IPhO 2016

International Physics Olympiad Switzerland Liechtenstein

E-2: Jumping beads – a model for phase transitions and instabilities

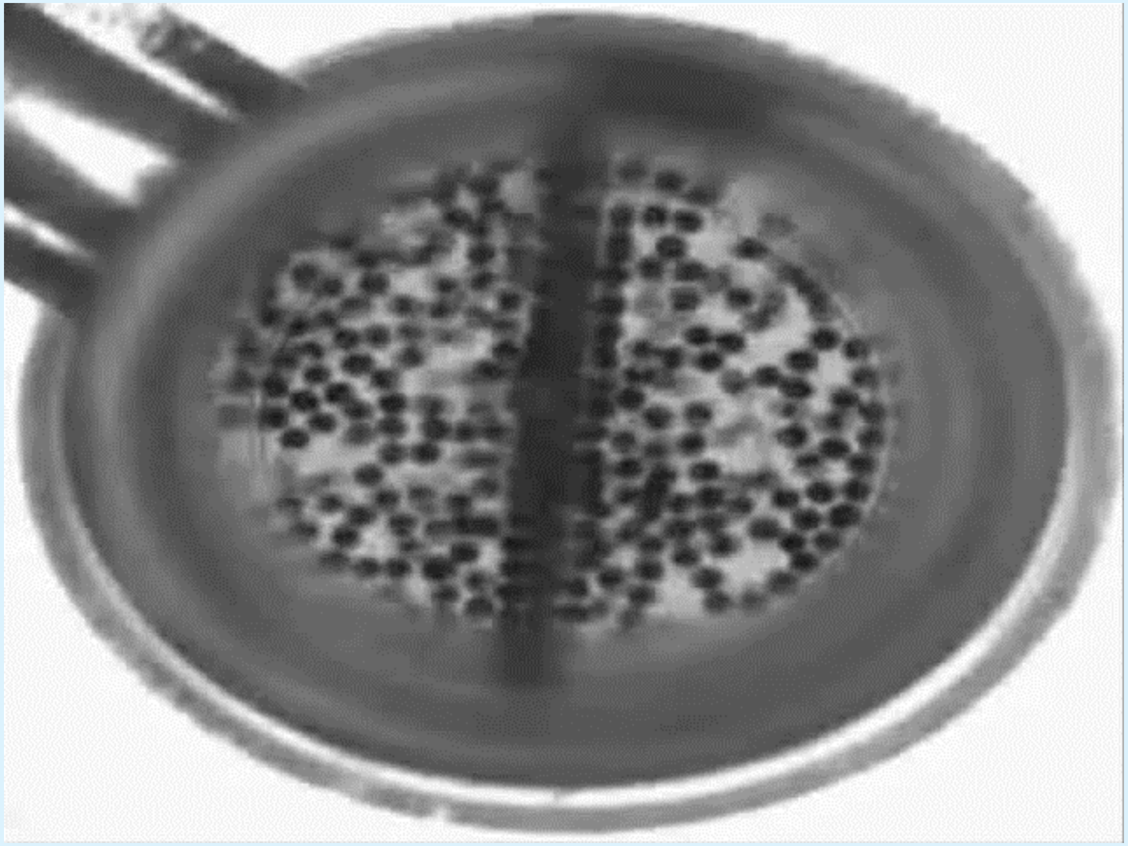
10 marks

Christof Aegerter and Alex Kish

Prototype setup by Hans-Peter Koch
Final setup design by Achim Vollhardt

Motivation

How do instabilities occur and what do they have in common with phase transitions?



