
IPhO
2016
International
Physics Olympiad
Switzerland
Liechtenstein

T2: Nonlinear Dynamics in Electric Circuits

10 points

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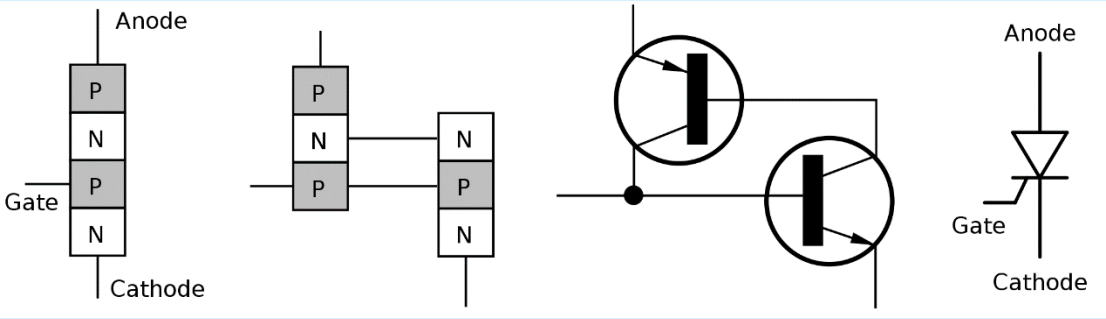
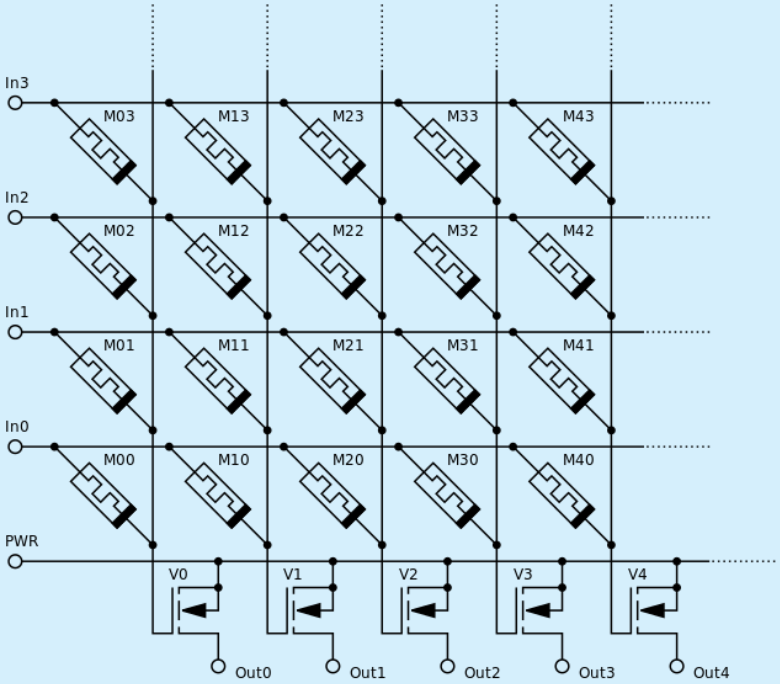
Motivation

