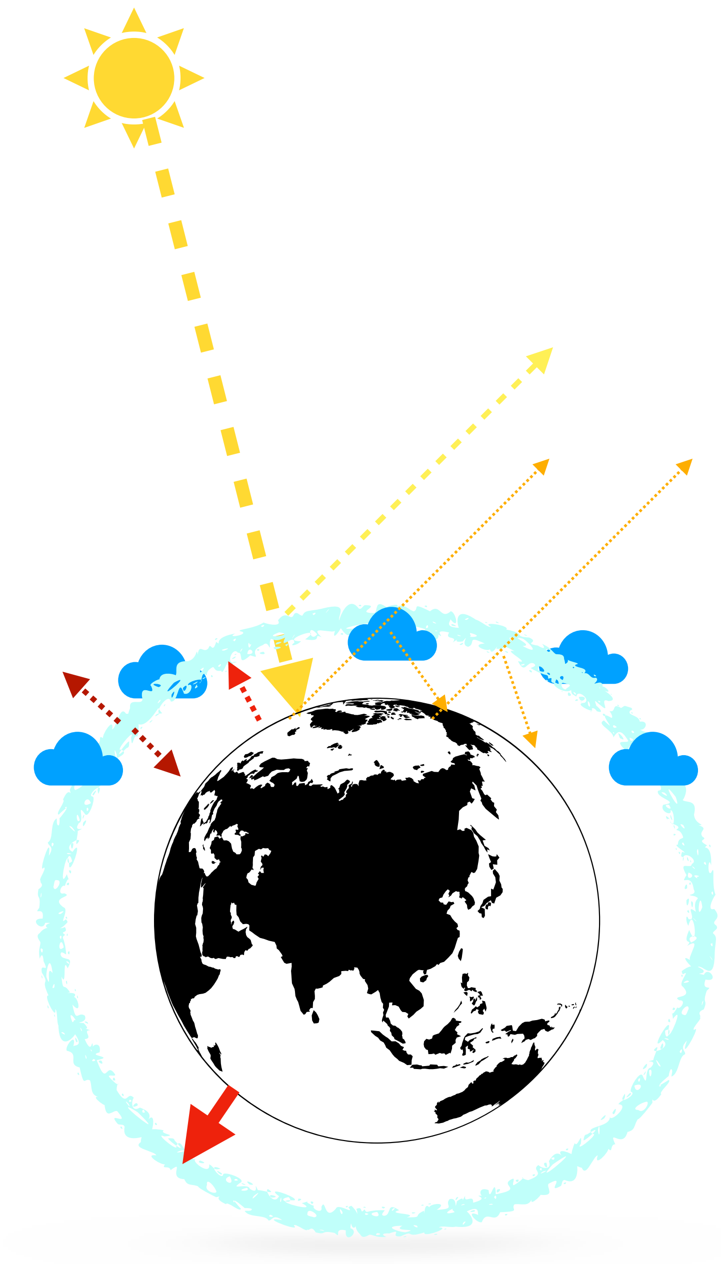


$$\frac{dT_E}{d\epsilon} = \frac{1}{4}\left[\frac{(1-\alpha)S_0}{2\sigma(2-\epsilon)}\right]^{\frac{1}{4}} \frac{1}{(2-\epsilon)}$$

$$dT_E = \left(\frac{dT}{d\epsilon}\right)d\epsilon = 0.488\ K$$

Part B

Assume $T_A = 245\ K$ and $T_E = 288\ K$. Now suppose that a non-radiative thermal flow $J_{NR} = k(T_E - T_A)$ is maintained from the Earth to the atmosphere, where k is a constant.



B.5. Calculate ϵ and k.