Objective

Describe the instability mathematically analogous to a phase transition

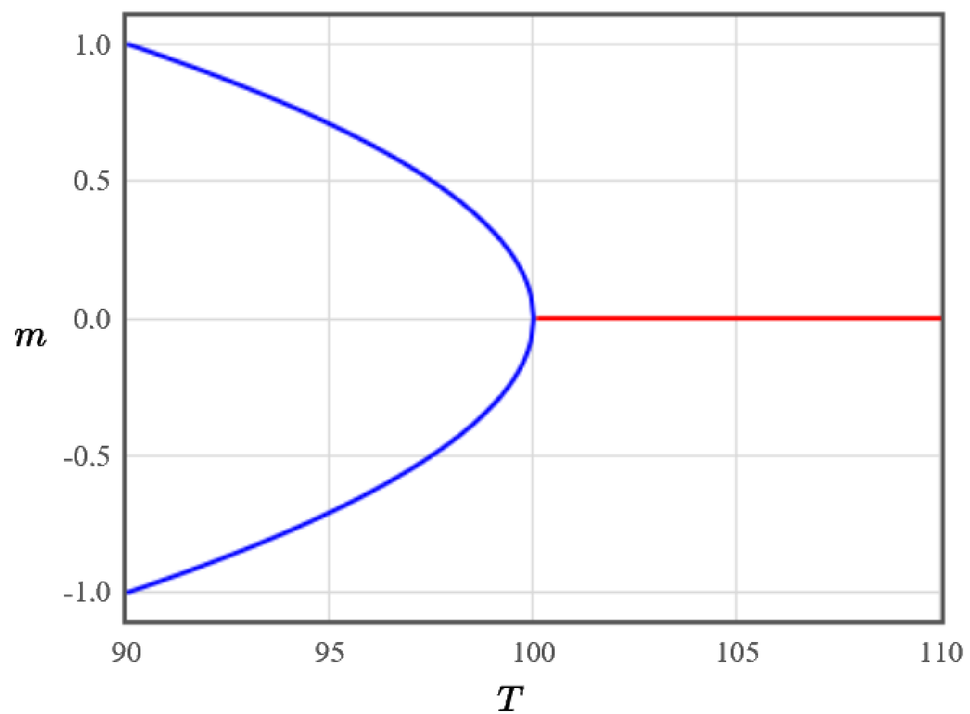
Right and left compartments can be seen as spin-up and spin down

The shaking excitation can be seen as temperature

Inelastic collisions lead to a clustering in one compartment

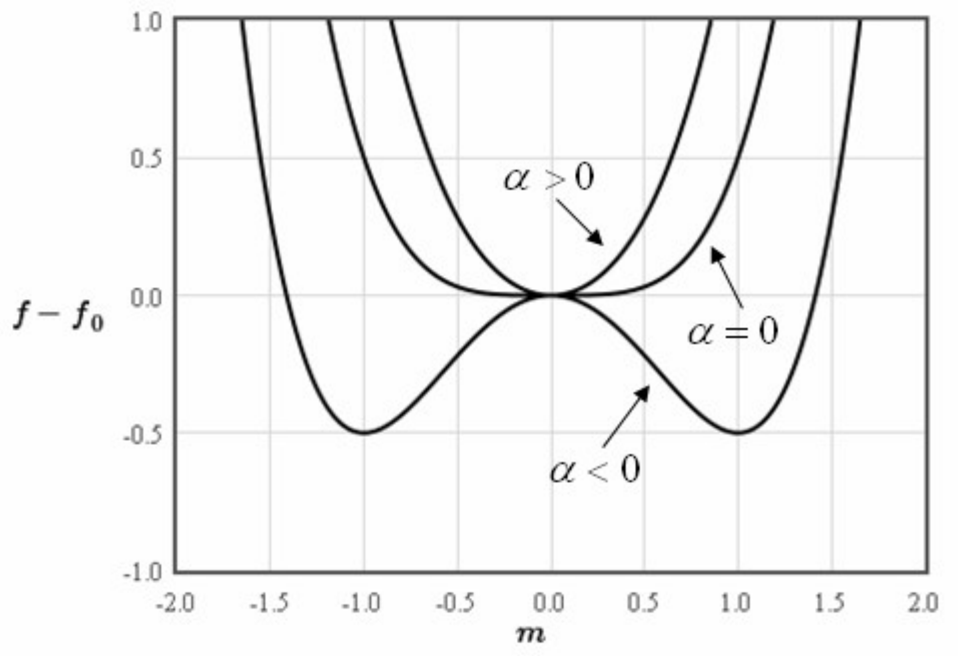
This is analogous to the macroscopic populations of spins in ferromagnetism