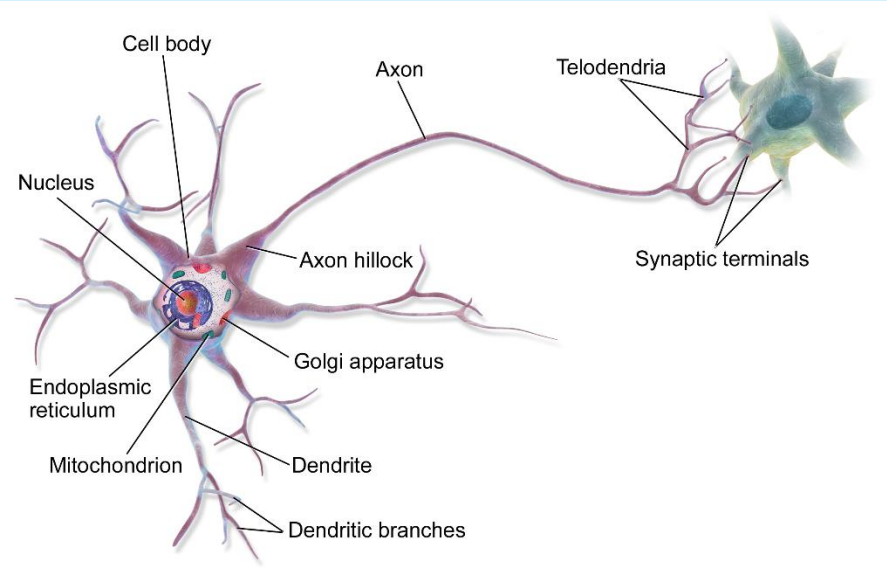


Bi-stable elements (e.g. thyristors) are used in controlling currents in high voltage power lines.

Motivation



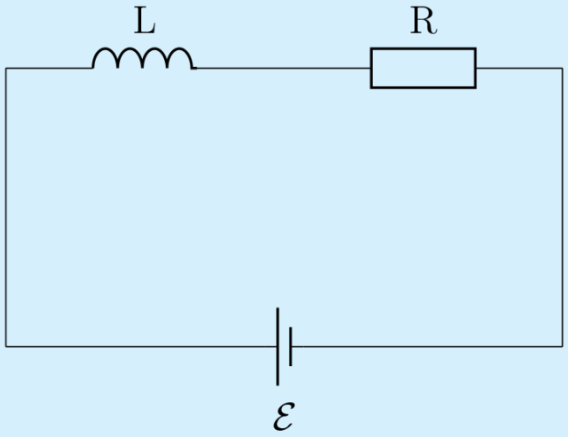
Bi-stable elements are used to model the behavior of neurons and neuron networks in the brain.



Objectives

- To study stability and instability of stationary states of a bi-stable element;
- To discover non-linear oscillations in a circuit with a bi-stable element
- To study a simple model of a neuristor

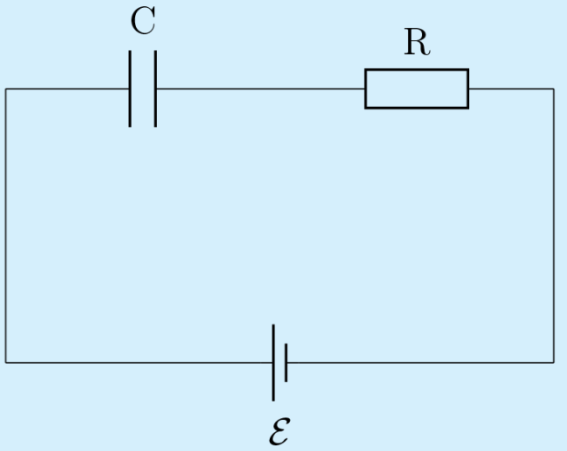
Theoretical background



- Dynamics of *RL*-circuit

$$RI + L\frac{dI}{dt} = \mathcal{E}$$

$$I(t) = \frac{\mathcal{E}}{R} + \left(I(0) - \frac{\mathcal{E}}{R}\right)e^{-\frac{R}{L}t}$$