Thermal current balance equation for the Earth's surface:

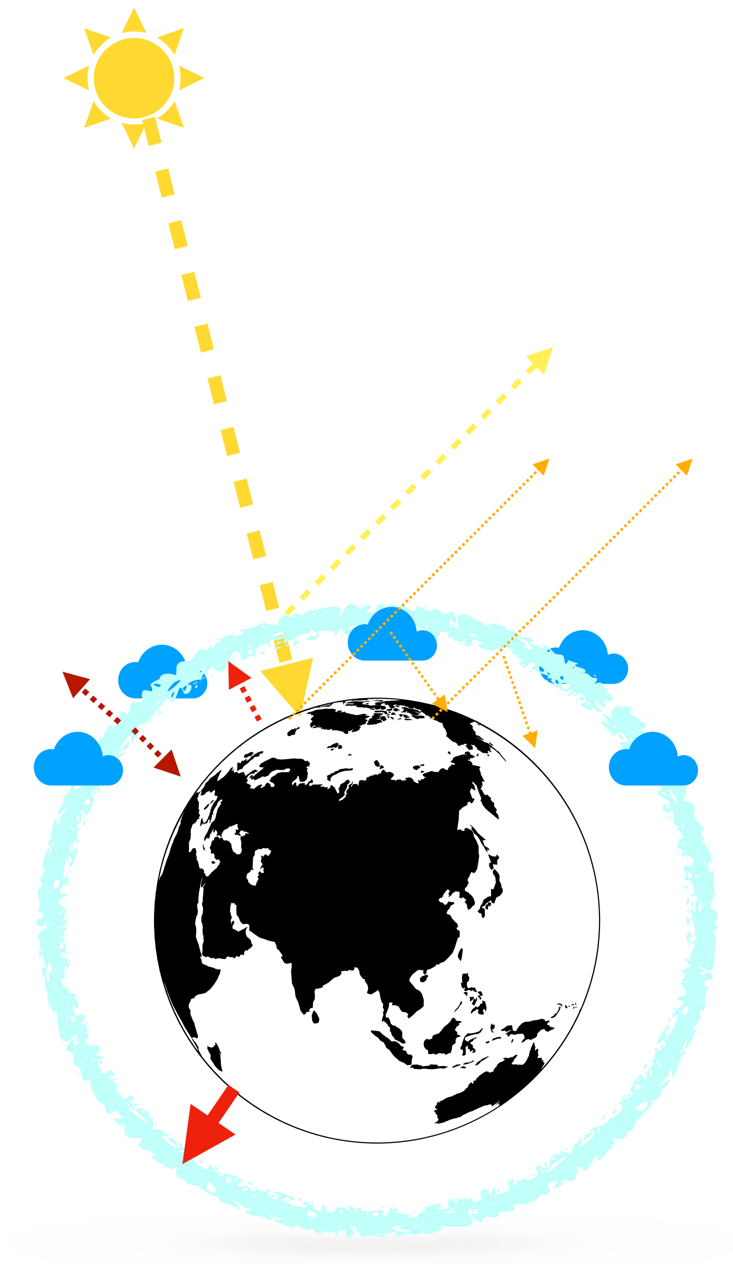
$$(4\pi R_E^2)(1-\alpha)S_0 + (4\pi R_E^2)\epsilon\sigma T_A^4 = (4\pi R_E^2)\sigma T_E^4 + (4\pi R_E^2)k(T_E - T_A)$$

Thermal current balance equation for the the atmosphere layer:

$$(4\pi R_E^2)\epsilon\sigma T_E^4 + (4\pi R_E^2)k(T_E - T_A) = 2(4\pi R_E^2)\epsilon\sigma T_A^4$$

$$\epsilon = \frac{\sigma T_E^4 - (1-\alpha)\frac{S_0}{4}}{\sigma(T_E^4 - T_A^4)} = 0.848 \qquad k = \frac{\epsilon\sigma(2T_A^4 - T_E^4)}{T_E - T_A} = 0.365\ w/m^2K$$

Part B



B.6. First find out what relations are satisfied by $\frac{dT_A}{d\epsilon}$ and , $\frac{dT_E}{d\epsilon}$ and then using them, find out the change in the Earth's temperature as a result of a one percent increase in the value of ϵ.

$$\epsilon\left[\frac{1}{T_E - T_A} + \frac{4T_E^3}{2T_A^4 - T_E^4}\right]\frac{dT_E}{d\epsilon} = 1 + \epsilon\left[\frac{8T_A^3}{2T_A^4 - T_E^4} + \frac{1}{T_E - T_A}\right]\frac{dT_A}{d\epsilon}$$