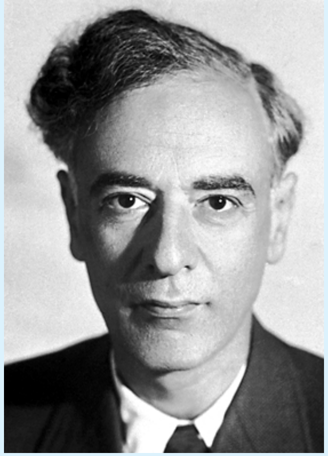
The difference between beads right and left corresponds to magnetization



Introduction to the problem: theoretical background

- Landau theory of phase transitions

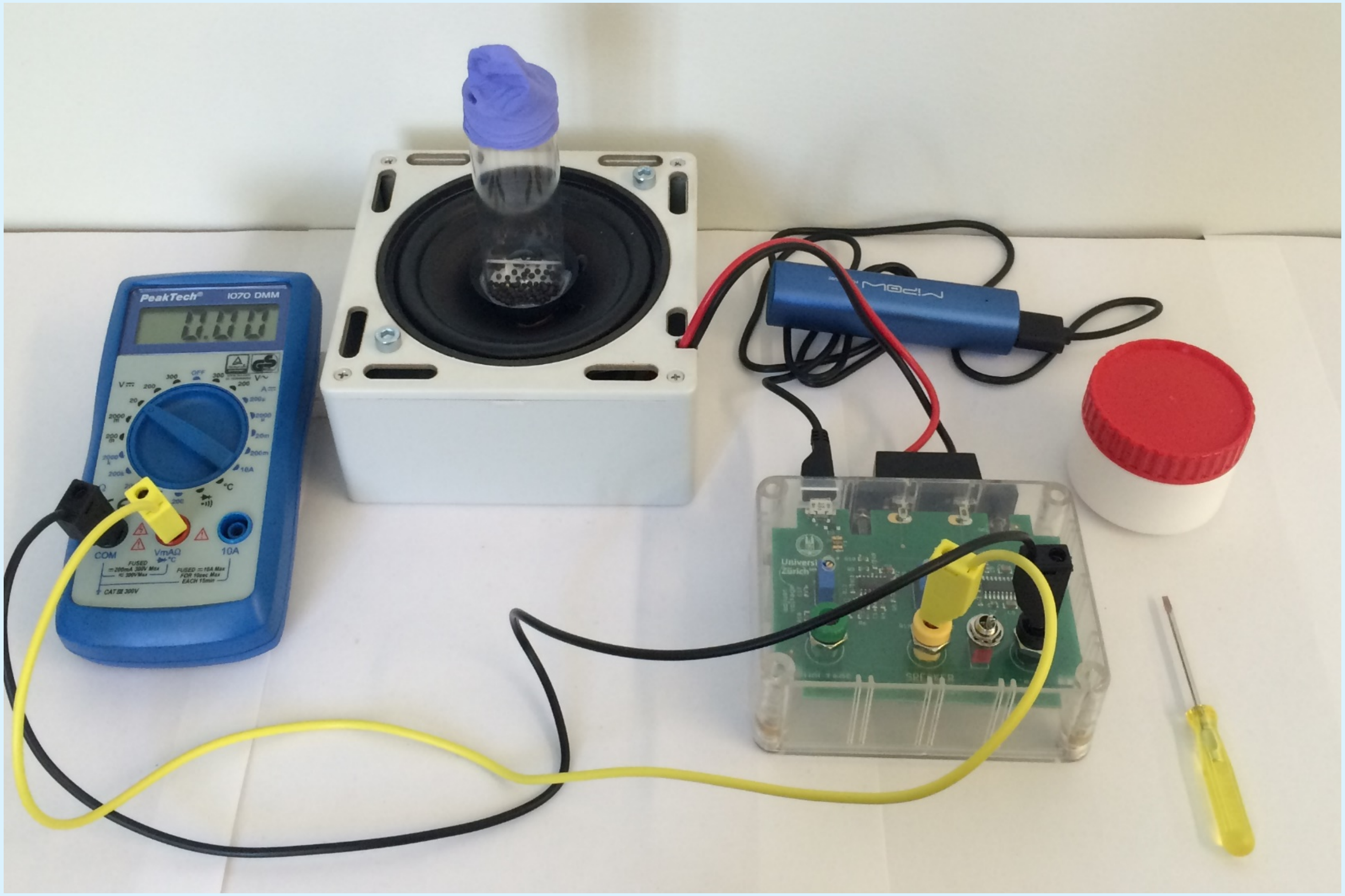