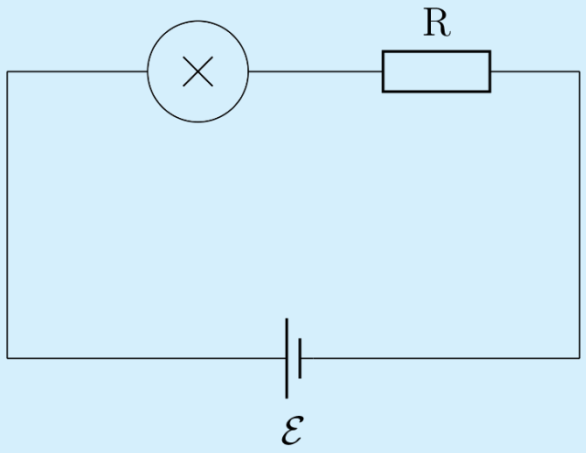


- Dynamics of *RC*-circuit

$$\frac{Q}{C} + R\frac{dQ}{dt} = \mathcal{E}$$

$$Q(t) = C\mathcal{E} + (Q(0) - C\mathcal{E})e^{-\frac{t}{RC}}$$

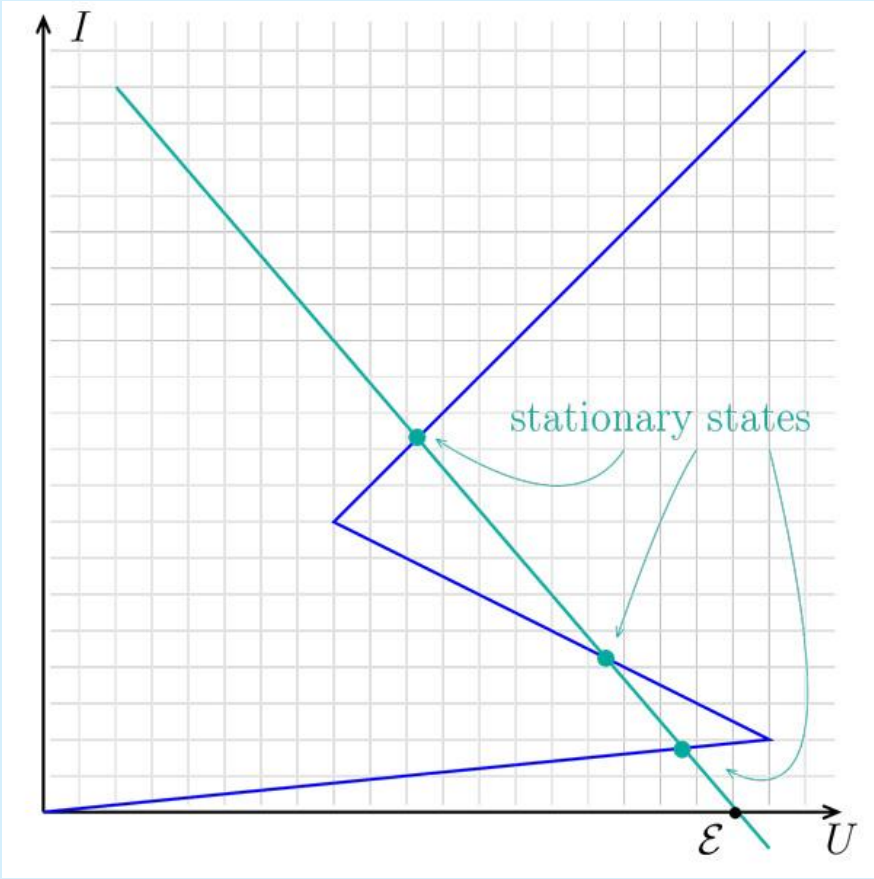
Theoretical background



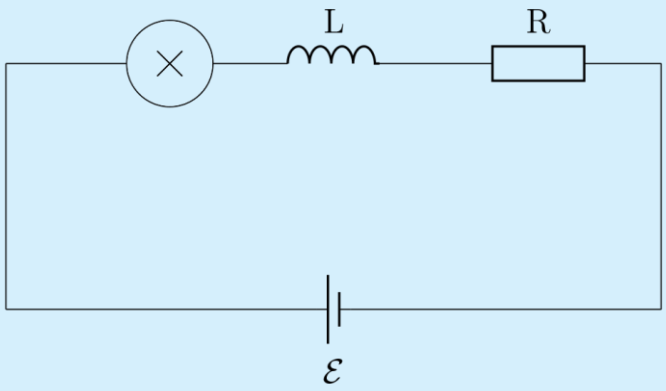
$$U_X + I_X R = \mathcal{E}$$

$$I_X = \frac{\mathcal{E} - U_X}{R}$$

stationary states are intersections of this line with the I-V graph



Task A (3 points)



