IPhO 2016

International Physics Olympiad Switzerland Liechtenstein

T-1: Mechanics

10 marks
Ben Kilminster, Christoph Keller

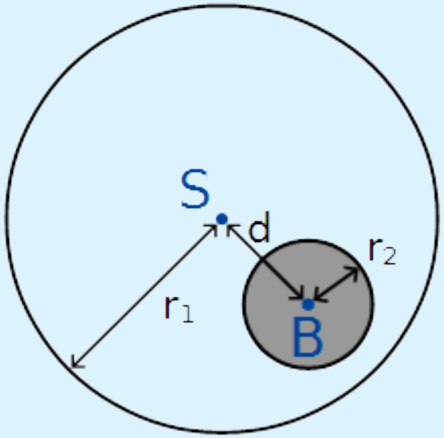
Andreas Schilling, Johan Runeson, Anton Alekseev

Overview

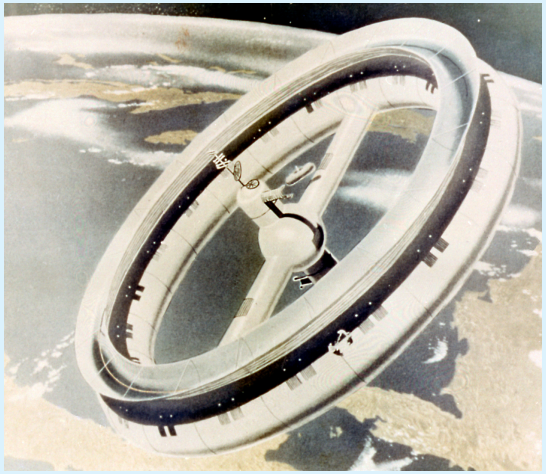
Two problems in Mechanics

The problem has two independent parts:

Part A: A metal disk in a cylinder



Part B: Artificial gravity in a rotating space station



Motivation

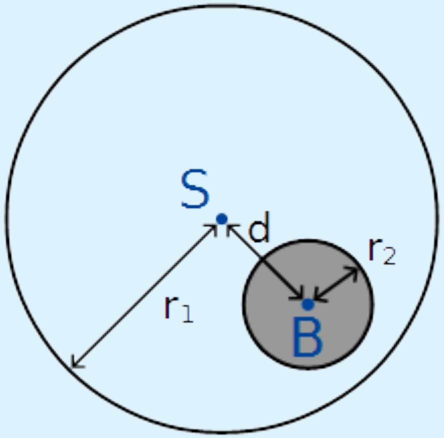
Part A:

- There is a metal disk hidden somewhere in a wooden cylinder
- Students are asked to find the size and position of the disk through indirect „measurements“

Part A – Objective

Find the size and position of the disk through three indirect measurements:

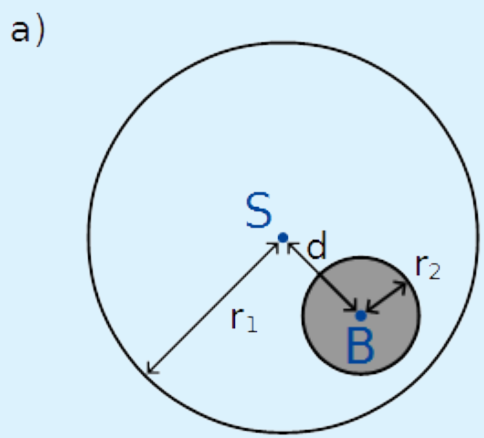
1) Measure total mass
2) Measure moment of inertia of system through oscillation period of gravity pendulum
3) Measure center of mass through equilibrium position on inclined plane

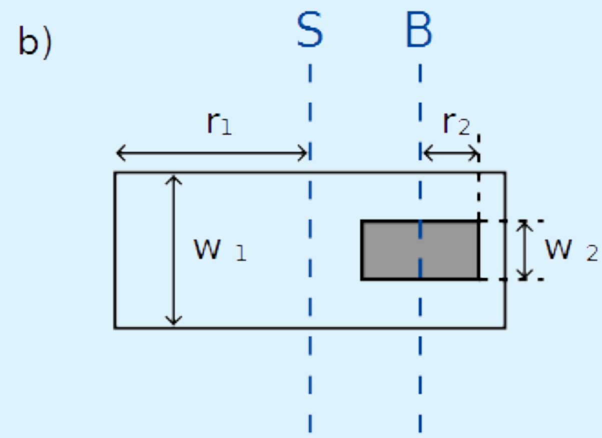


Part A (3.5 marks)

We are looking for:

