2. Diffraction from Phase Steps

Experimental Examination

Abolfazl Ebrahimi

Dariush Ghaemi

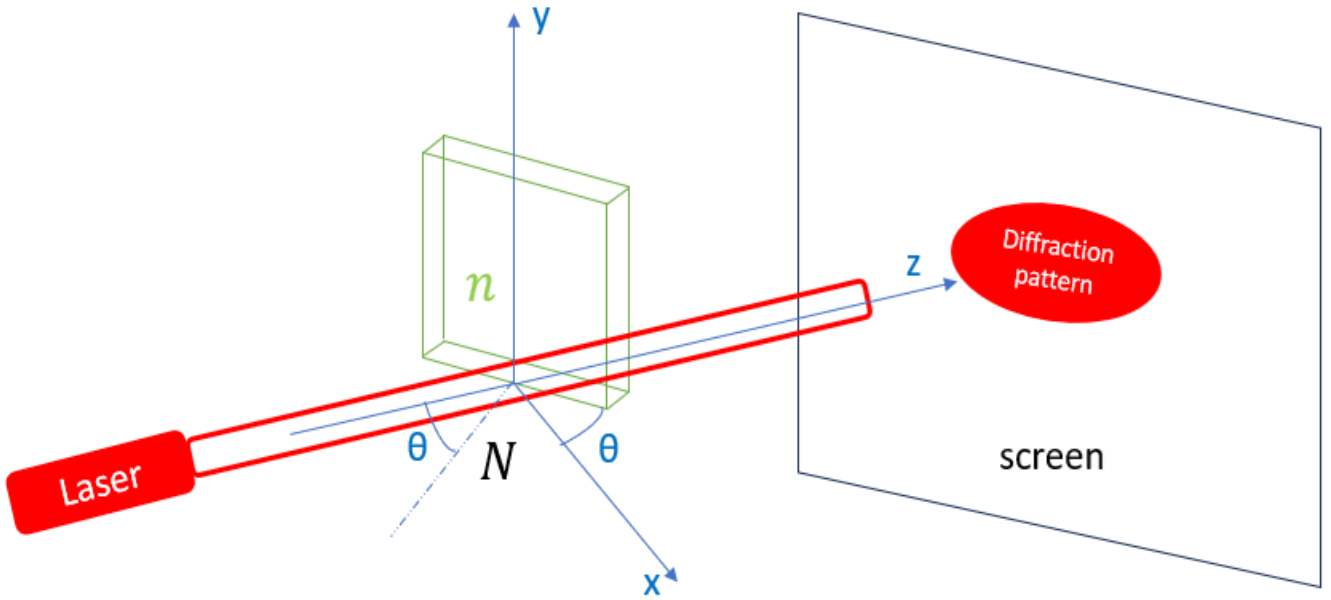
Introduction: Phase Step

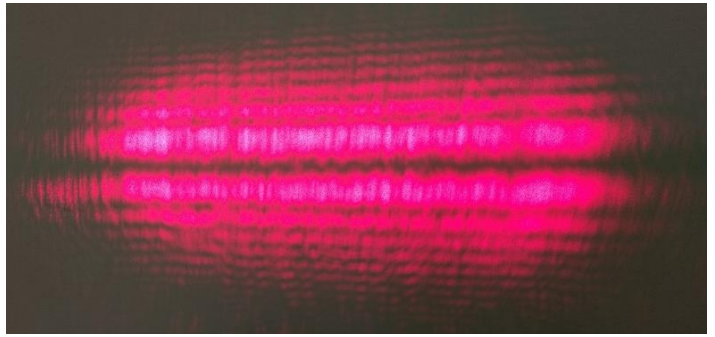
Laser beam shine the edge of a transparent slide

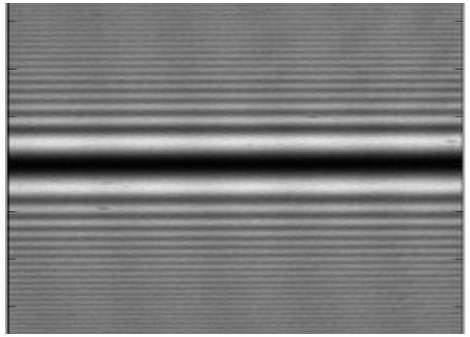
different refractive index

Phase difference (travel through the slide and air)

Diffraction pattern







Idea from Iranian physicist

April 15, 2012 / Vol. 37, No. 8 / OPTICS LETTERS 1

High precision refractometry based on Fresnel diffraction from phase plates

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Received December 12, 2011; revised February 5, 2012; accepted February 20, 2012; posted February 22, 2012 (Doc. ID 159839); published 0 MONTH 0000

When a transparent plane-parallel plate is illuminated at a boundary region by a monochromatic parallel beam of light, Fresnel diffraction occurs because of the abrupt change in phase imposed by the finite change in refractive index at the plate boundary. The visibility of the diffraction fringes varies periodically with changes in incident angle. The visibility period depends on the plate thickness and the refractive indices of the plate and the surrounding medium. Plotting the phase change versus incident angle or counting the visibility repetition in an incident-angle interval provides, for a given plate thickness, the refractive index of the plate very accurately. It is shown here that the refractive index of a plate can be determined without knowing the plate thickness. Therefore, the technique can be utilized for measuring plate thickness with high precision. In addition, by installing a plate with known refractive index in a rectangular cell filled with a liquid and following the described procedures, the refractive index of the liquid is obtained. The technique is applied to measure the refractive indices of a glass slide, distilled water, and ethanol. The potential and merits of the technique are also discussed.

OCIS codes: 120.3940, 260.0260, 050.1940.

Prof. Mohammad Taghi Tavassoly

The idea of using diffraction from phase steps for measuring physical quantities was first developed by an Iranian scientists.