- Study of stability of a circuit with a non-linear element

$$U_X + L\frac{dI_X}{dt} + RI_X = \mathcal{E}$$

- Stationary state on the middle branch

$$U_X = R_{int}(I_0 - I_X)$$

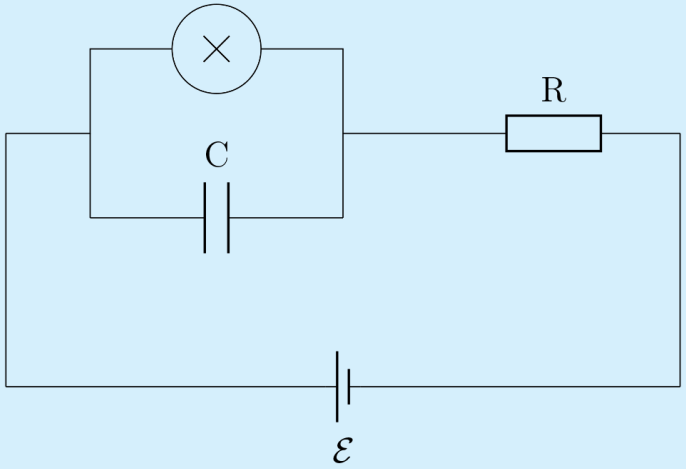
$$\Rightarrow L\frac{dI_X}{dt} = (\mathcal{E} - R_{int}I_0) + (R_{int} - R)I_X$$

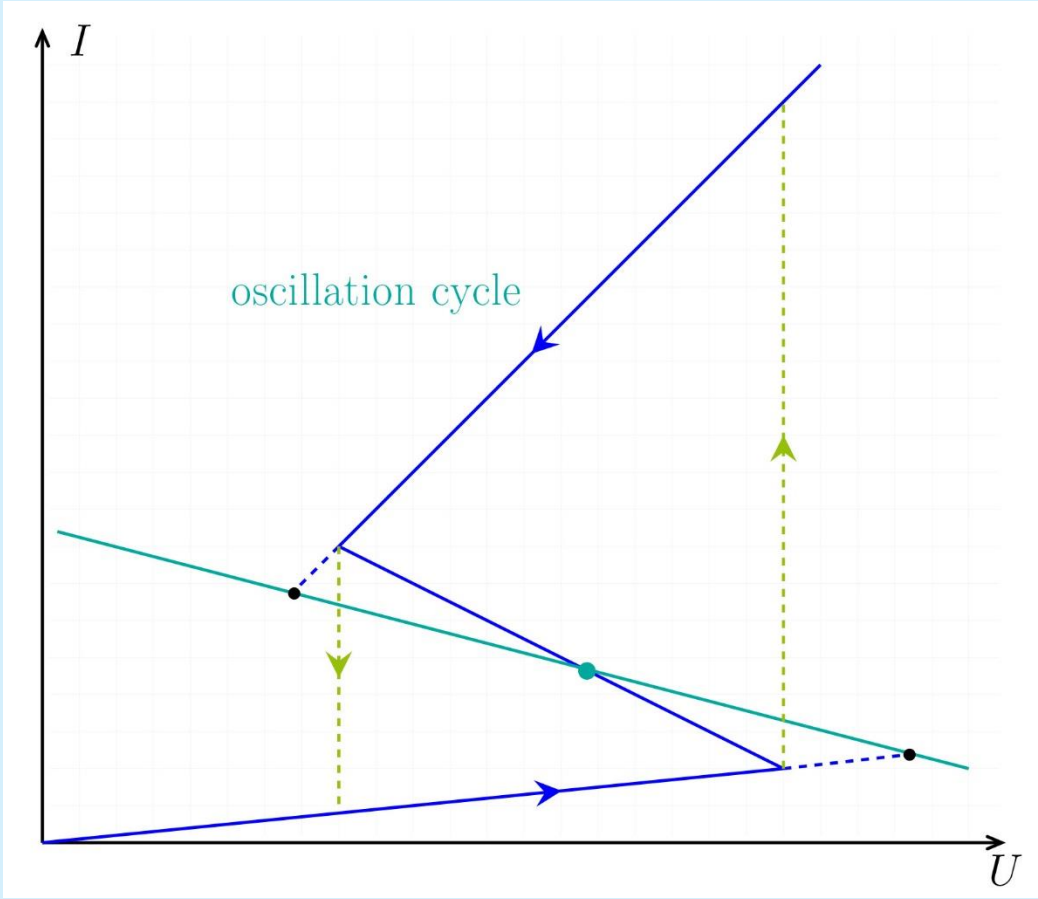
$$R_{int} - R < 0 \quad \Rightarrow \quad \text{stable}$$