- radius of disk r_2
- thickness w_2
- position of center d

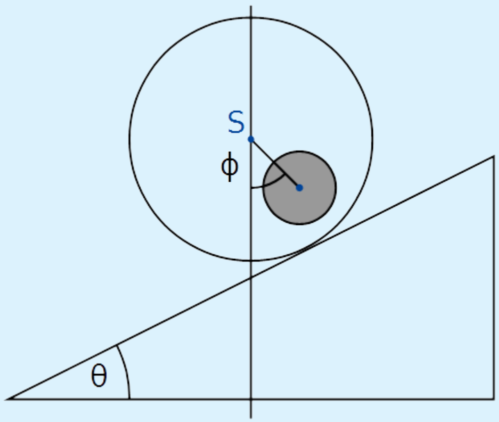




We „measure“:

1) Total mass M
2) Equilibrium position on inclined plane
3) Oscillation period of suspended cylinder

Part A (continued)



First two subtasks:

A1: Obtain center of mass b from equilibrium angle ϕ
(Use combined force and torque equilibrium)
A2: Obtain moment of inertia I_S from oscillation period of gravity pendulum

Then: Combine these results to obtain $r_2,\ w_2\ , d$:

A3: Find position d from b (center of mass equation)
A4: Compute moment of inertia I_S in terms of $r_2,\ w_2\ , d$ (moment of inertia of disks, Steiner's theorem)
A5: Combine A1-A4 to obtain $r_2,\ w_2\ , d$

Motivation

Part B:

- Alice and Bob live in a rotating space station. The centrifugal force provides artificial gravity.
- Bob doesn't believe that they are on a space station. Alice uses physics to convince him that they are.

Part B – Objective

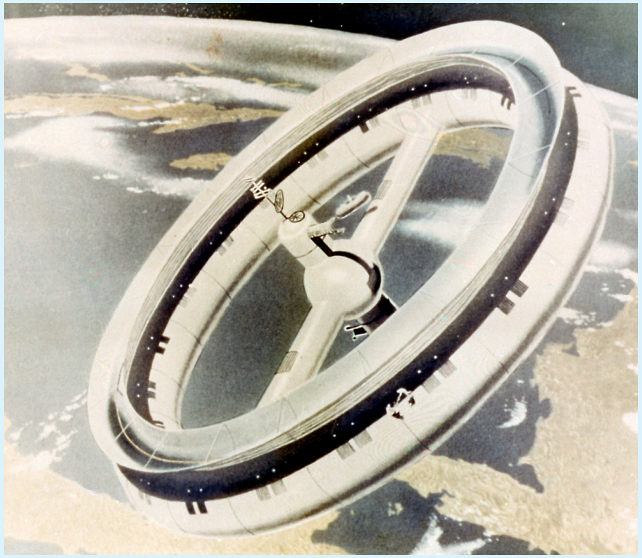
Explore the difference between artificial gravity on the rotating space station and actual gravity on the earth:

1) Compare the behavior of a harmonic oscillator (mass on a spring) in the gravitational field of earth and the artificial gravity on the space station.
2) Coriolis force: Find the action of the Coriolis force on free falling objects
3) Find the behavior of the harmonic oscillator under the influence of the Coriolis force

NB: We give the expression for the Coriolis force.

$$F_C = 2mv\omega_{ss}\sin\phi \ ,$$

Part B (6.5 marks)



B1: Rotating space station: artificial gravity due to centrifugal force

Harmonic oscillator on a spring:

B2: If gravity is constant, frequency is independent of g
B3: centrifugal force is proportional to radius => on space station, frequency is shifted
B4: On earth, to first approximation gravity also depends linearly on height => same effect, frequency is also shifted

Part B (6.5 marks)

Two ways to think about Coriolis force: Use formula, or go to inertial frame.

B6: Drop mass from small tower. Using the formula for the Coriolis force, compute horizontal displacement.

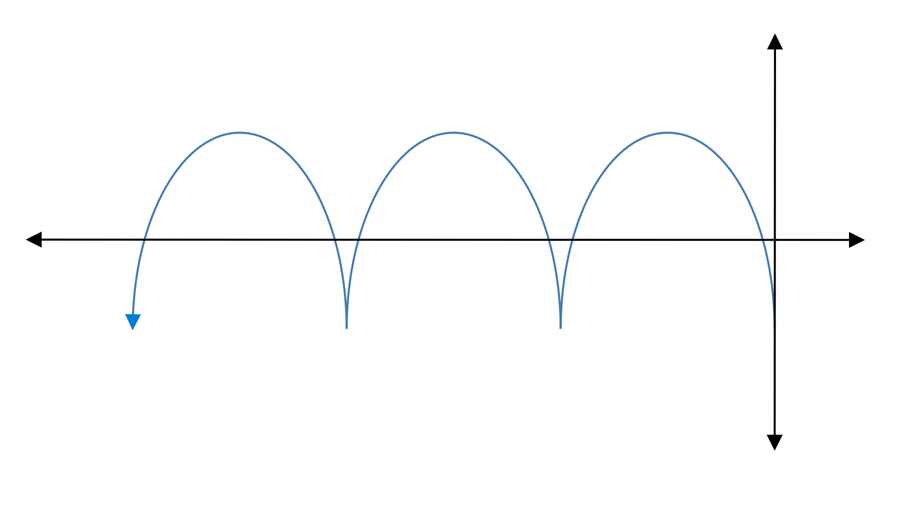
B7: Drop mass from tall tower. To get correct result, go to inertial frame. If tower is high enough, space station completes one (or more!) rotations before mass hits the ground.