$$m = \pm\sqrt{\frac{\alpha_0(T_c - T)}{\beta}} \qquad T < T_c$$

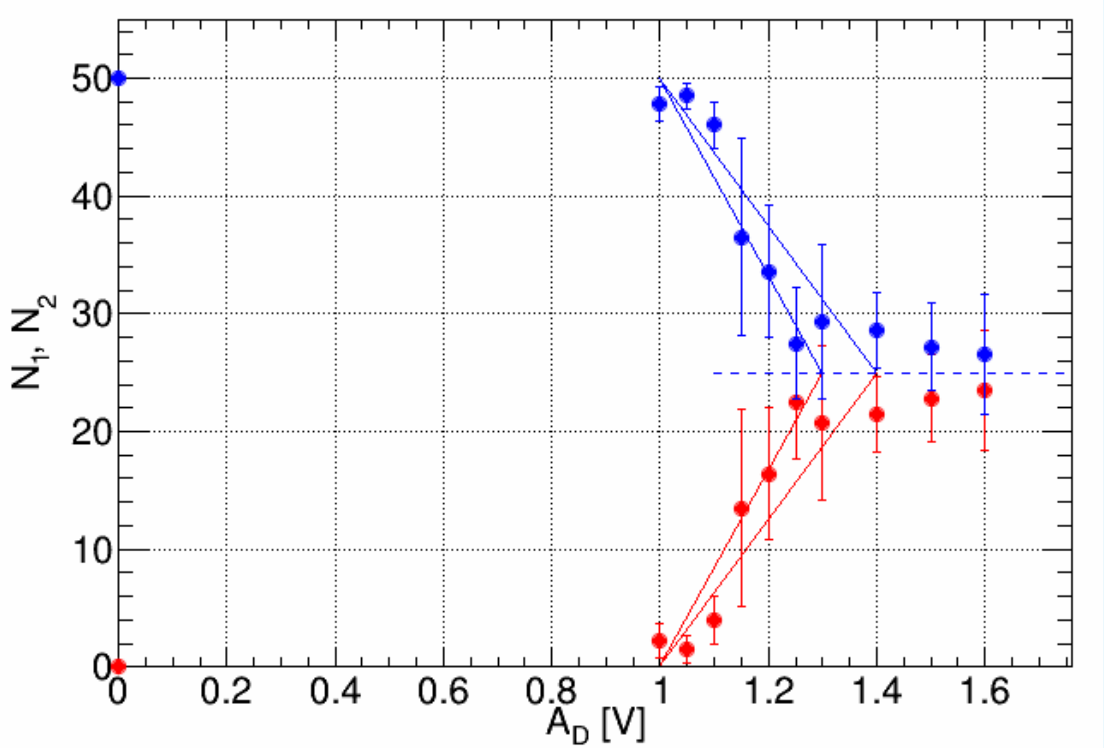
$$f(T) = f_0(T) + \alpha_0(T - T_c)m^2 + \frac{1}{2}\beta m^4 \qquad \alpha_0 > 0, \qquad \beta > 0.$$