$$R_{int} - R > 0 \quad \Rightarrow \quad \text{unstable}$$

Task B (5 points)

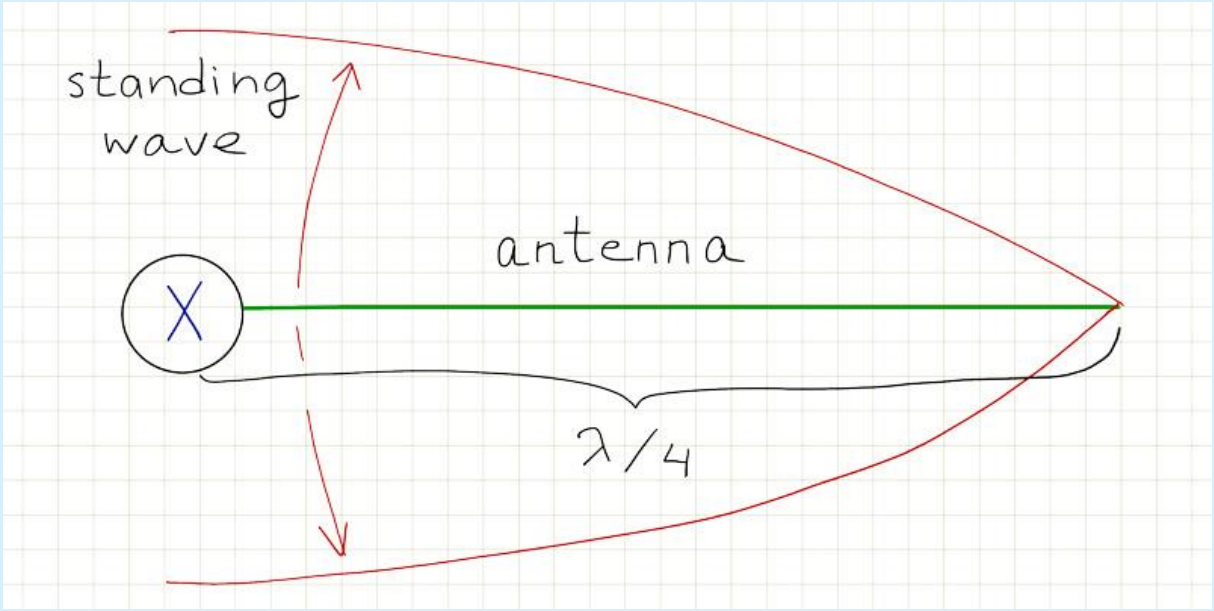
- Study of non-linear oscillations in a circuit with a bi-stable element