➡ Need to find approximate solution to $$\frac{H}{R} = 1 - \cos\left(\frac{\sqrt{1-(1-H/R)^2}}{(1-H/R)}\right)$$

Part B (6.5 marks)

Now combine everything:

B8: Movement of harmonic oscillator under influence of Coriolis force. Oscillator can freely move in the x direction, oscillates in y direction.



Summary

Part A (3.5 marks):

- „Measure“ moment of inertia and equilibrium point
- Find size of position of metal disk from those measurements

Part B (6.5 marks):

- Find behavior of harmonic oscillator in non-constant gravitational field
- Coriolis force acting on freely falling mass
- Coriolis force acting on harmonic oscillator

What the students are being tested for

Knowledge

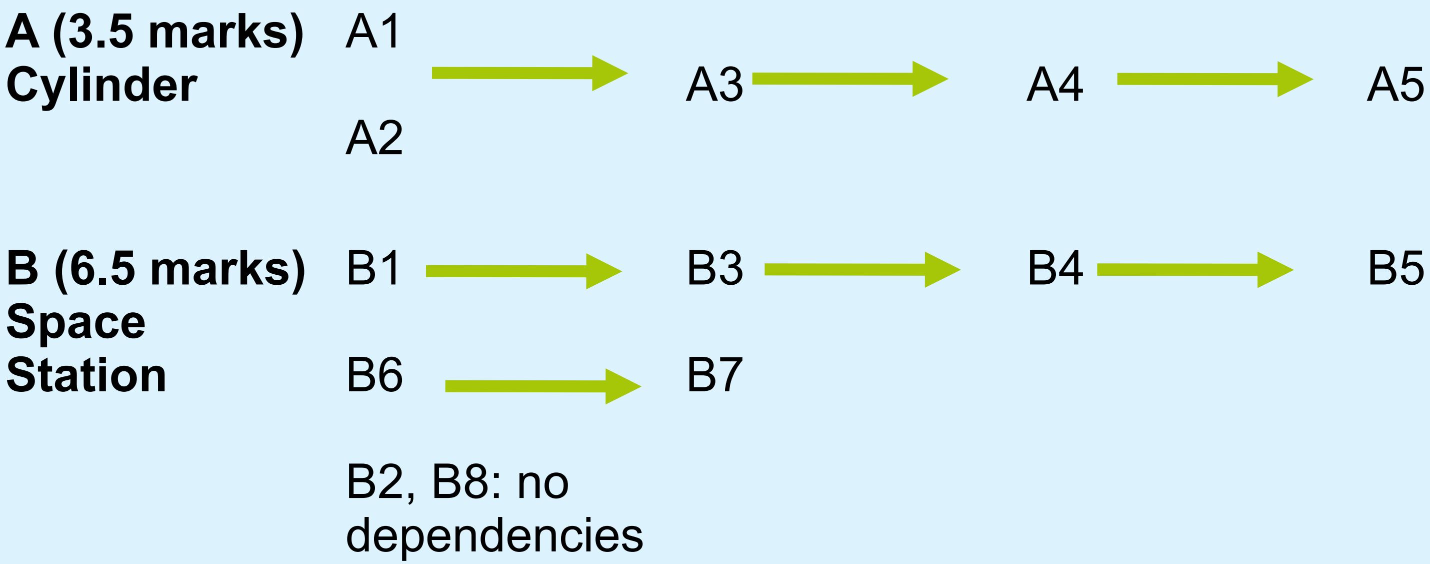
2.2.2 Statics: Torque, force equlibria (A1)
2.4.1 Single oscillator (A2,B8)

2.2.1 Kinematics (B1,B2,B3,B7)
2.2.3 Dynamics (B8)
2.2.4 Celestial mechanics (B1,B4)

Skills

4.2 Functions
4.7 Calculus (B4,B6)
4.8 Approximate and numerical methods (B4,B7)

Dependencies



Note: Dependencies between sub-tasks are indicated by arrows.

Thank you!

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