Setup



Task A: critical driving amplitude (3 marks)

- Determine number of beads in different halves of the container
- Plot the bifurcation diagram
- Determine the critical amplitude, i.e. where particles are mixed

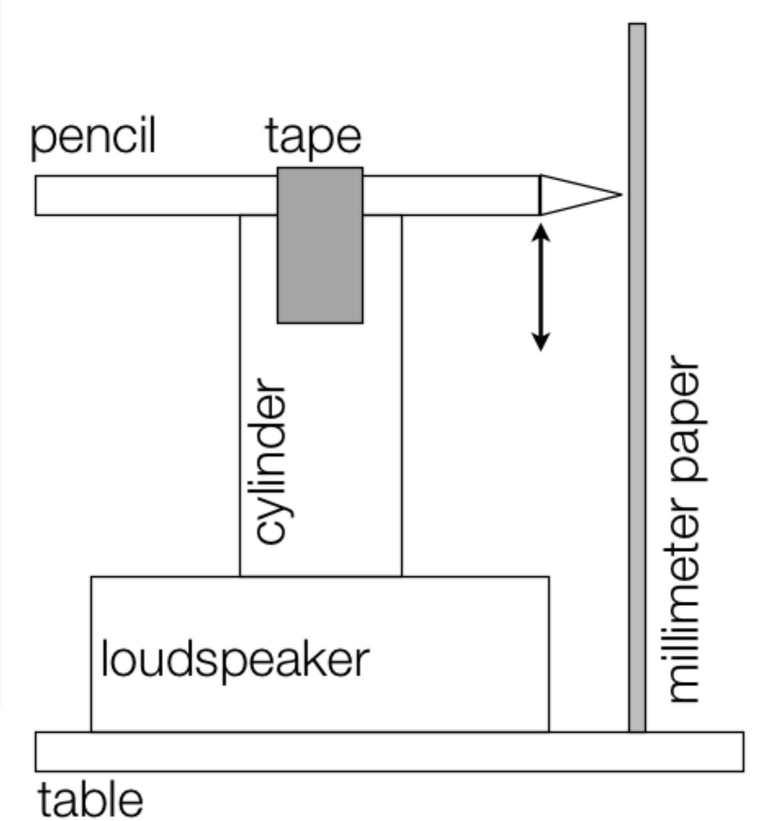


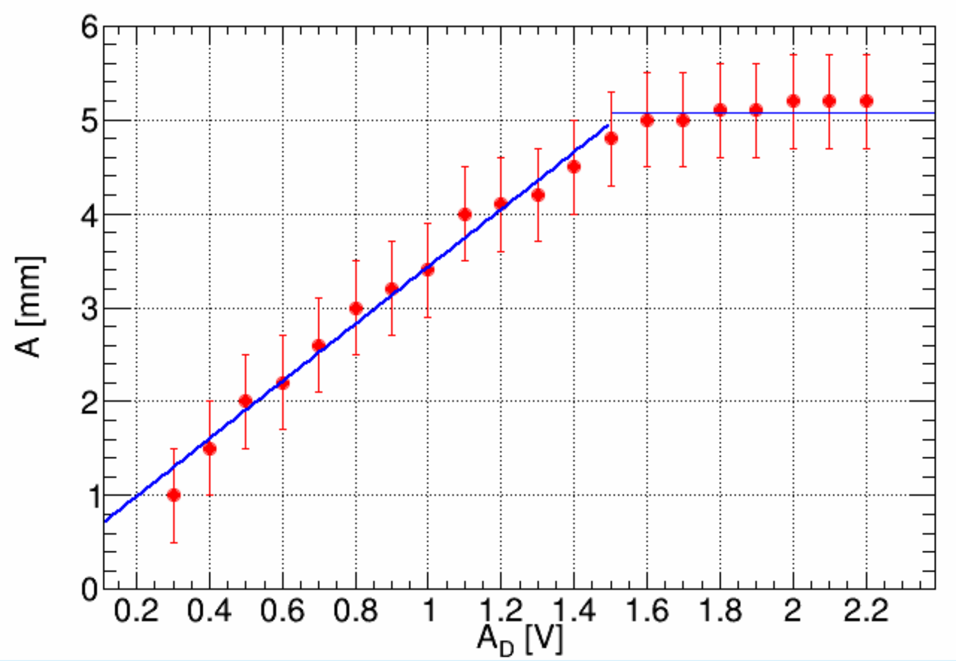
$A_{D,crit}$ = 1.3 +/- 0.05 V

Systematic variation between setups 20%, i.e. 0.25 V

Task B: calibration of the amplitude (2.5 marks)