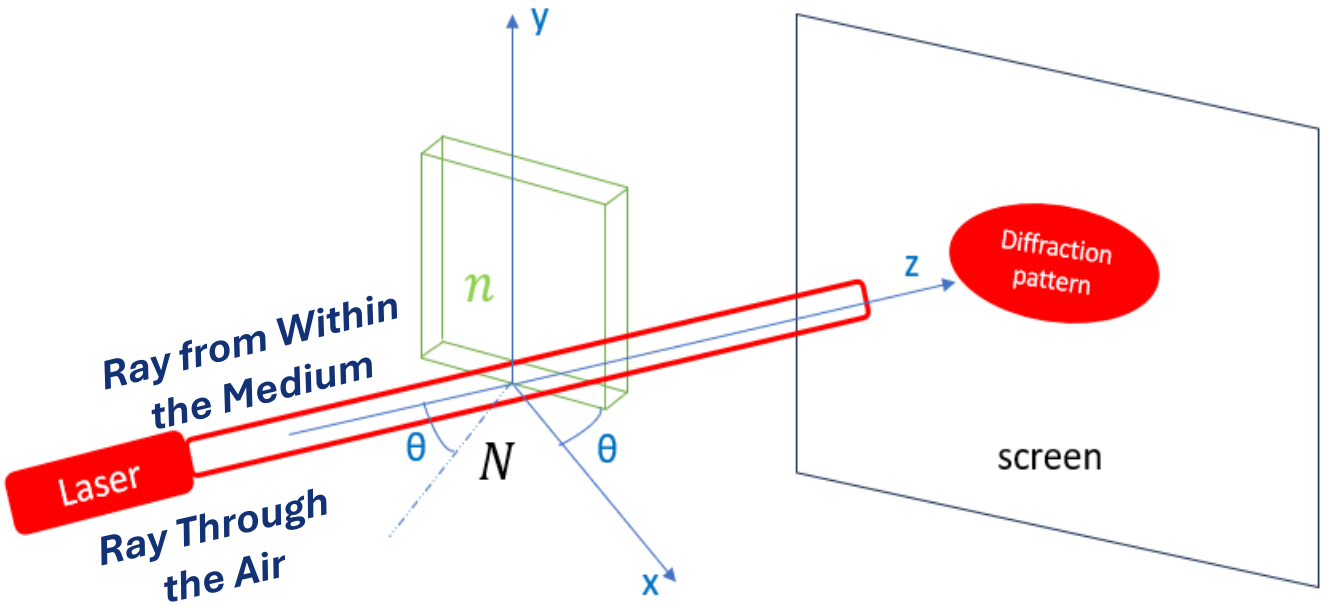
Applications of the Method

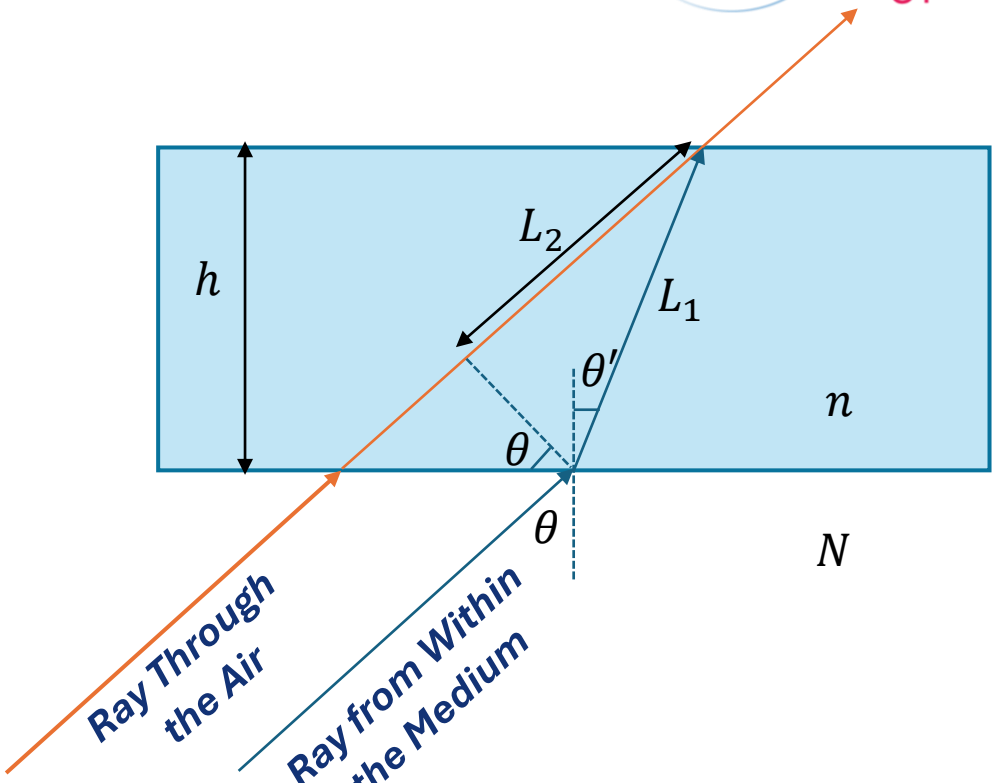
- High precision measurement of:
  - Thickness of layers
  - Refractive index of transparent plates
  - Refractive index of liquids
  - Wavelength of light
  - Coherence length of lasers
  - Etc.

Articles based on diffraction from phase step

- Tavassoly, M. T., et al. "***Fresnel diffraction from a step in reflection and transmission***." (2001): 237-246.
- Amiri, M., and M. T. Tavassoly. "***Fresnel diffraction from 1D and 2D phase steps in reflection and transmission modes***." Optics communications 272.2 (2007): 349-361.
- Tavassoly, M. Taghi, et al. "***High precision refractometry based on Fresnel diffraction from phase plates***." Optics letters 37.9 (2012): 1493-1495.
- Tavassoly, Mohammad Taghi, Iman Moaddel Haghighi, and Khosrow Hassani. "***Application of Fresnel diffraction from a phase step to the measurement of film thickness***." Applied optics 48.29 (2009): 5497-5501.
- Tavassoly, M. Taghi, et al. "***Applications of Fresnel diffraction from the edge of a transparent plate in transmission***." Applied optics 51.30 (2012): 7170-7175.
- Siavashani, Morteza Jafari, et al. "**Optical diffractometry by rough phase steps**." Scientific Reports 13.1 (2023): 13155.
- Etc.

