A1 0.50 At what angle α_1 to the X axis should the body velocity vector be for the absolute value of the power of the friction force be at maximum?

Power of the friction force is given by the equation:

$$P = (F, v) = -\left(\mu_x mg\cos^2\alpha + \mu_y mg\sin^2\alpha\right)v$$

since $\mu_x > \mu_y$, then maximum of the power is reached when $\alpha = 0$

Ответ:

$$\alpha = 0$$

A2 0.50 At what angle α_2 to the X axis should the body velocity vector be for the absolute value of the power of the friction force be 1.2 times less than maximum?

From the previous point it follows that:

$$\mu_x mgv\cos^2\alpha + \mu_y mgv\sin^2\alpha = \frac{1}{1.2}\mu_x mgv$$

From where we get:

$$\cos\alpha_2 = \pm\sqrt{\frac{5\mu_x - 6\mu_y}{6(\mu_x - \mu_y)}} = \pm\frac{1}{\sqrt{2}}$$
$$\sin\alpha_2 = \pm\sqrt{\frac{\mu_x}{6(\mu_x - \mu_y)}} = \pm\frac{1}{\sqrt{2}}$$

Ответ:

$$\alpha_2 = \pm\pi/4; \pm 3\pi/4$$

A3 1.00 Let the initial velocity have components $v_{0x} = 1\mathrm{m/s}$ and $v_{0y} = 1\mathrm{m/s}$. After some time the velocity component along the Y axis equals $v_{1y} = 0{,}25\mathrm{m/s}$. What is the velocity magnitude at this moment?

Newton's equation:

$$\begin{cases} m\dfrac{dv_x}{dt} &= -\mu_x mg\dfrac{v_x}{v} \\ m\dfrac{dv_y}{dt} &= -\mu_y mg\dfrac{v_y}{v} \end{cases}$$

Dividing one equation by another, we get:

$$\frac{dv_x}{dv_y} = \frac{\mu_x v_x}{\mu_y v_y}$$

Integrating these equations, we obtain:

$$\frac{v_x^{\mu_y}}{v_y^{\mu_x}} = Const$$

From the initial conditions we find:

$$v_x = 0.125\mathrm{m/s}$$

So the absolute value of velosity is

Ответ:

$$v = 0.28\mathrm{m/s}$$

A4 [1.00] Let the velocity be $v_2 = 1.0\mathrm{m/s}$. At what angle α_3 to the X axis should the velocity vector be for the radius of curvature of the trajectory be minimum? What is this radius equal to? The free fall acceleration is $g = 9{,}8\mathrm{m/s}^2$.

Projection of the friction force onto the direction perpendicular to the velocity is given by equation:

$$F_n = mg(\mu_x - \mu_y)\sin\alpha\cos\alpha$$

Newton's second law projected onto the direction perpendicular to the velocity has the form:

$$\frac{mv^2}{R} = mg(\mu_x - \mu_y)\sin\alpha\cos\alpha,$$

where R is the radius of curvature of the trajectory. It's obvious that

$$R = \frac{v^2}{g(\mu_x - \mu_y)\sin\alpha\cos\alpha} = \frac{2v^2}{g(\mu_x - \mu_y)\sin 2\alpha}$$

The minimum radius of curvature will be when:

Ответ:

$$\alpha = \pi/4$$

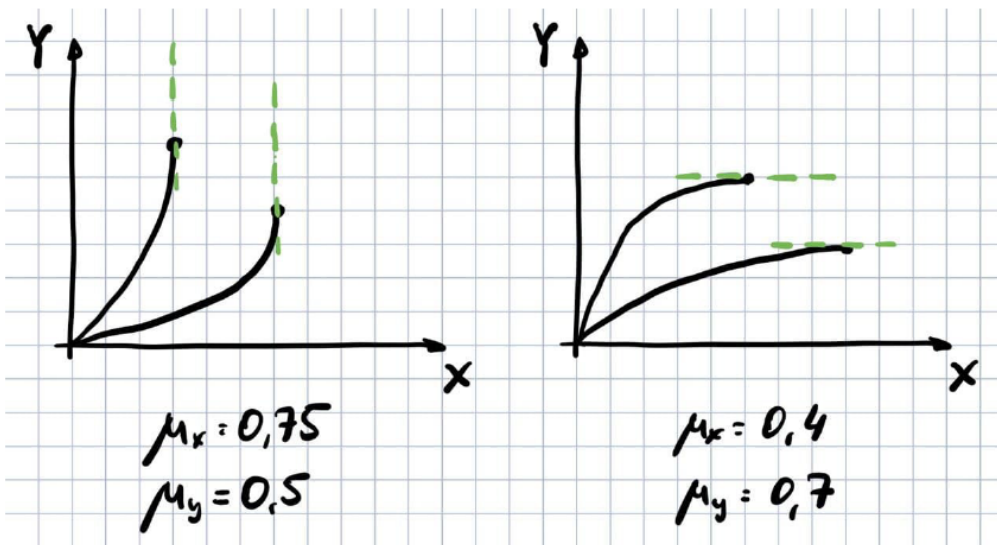
and

$$R_{min} = \frac{2v^2}{g(\mu_x - \mu_y)}$$

A5 [1.00] In a single diagram on the XY plane, sketch the trajectories of the body launched at the angles $\alpha_4 = \pi/6$ and $\alpha_5 = \pi/3$ for the friction coefficients specified above. The magnitudes of initial velocities are the same. Solve the same problem for the friction coefficients $\mu_x = 0{,}4$ and $\mu_y = 0{,}7$.