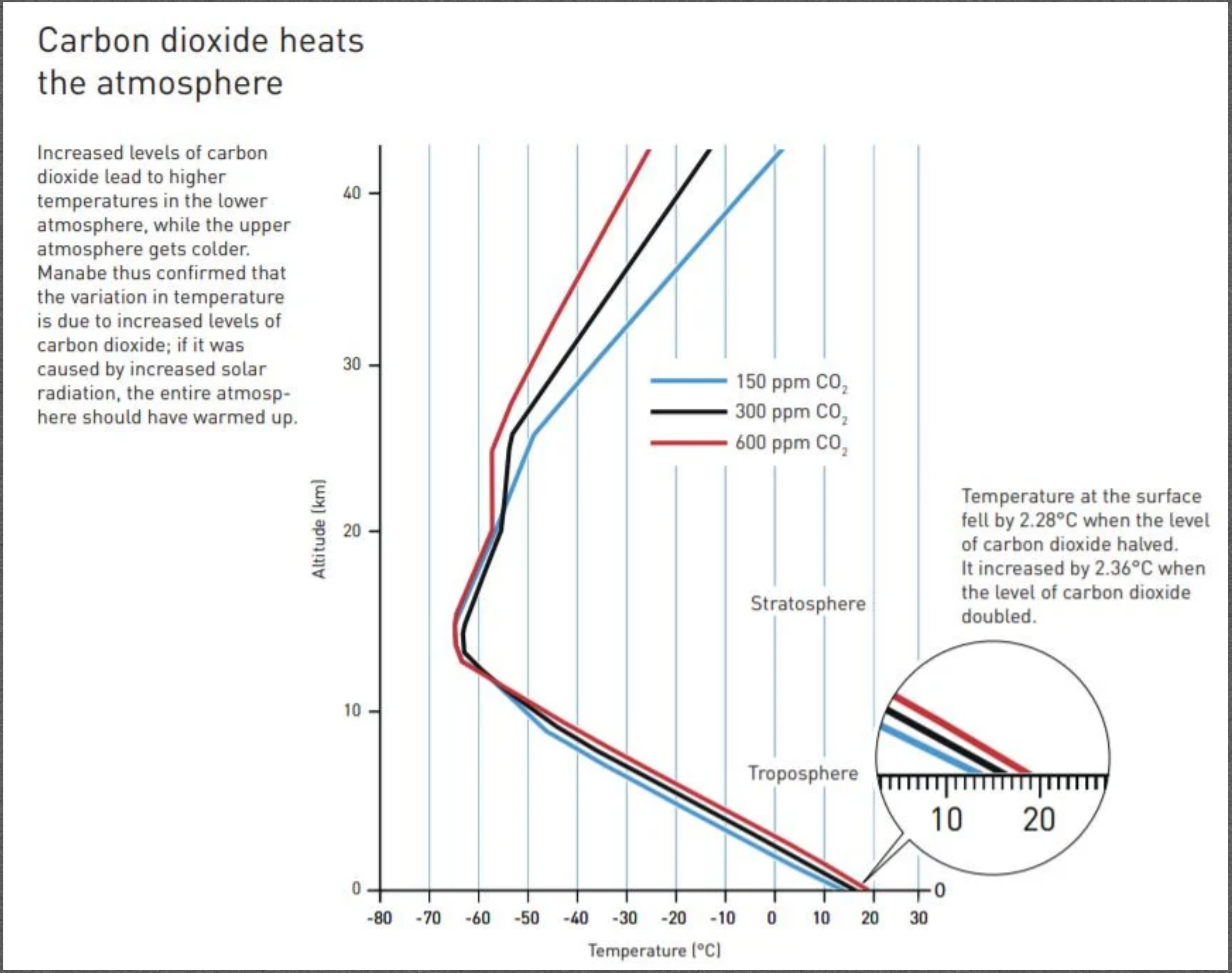
$$1 + \epsilon\left[\frac{4T_E^3}{T_E^4 - T_A^4} - \frac{4\sigma T_E^3}{\sigma T_E^4 - (1-\alpha)\frac{S_0}{4}}\right]\frac{dT_E}{d\epsilon} = \frac{4T_A^3}{T_E^4 - T_A^4}\epsilon\frac{dT_A}{d\epsilon}$$