- Come up with a setup for calibration using provided material and sketch this
- Determine travel amplitude as a function of applied voltage and plot data
- Describe the results mathematically, i.e. fit a straight line
- Apply the calibration to the determination of the critical amplitude





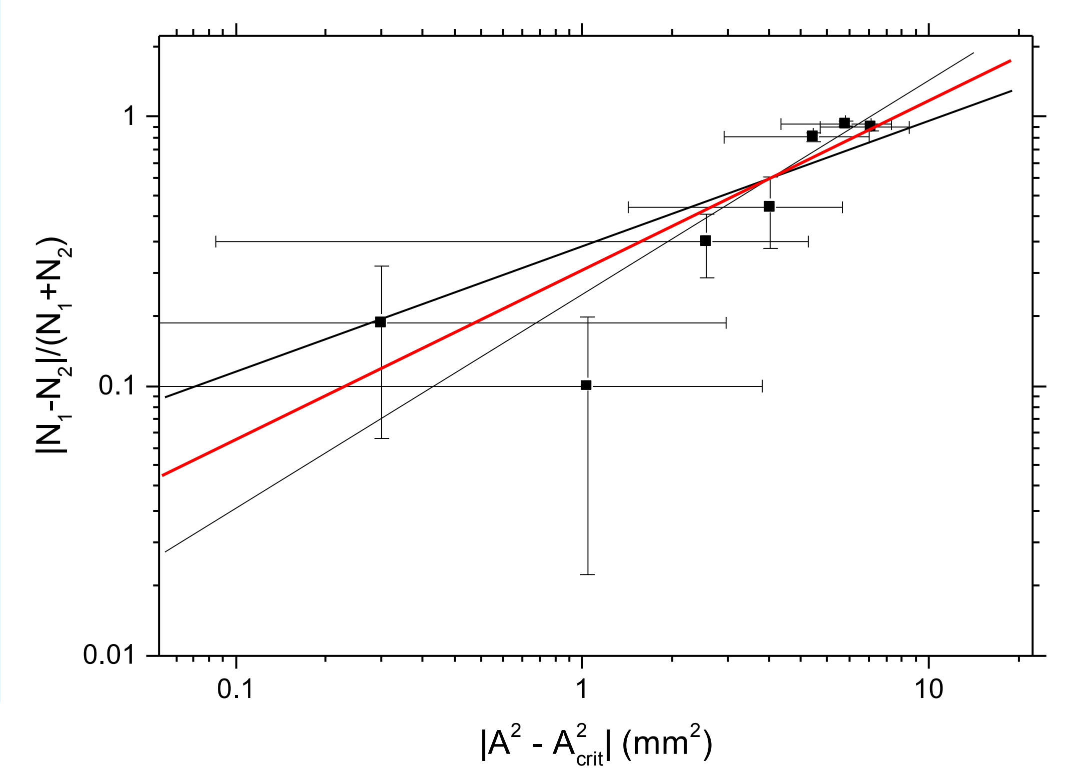
$$A_{crit} = 4.2(2)\ \text{mm}$$

$$A = 0.3 + 3.1\,A_D$$

IPhO 2016

Task C: critical exponent (2.5 marks)