If $\mu_x = \mu_y$, then the accelerations along the x and y axes will be proportional to the ratio of the initial velocities and the body will move in a straight line. If $\mu_x > \mu_y$, then the speed along the x-axis will decrease faster than in the previous case and the body will deviate from a linear motion as shown in the figure. The direction of deflection does not depend on how the initial velocity is directed. If $\mu_x < \mu_y$, then the body will deviate in the opposite direction.

Ответ:



B1 [2.00] A body of mass m is at rest at the origin. A force has been applied to it at an angle α to the X axis. The force magnitude $F(t) = \gamma t$ linearly grows with time. Find the dependence of the moment the body starts moving on α. Ignore the stagnation phenomenon.

Let us notice that the body will start the motion with the angle $\varphi \neq \alpha$. Then projections of the forces at this moment are given by

$$\begin{cases} F_{\mathrm{fr},x} = \mu_x mg\cos\varphi = \gamma t\cos\alpha \\ F_{\mathrm{fr},y} = \mu_y mg\sin\varphi = \gamma t\sin\alpha \end{cases}$$

Dividing these equations by $\mu_x mg$ and $\mu_y mg$ correspondingly and summing up squares, we get

$$(mg)^2 = (\gamma t)^2\left[\left(\frac{\cos\alpha}{\mu_x}\right)^2 + \left(\frac{\sin\alpha}{\mu_y}\right)^2\right]$$

So the answer is

Ответ:

$$t=\frac{\mu_x\mu_y mg}{\gamma\sqrt{\mu_y^2\cos^2\alpha+\mu_x^2\sin^2\alpha}}$$

C1 1.50 For a given initial velocity v_0 find the dependence of its velocity v on the angle of rotation of the rod φ assuming that the other body remains at rest.

Newton's law projected onto the direction of motion of a point-like mass is:

$$m\frac{dv}{dt}=-mg(\mu_x\cos^2\varphi+\mu_y\sin^2\varphi)$$

Since $v=L\varphi$, then:

$$L\ddot{\varphi}=-g(\mu_x\cos^2\varphi+\mu_y\sin^2\varphi)$$

Multiply this equation by $\dot{\varphi}\,dt$ we get

$$L\dot{\varphi}d\dot{\varphi}=-g(\mu_x\cos^2\varphi+\mu_y\sin^2\varphi)d\varphi$$

Integrating this equation, we obtain:

Ответ:

$$v^2+gL\left((\mu_x+\mu_y)\varphi+(\mu_x-\mu_y)\frac{\sin 2\varphi}{2}\right)=v_0^2$$

C2 1.50 Find the maximum value of the initial velocity $v_{0\max}$ at which the other body will remain at rest.

Newton's law for a moving body in projection onto a rod has the form:

$$T+F_{\text{fr}}\sin\beta=m\dot{\varphi}^2L$$

where β is the angle between frictional force and direction of velocity.
Using the result from C1, we find that:

$$T=\frac{mv_0^2}{L}-mg(\mu_x+\mu_y)\varphi-mg(\mu_x-\mu_y)\sin 2\varphi$$

Using the result from B1, we find that:

$$\frac{mv_0^2}{L}-mg(\mu_x+\mu_y)\varphi-mg(\mu_x-\mu_y)\sin 2\varphi\le\frac{\mu_x\mu_y}{\sqrt{\mu_y^2\sin^2\varphi+\mu_x^2\cos^2\varphi}}mg$$

Note that with increasing φ, the left hand side decreases, and the right hand side increases, so the body starts to move right at $\varphi=0$. So

Ответ:

$$v_{0\max}=\sqrt{\mu_y gL}=2.2\mathrm{m/s}$$

C3 1.00 What distance will the body travel until it stops completely if the initial velocity is $v_{0\max}$?

Substituting $v_{0\max}$ from the previous section in the final equation of the C1, we get the equation for the φ at the stopping point $(v=0)$:

$$gL\left((\mu_x+\mu_y)\varphi+(\mu_x-\mu_y)\frac{\sin 2\varphi}{2}\right)=\mu_y gL$$

Numerically solving this equation, we get the answer for the φ, and so the travelled distance is equal to

Ответ:

$$L\varphi = 0.34\mathrm{m}$$