Theory: Phase Difference





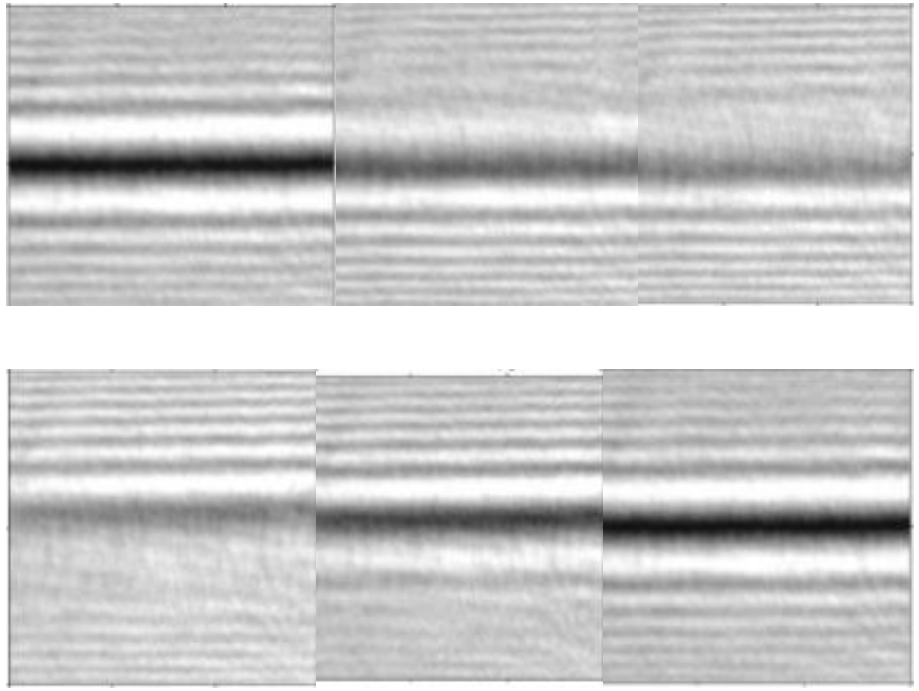
$$\Delta\phi = \frac{2\pi}{\lambda}(nL_1 - NL_2) \quad \Rightarrow \quad \Delta\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta} - Ncos\theta\right)$$

Theory: Fringe Shift

$$\Delta\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta} - Ncos\theta\right)$$

- Continuously Change the Angle
  - ➔ Phase Difference Increases
    Pattern Shape Changes

- Phase Difference Reaches ***2π***
  - ➔ Pattern reverts to its initial shape
  - ➔ ***One Fringe shift Cycle***



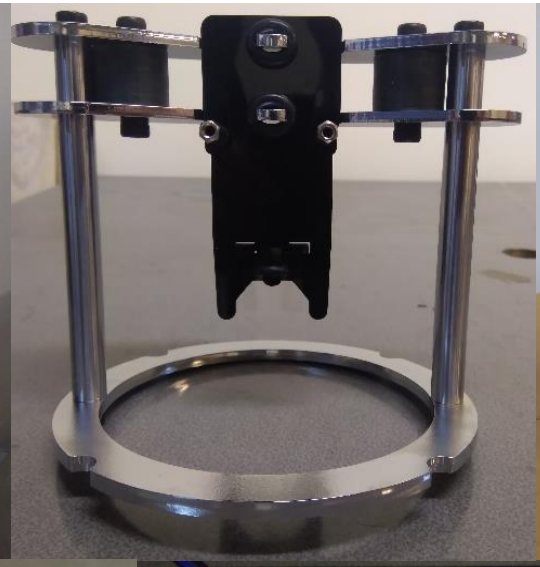
Theory: Fringe Number Equation

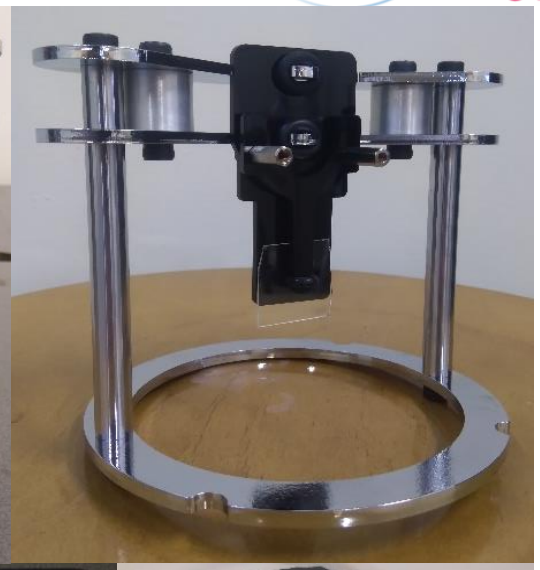
We can start from $\theta = 0$ and gradually increase the angle, to observe the shift of the fringes. If after turning the slide by $\theta = \theta_m$, the fringes shift m times, we will have:

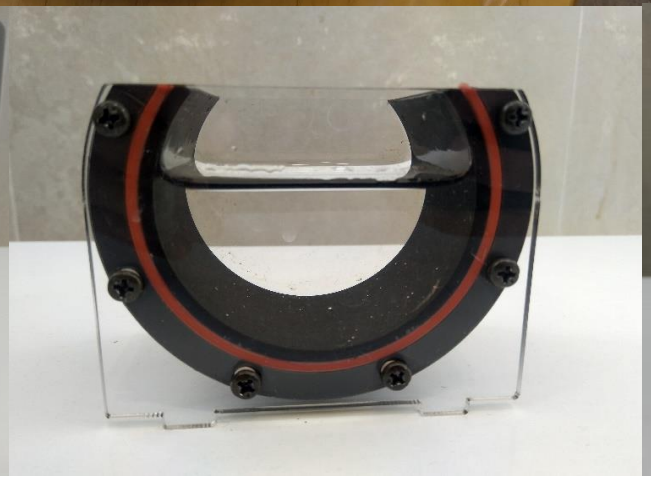
$$\Delta\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - N cos\theta_m\right) = 2\pi m + \phi_0$$

By recording the data corresponding to the number of fringe shifts m, versus the angle θ_m, and using the above equations, one can investigate the relation between the various physical quantities involved in the problem.

