Marking Scheme Q1 (10 points)

Part A (3.0 pt) If the final answer is written then the complete point will be achieved

A-1	$S_0 = \sigma T_S^4 . \left(\frac{R_S}{d}\right)^2$ (0.4pt), [Realizing energy conservation (0.1pt)] Numerical value of $S_0 = 1.35 \times 10^3$ W/m^2 (0.2pt) [more than 4 significant figures (0.1pt)]	0.6 pt
A-2	$T_E = \left(\frac{S_0}{4\sigma}\right)^{\frac{1}{4}} = \sqrt{\frac{R_S}{2d}} T_S$ (0.4pt), [realizing energy balance (0.1pt)] Numerical value of $T_E = 278$ K (0.2pt) [more than 4 significant figures (0.1pt)]	0.6 pt
A-3	$f(x) = 5(1 - e^{-x}) - x$	0.4 pt
A-4	$x_m = \{4.96, 4.97\}$ (0.3 pt), [more than 4 significant figures (0.2pt)] Numerical value of $b = [2.89, 2.90] \times 10^6$ nm. K (0.1 pt) [more than 4 significant figures (0.1pt)]	0.4 pt
A-5	$\lambda_{max}^{Sun} = [5.01,\ 5.02] \times 10^2$ nm(0.1 pt), $\lambda_{max}^{Earth} = 1.04 \times 10^4$ nm(0.1 pt) [more than 4 significant figures (0.1pt)]	0.2 pt
A-6	$\gamma = \left(\frac{d}{R_S}\right)^2 \times \left(\frac{T_E}{T_S}\right)^5 = \left(\frac{\lambda_S}{\lambda_E}\right)^5 \times \left(\frac{d}{R_S}\right)^2$ (0.6 pt), [realizing $\tilde{u}_S = \left(\frac{R_S}{d}\right)^2 u_S(\lambda)$(0.3pt)] Numerical value of $\gamma = [1.20, 1.21] \times 10^{-2}$ (0.2 pt) [more than 4 significant figures (0.1pt)]	0.8 pt

Part B (7.0 pt)

B-1	$T_A = \left(\frac{(1-r_A)\frac{S_0}{4}}{\sigma}\right)^{\frac{1}{4}}$ $T_E = \left(\frac{(1-r_A)\frac{S_0}{2}}{\sigma}\right)^{\frac{1}{4}}$ Two correct expressions (0.8 pt) [One correct expression (0.6 pt)] [no correct expression: for each energy balance relation (0.2pt)] Numerical value of $T_A = 2.58 \times 10^2$ K (0.1 pt) Numerical value of $T_E = 3.07 \times 10^2$ K (0.1 pt) [more than 4 significant figures (0.1pt)]	1.0 pt
B-2	$\alpha = r_A + \frac{(1-r_A)^2 r_E}{1-r_A r_E}$ (1.4pt) [$\tilde{S}_0 = r_A S_0$ (0.1 pt) $\tilde{S}_1 = (1-r_A)^2 r_E S_0 = \frac{(1-r_A)^2}{r_A} r_E \tilde{S}_0$ (0.3 pt)	1.6 pt

	$\tilde{S}_n = \frac{\tilde{S}_{n-1}}{1-r_A} r_A r_E \times (1 - r_A) = r_A r_E \tilde{S}_{n-1} = (r_A r_E)^{n-1} \tilde{S}_1$ (0.5 pt) $\tilde{S} = \sum_{n=0}^{\infty} \tilde{S}_n = \tilde{S}_0 + \tilde{S}_1 \sum_{n=1}^{\infty} (r_A r_E)^{n-1}$ (0.3 pt)] Numerical value of $\alpha = 3.13 \times 10^{-1}$ (0.2pt) [more than 4 significant figures (0.1pt)]	
B-3	$T_E = \left[\frac{(1-\alpha)}{2\sigma(2-\epsilon)} S_0\right]^{\frac{1}{4}}$ (0.6pt) Numerical value of $\epsilon = [8.07, 8.11] \times 10^{-1}$ (0.4pt) [wrong numerical value: correct expression for ϵ (0.2pt)] [more than 4 significant figures (0.3pt)]	1.0 pt
B-4	$\frac{dT_E}{d\epsilon} = \frac{1}{4}\left[\frac{(1-\alpha)S_0}{2\sigma(2-\epsilon)}\right]^{\frac{1}{4}} \frac{1}{(2-\epsilon)}$ (0.6 pt), Numerical value of $\delta T_E = [4.87, 4.92] \times 10^{-1}$ K (0.2pt) [more than 4 significant figures (0.1pt)]	0.8 pt
B-5	$\epsilon = \frac{\sigma T_E^4 - (1-\alpha)\frac{S_0}{4}}{\sigma(T_E^4 - T_A^4)}$ (0.6pt) $k = \frac{(2T_A^4 - T_E^4) \times \left[\sigma T_E^4 - (1-\alpha)\frac{S_0}{4}\right]}{(T_E^4 - T_A^4) \times (T_E - T_A)}$ (0.6pt) [Correct relations for balance of energy (0.3+0.3 pt)] Numerical value of $\epsilon = [8.47, 8.52] \times 10^{-1}$ (0.2pt) Numerical value of $k = [3.57, 3.66] \times 10^{-1}$ W/m^{2}K (0.2pt) [more than 4 significant figures for each one (0.1pt)]	1.6 pt
B-6	(a) (0.4+0.4) $\begin{cases} \epsilon\left[\frac{1}{T_E - T_A} + \frac{4T_E^3}{2T_A^4 - T_E^4}\right]\frac{dT_E}{d\epsilon} = 1 + \epsilon\left[\frac{8T_A^3}{2T_A^4 - T_E^4} + \frac{1}{T_E - T_A}\right]\frac{dT_A}{d\epsilon} \\ 1 + \epsilon\left[\frac{4T_E^3}{T_E^4 - T_A^4} - \frac{4\sigma T_E^3}{\sigma T_E^4 - (1-\alpha)\frac{S_0}{4}}\right]\frac{dT_E}{d\epsilon} = \frac{4T_A^3}{T_E^4 - T_A^4}\epsilon\frac{dT_A}{d\epsilon} \end{cases}$ (0.6 pt) (b) $\delta T_E = [5.21, 5.28] \times 10^{-1}$K (0.2pt) [more than 4 significant figures for each one (0.1pt)]	1.0 pt