$$dT_E = 0.523\ K$$

Thanks for your attention

Manabe's climate model

Temperature profile of atmosphere

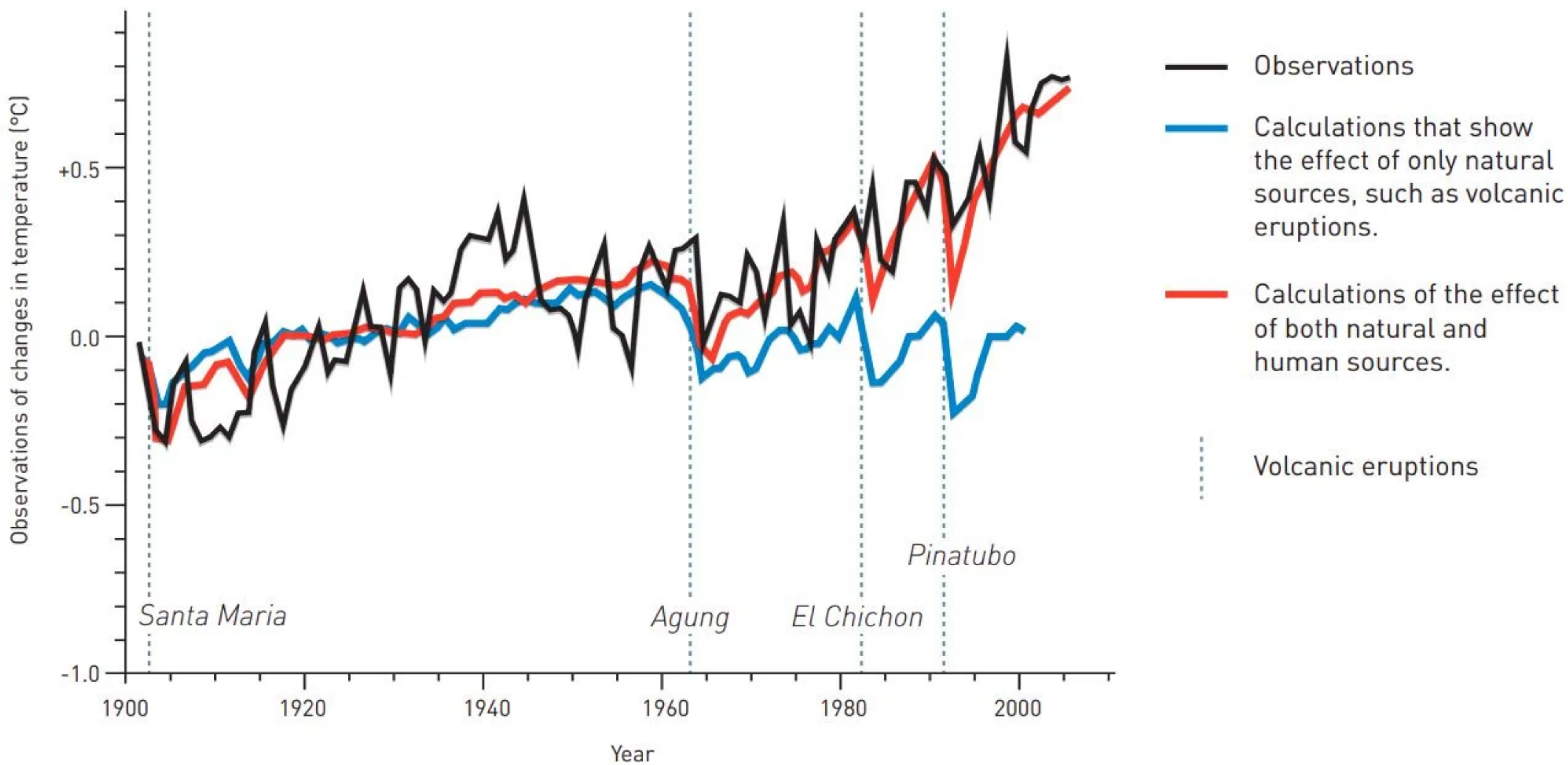


Manabe and Wetherald (1967) Thermal equilibrium of the atmosphere with a given distribution of relative humidity, Journal of the atmospheric sciences, Vol. 24, Nr 3, May.

Impact of human on the global warming

Identifying fingerprints in the climate

Klaus Hasselmann developed methods for distinguishing between natural and human causes (fingerprints) of atmospheric heating. Comparison between changes in the mean temperature in relation to the average for 1901–1950 (°C).



Hegerl and Zweirs (2011) Use of models in detection & attribution of climate change, WIREs Climate Change