Experimental Setup

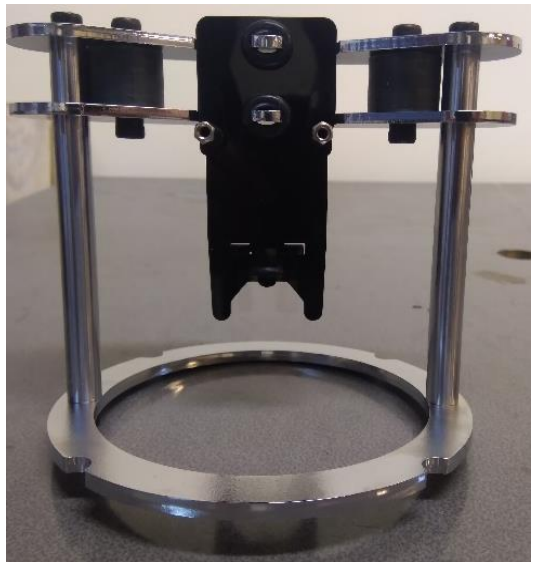






Experimental Setup

The main platform which consists of: Laser, Protractor, Magnifying lens

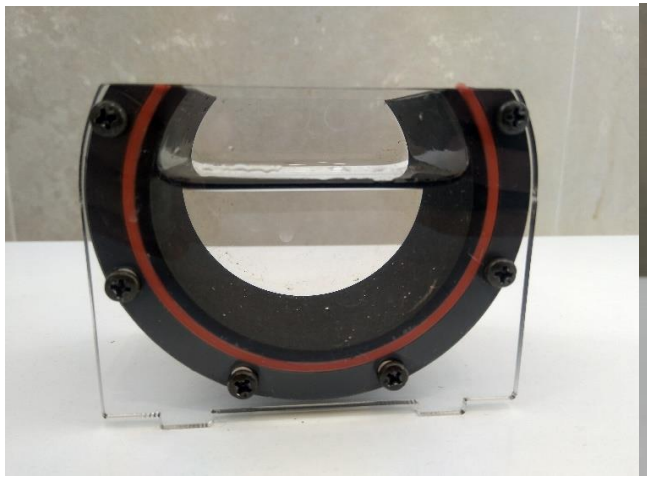


The holder S1 (the thin microscope slide)

The holder S2 (the thick microscope slide)