- Obtain Number vs. Amplitude from the calibration – take more data close to transition
- Plot the data around the critical driving amplitude on log-log paper
- Determine the critical exponent b from slope of the curve



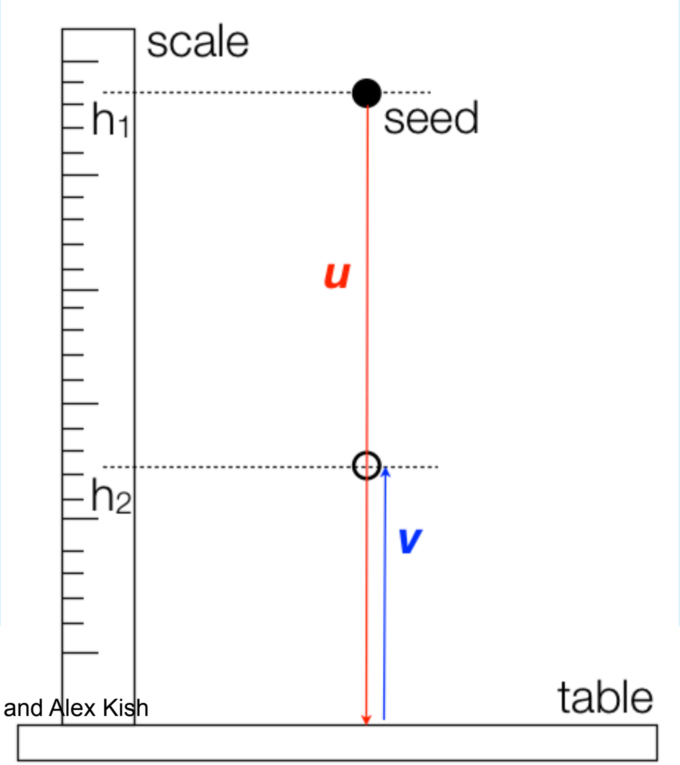
b = 0.6 +/- 0.2

Theoretical value: b = 0.5

Task D: coefficient of restitution (2 marks)

- Come up with a setup for determining the coefficient of restitution and sketch this
- Measure particle recoil to determine coefficient of restitution
- Determine the coefficient of restitution from these measurements
- Estimate the uncertainty of the value

$$e = \left|\frac{v}{u}\right| = \sqrt{\frac{h_2}{h_1}}$$



e = 0.6 +/- 0.2

Summary

Part A (3 marks):

- Measure the instability as a function of driving (A1-A2)
- Understand what the critical point is (A3)

Part B (2.5 marks):

- Calibrate the measuring equipment in terms of driving amplitude (B1-B4)
- Apply the calibration to previous measurements (B5)

Part C (2.5 marks):

- Determine the critical exponent of the pitchfork bifurcation (C1-C3)

Part D (2 marks):

- Study the physics behind the instability; Measure the coefficient of restitution of the particles (D1-D4)