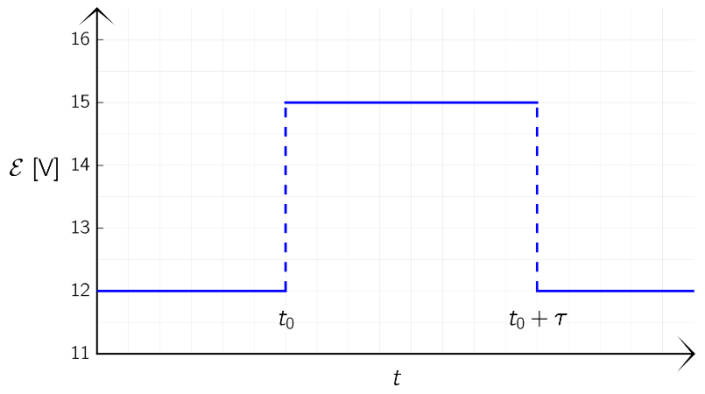


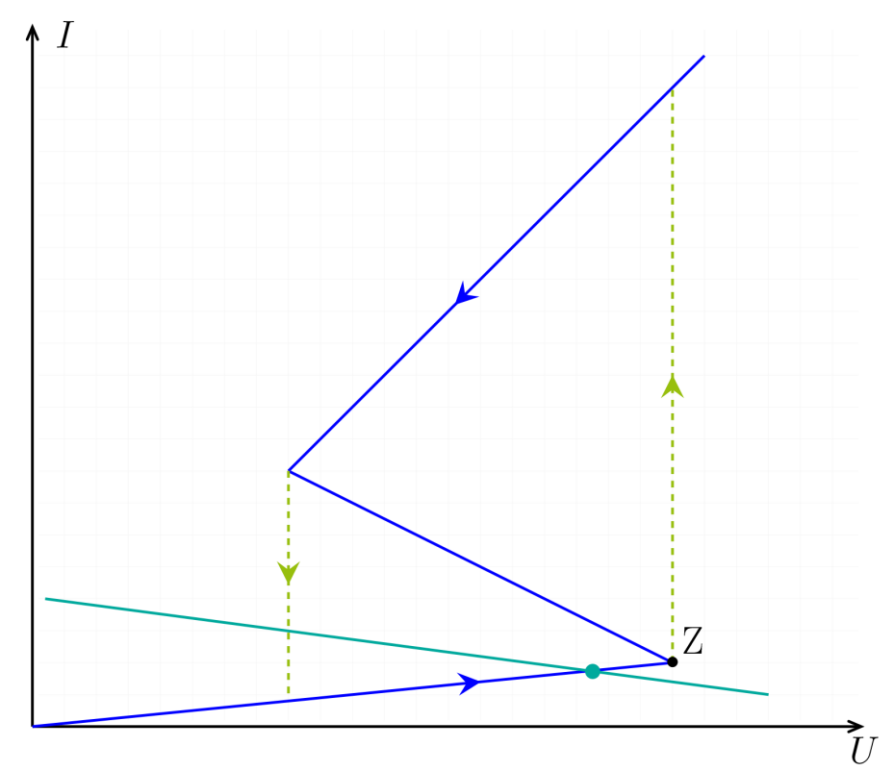
Task B (5 points)

- Compute oscillation period
- Estimate power dissipated by the element X
- Build a radio transmitter



Task C (2 points)