Experimental Setup

The observation screen

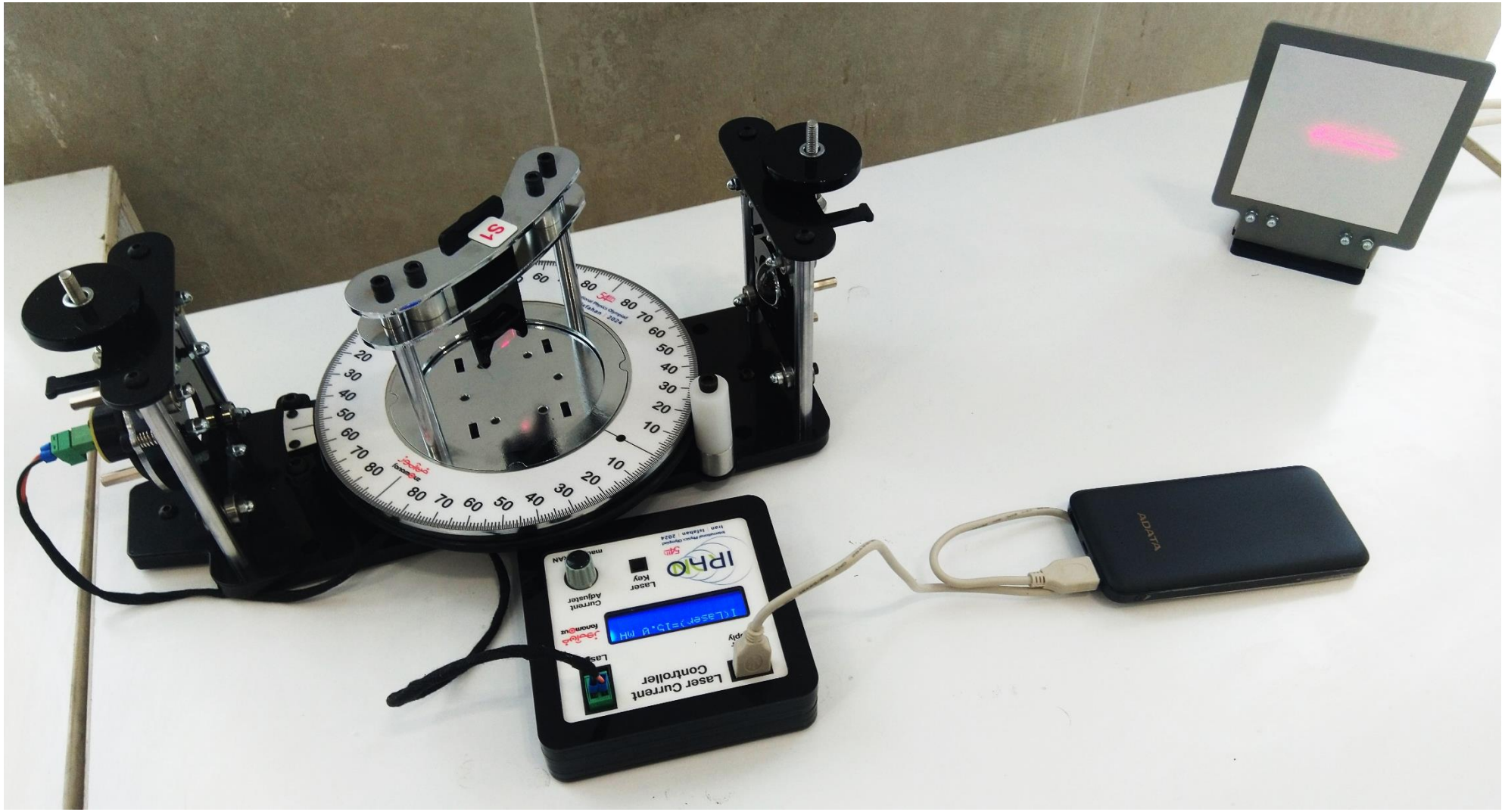


The unknown liquid and container

The laser electronic board

Power bank and electrical cables.

Experimental Setup in Operation



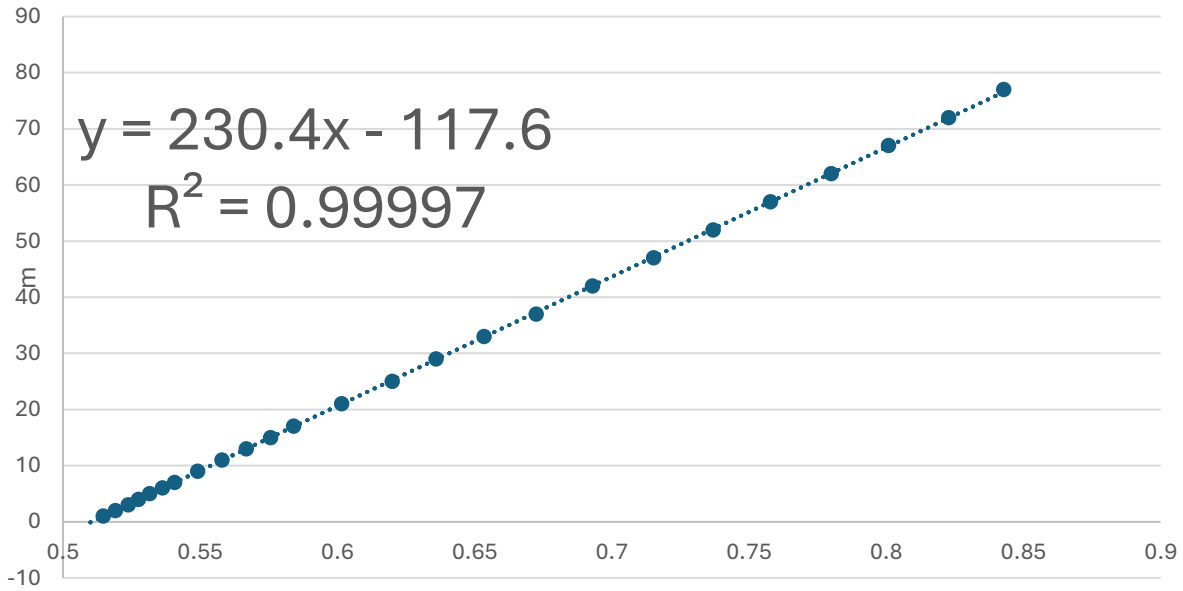
Part A: Thickness of the Thin Slide (2.0 Points)

By recording the number of fringe shifts m, and the angle, θ_m, and drawing appropriate diagrams, students must find the thickness of the thin slide.

A.1	Starting with zero degrees, rotate the protractor and go up to 70 degrees. Watch the number of fringe shifts and write down the angle θ_m corresponding to each fringe shift number m. Take at least 25 data points and fill out the table.	0.8 pt
A.2	Draw the appropriate curve.	0.3 pt
A.3	Find the slope (B) and the vertical axis intercept (A).	0.1 pt
A.4	Using the slope, find the thickness of the thin slide	0.8 pt

Part A Solution

By recording the number of fringe shifts m, and the angle, θ_m, and drawing appropriate diagrams, students must find the thickness of the thin slide.



$$\sqrt{n^2 - sin^2\theta_m} - cos\theta_m$$

$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - sin^2\theta_m} - cos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

m → Y; $\sqrt{n^2 - sin^2\theta_m} - cos\theta_m$ → X

Thickness of the thin slide

Final answer:	h = $(149.8 \pm 0.2)\ \mu m$
First interval:	Less than 2% error
second interval:	Less than 3% error
Third interval:	Less than 4% error

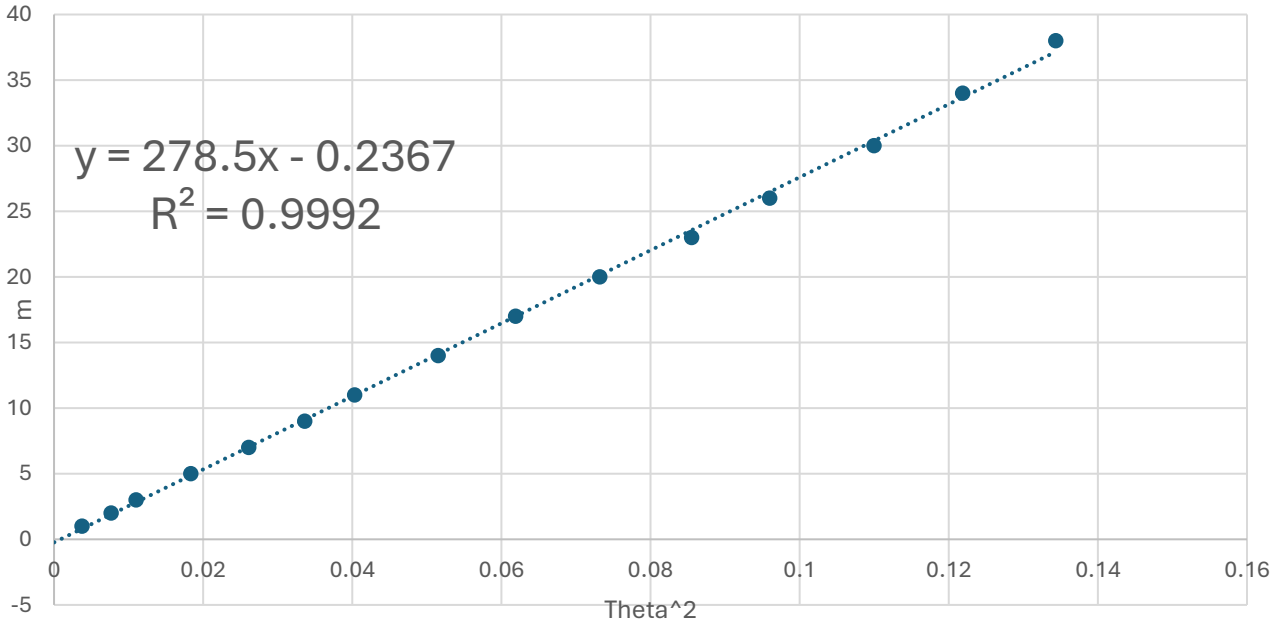
Part B: Thickness of the Thick Slide (1.6 Points)

By recording the number of fringe shifts m, and the angle, θ_m, and drawing appropriate diagrams, students must find the thickness of the thick slide.

B.1	Repeat the task **A.1** for θ between 0 and 20 degrees and record at least 15 data points.	0.6 pt
B.2	Assuming θ_m in Equation 4 is small enough, expand the relation to the order θ_m^2 and find a linear relation between the fringe shift number and θ_m^2 (assume $N = 1.00$).	0.1 pt
B.3	Draw an appropriate curve.	0.2 pt
B.4	Find the slope and the vertical axis intercept.	0.1 pt
B.5	Using the slope, find the thickness of the thick slide.	0.6 pt

Part B Solution

By recording the number of fringe shifts m, and the angle, θ_m, and drawing appropriate diagrams, students must find the thickness of the thick slide.



$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - sin^2\theta_m} - cos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

Taylor series → $m = \frac{h}{2\lambda}\left(1 - \frac{1}{n}\right)\theta_m^2$

Thickness of the thick slide

Final answer:	H = $(1.072 \pm 0.004)mm$
First interval:	Less than 3% error
second interval:	Less than 5% error

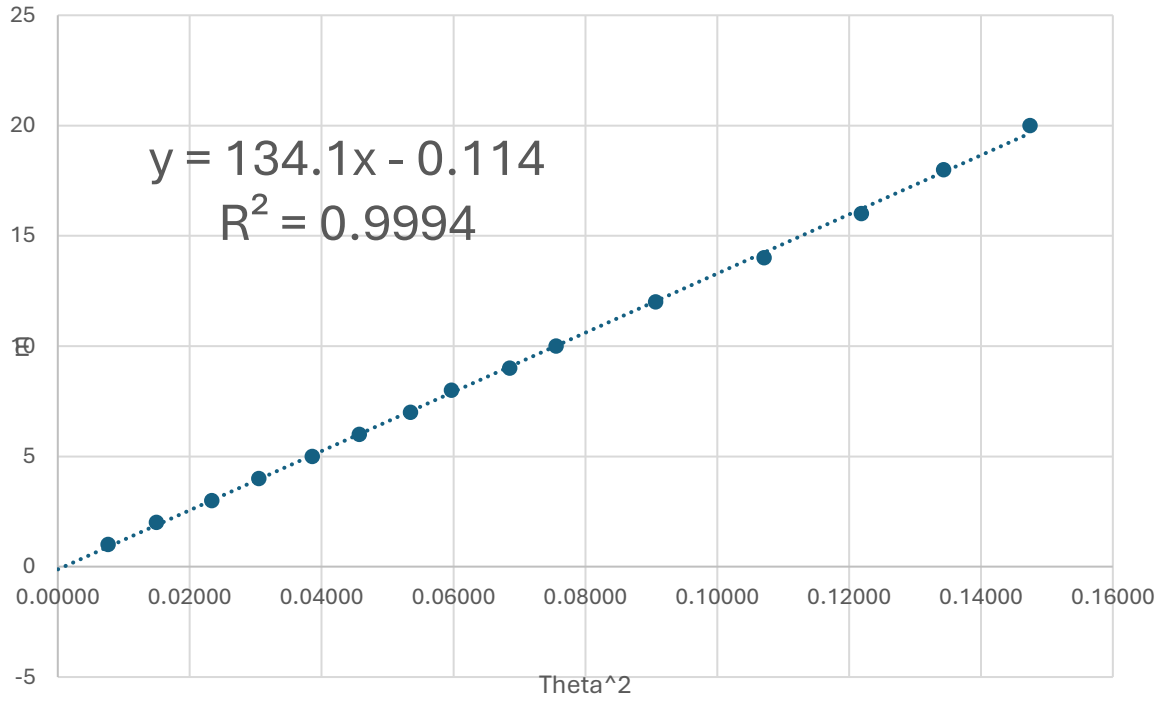
Part C: Finding refractive index using thick slide (1.6 points)

Pour the unknown liquid into the container. Place the container at the center of the protractor and gently place the S2 Holder back onto the protractor in such a way that the microscope slide is immersed inside the liquid. Again, a diffraction pattern will be observed on the screen.