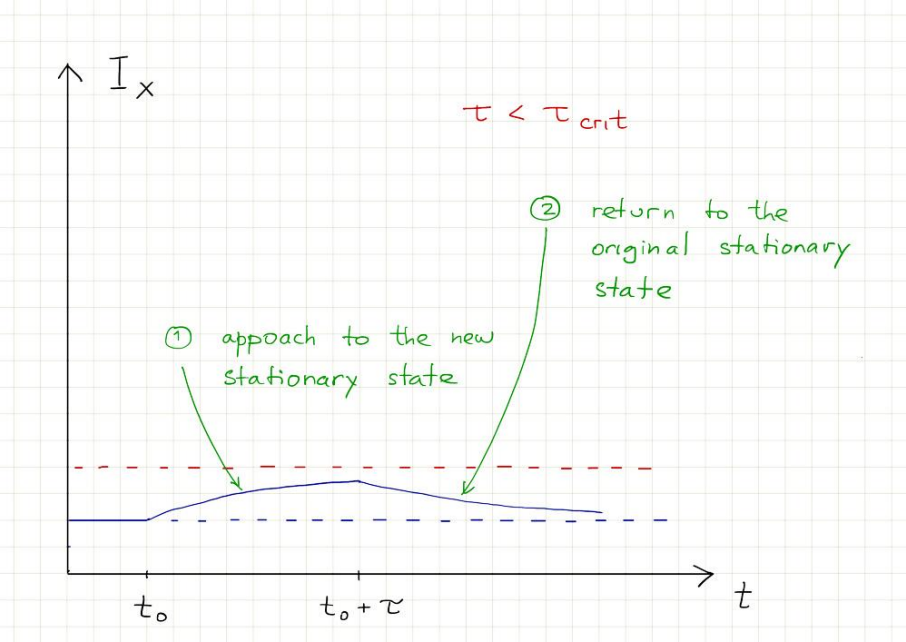


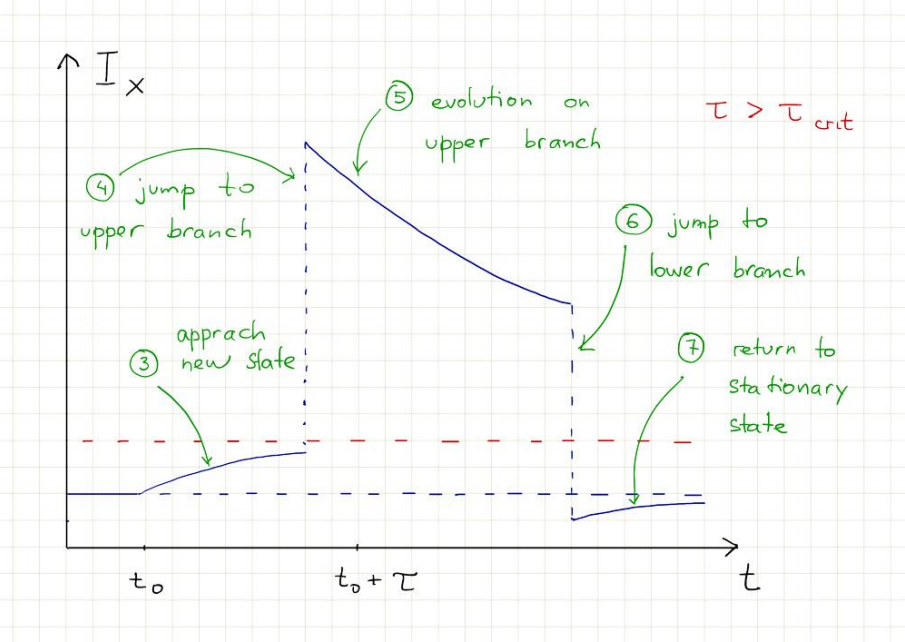


- If the system reaches Z $\Longrightarrow$ one oscillation $\Longrightarrow$ it's a neuristor
- If the system doesn't reach Z $\Longrightarrow$ returns to stationary state

Task C (2 points)

The time dependence of the voltage $U_X(t)$ for $\tau < \tau_{crit}$ and for $\tau > \tau_{crit}$





Summary

Part A (3 points)

- Stationary states
- Stability

Part B (5 points)

- Non-linear oscillations
- Power estimate
- Radio transmitter

Part C (2 points)

- Neuristor

What the students are being tested for

Knowledge

2.3.1 Kirchoff's current and voltage laws

2.3.4 Ohm's law, Jule's law, nonlinear elements of given V-I characteristic, time constants of RL and RC circuits

2.4.1 Positive feedback as a source of instability

2.4.3 Wave length

2.4.4 Standing waves

Skills

4.2 Solving simple equations involving logarithmic and exponential functions
4.7 Calculus
4.8 Approximation

Modularity