C.1	Repeat the task **B.1** (15 data points up to 20 degrees).	0.6 pt
C.2	Repeat the task **B.2** for arbitrary N.	0.1 pt
C.3	Draw an appropriate curve.	0.2 pt
C.4	Find the slope and the vertical intercept.	0.1 pt
C.5	Find the refractive index of the unknown liquid (N).	0.6 pt

Part C Solution

By recording the number of fringe shifts m, and the angle, θ_m, and drawing appropriate diagrams, students must find the thickness of the thick slide.



$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - N cos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

Taylor series → $$m = \frac{h}{2\lambda} N \left(1 - \frac{N}{n}\right) \theta_m^2$$

Refractive index of unknown liquid

Final answer:	N =1.325 ± 0.002
First interval:	1.31 < N < 1.34
second interval:	1.30 < N < 1.35

Part D: Finding refractive index using thin slide (4.8 points)

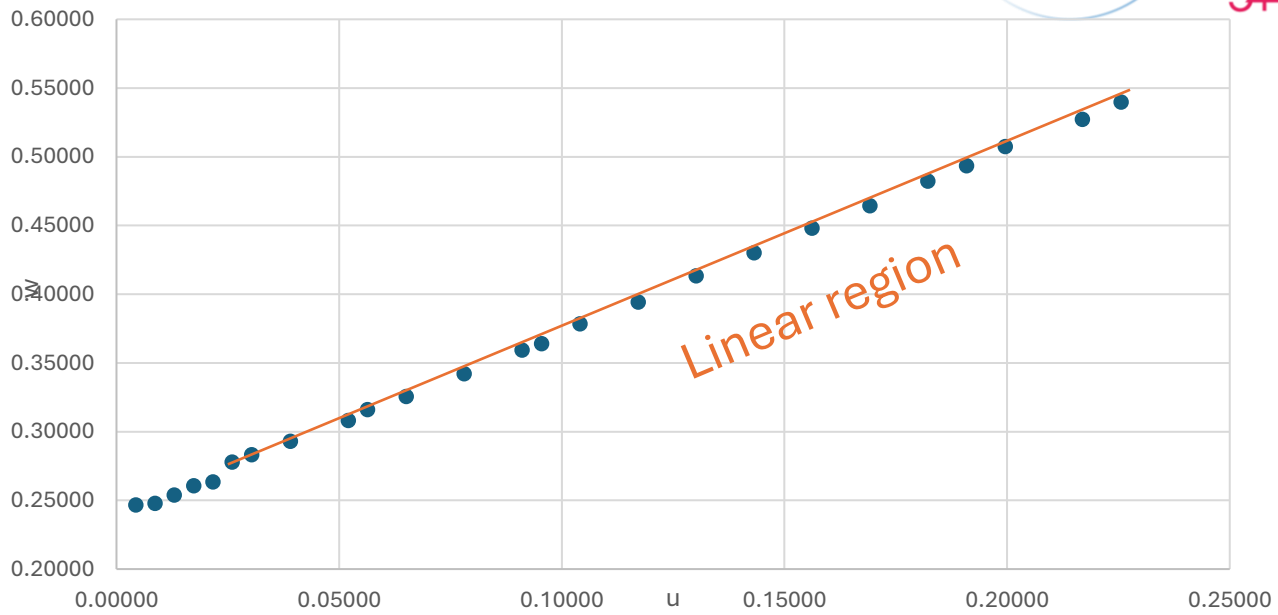
D.1	Repeat the task **A.1** for this case (25 data points up to 70 degrees).	0.7 pt
D.2	Do a simple calculation and eliminate ϕ_0 from Equations 1 and 4 to obtain a relation like $N(n-N)+uN=w$. Find u and w in terms of m, n, h, λ and θ.	0.8 pt
D.3	Use your calculator to determine u and w.	1.2 pt
D.4	Draw w versus u.	0.3 pt
D.5	In the previous graph, find the linear region and calculate the slope and the vertical axis intercept of the curve.	0.2 pt
D.6	Find the refractive index, first using the slope (N_B), and then using the vertical intercept (N_A).	1.6 pt

Part D: Refractive index of the unknown liquid

$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - N cos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

$$\frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m} = N(n - N) + \left(\frac{m\lambda}{h}\right)N$$

$$w = N(n - N) + uN$$



Using intercept

Final answer: $N_B = 1.325 \pm 0.005$

First interval: $N_B = (1.31 - 1.34)$

second interval: $N_B = (1.29 - 1.36)$

Using slope

Final answer: $N_A = 1.3292 \pm 0.0006$

First interval: $N_A = (1.31 - 1.34)$

second interval: $N_A = (1.29 - 1.36)$

Syllabus

Syllabus – Theoretical Skills

Optics Physics

- 2.7.1. Wave optics
  - Concepts of wave number, phase of light and diffraction.

Syllabus – Experimental Skills (1/2)

- 3.3 Measurement techniques and apparatus
  - Height adjustment of mechanical parts of setup.
  - Measurement of angle by protractor.
  - Fringe shift counting.

Syllabus – Experimental Skills (2/2)

- 3.5 Experimental uncertainty analysis
  - Identification of dominant error sources, and reasonable estimation of the magnitudes of the experimental uncertainties of direct measurements
  - Finding absolute and relative uncertainties of a quantity determined as a function of measured quantities using any reasonable method

- 3.6. Data analysis
  - Transformation of a dependence to a linear form by appropriate choice of variables and fitting a straight line to experimental points
  - Finding the linear regression parameters either graphically, or using the statistical functions of a calculator
  - Selecting optimal scales for graphs and plotting data points with error bars

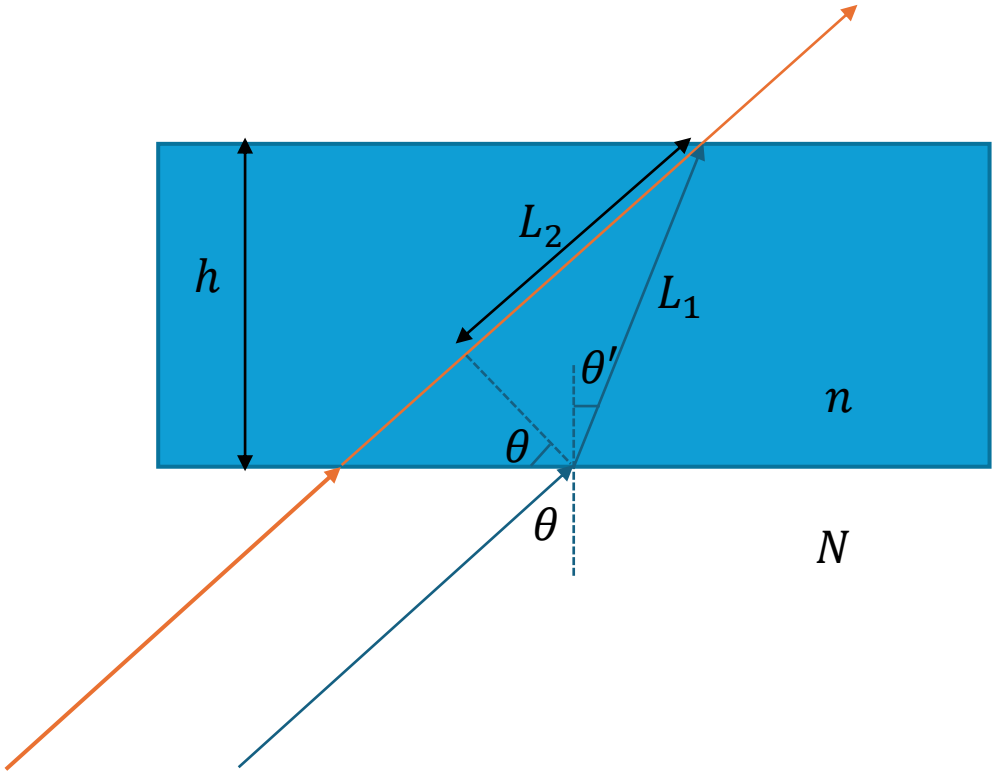
Syllabus – Mathematics

- 4.1. Algebra
  - Taylor series expansion
- 4.2. Functions
  - Basic properties of linear functions

- 4.8 Approximate and numerical methods
  - Using linear and polynomial approximations based on Taylor series.

Thank You

Theory: Phase difference



$$n \sin\theta' = N \sin\theta$$

$$L_1 = \frac{h}{\cos\theta'} = \frac{h}{\sqrt{1 - \frac{N^2 \sin^2\theta}{n^2}}}$$

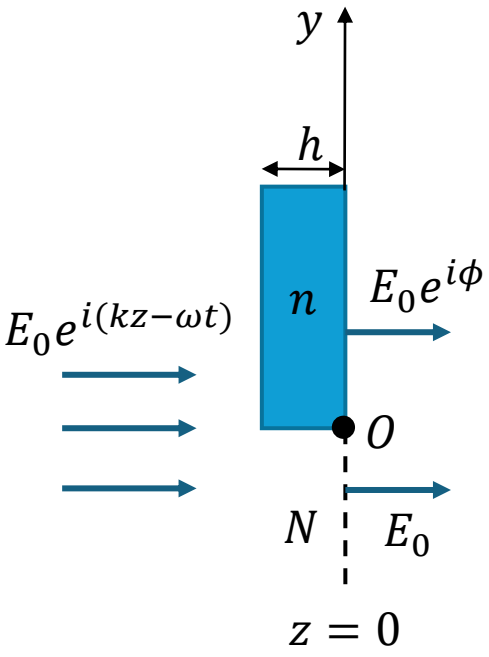
$$L_2 = L_1 \cos(\theta - \theta') = \frac{h}{\cos\theta'} \cos(\theta - \theta') = h\,(\cos\theta + \sin\theta\ \tan\theta')$$

$$\phi = \frac{2\pi}{\lambda}(nL_1 - NL_2)$$

$$\phi = \frac{2\pi}{\lambda}\left(\frac{n}{\sqrt{1 - \frac{N^2 \sin^2\theta}{n^2}}} - N\left(\cos\theta + \frac{N\ \sin^2\theta}{\sqrt{n^2 - N^2 sin^2\theta}}\right)\right)$$

$$\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta} - N cos\theta\right)$$

Theory



(x, y, z) ●

Fresnel diffraction:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z}\int dx'dy'E(x',y',0)e^{\frac{ik}{2z}[(x-x')^2+(y-y')^2]}$$

$$E(x,y,z) = \frac{E_0 e^{ikz}}{i\lambda z}\{e^{i\phi}\int_{y'=0}^{\infty} dx'\,dy'e^{\frac{ik}{2z}[(x-x')^2+(y-y')^2]} + \int_{y'=-\infty}^{0} dx'\,dy'e^{\frac{ik}{2z}[(x-x')^2+(y-y')^2]}\}$$

$$E(x,y,z) = \frac{E_0 e^{ikz}}{i\lambda z}\left(\frac{2\pi z}{ik}\right)^{\frac{1}{2}}\{e^{i\phi}\int_{y'=0}^{\infty} dy'\,e^{\frac{ik}{2z}(y-y')^2} + \int_{y'=-\infty}^{0} dy'\,e^{\frac{ik}{2z}(y-y')^2}\}$$

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Fresnel integral

After simple calculation →

$$E(x,y,z) = E_0 e^{ikz}\{\cos\frac{\phi}{2} + 2\sqrt{i/\pi}\sin\frac{\phi}{2}(C_{(x)} + iS_{(x)})\}$$

$$\text{Intensity} \propto |E(x,y,z)|^2$$