What the students are being tested for

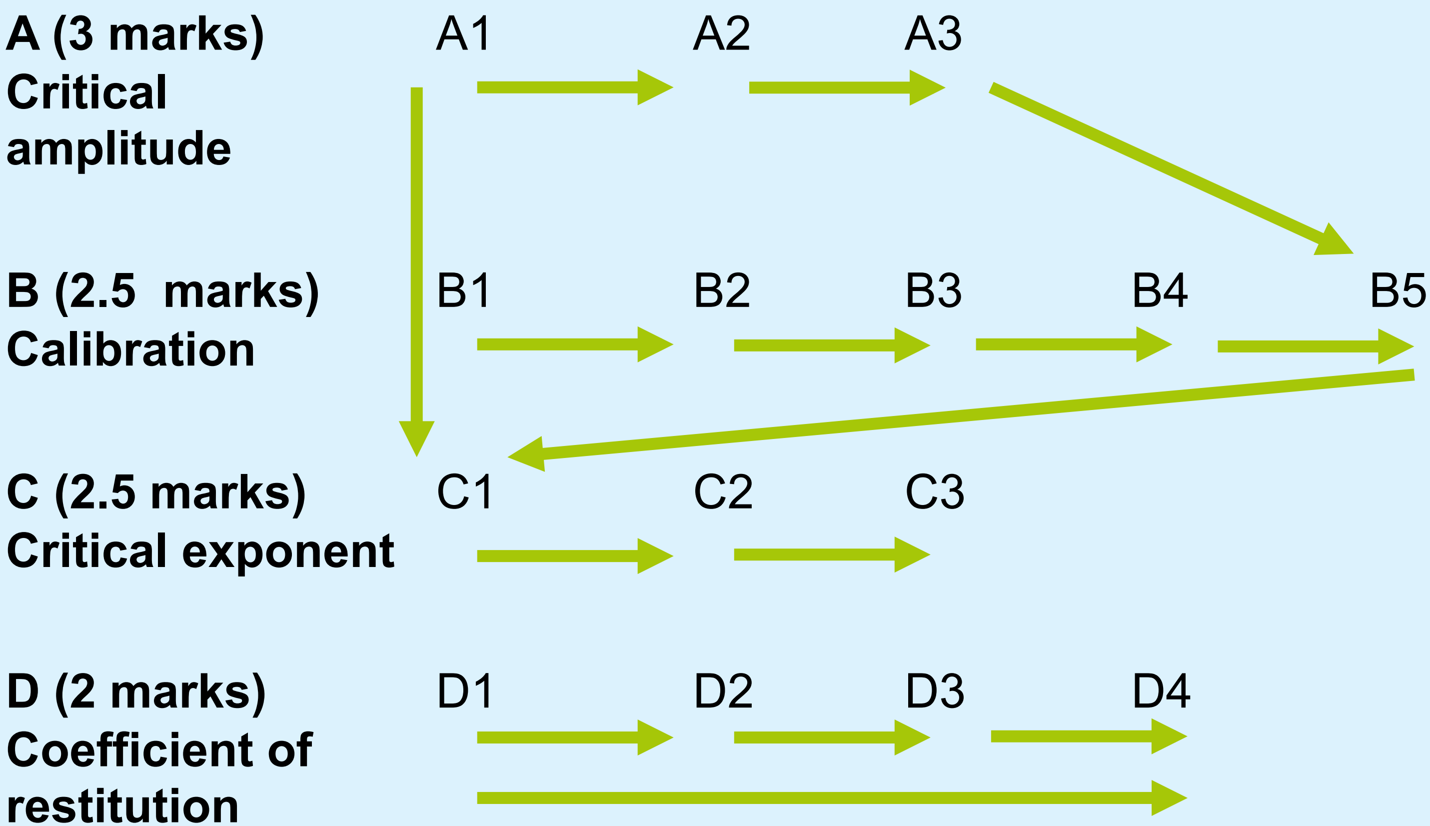
Knowledge

- *Statistics and uncertainties, 4.6 – tasks A and B*
- *Power-Laws and double logarithmic scales; functions, 4.2 - task C*
- *Classical mechanics, 2.2.3 – task D*

Skills

- *Error propagation; uncertainty analysis, 3.5 – tasks C, D*
- *Error estimation; accuracy, 3.4 – tasks A,B, D*
- *Plotting graphs of functions; data analysis, 3.6 – tasks A,C*
- *Designing experimental setups; measurement techniques 3.3 – tasks B, D*

Modularity


A (3 marks)
Critical amplitude
A1
A2
A3
B (2.5 marks)
Calibration
B1
B2
B3
B4
B5
C (2.5 marks)
Critical exponent
C1
C2
C3
D (2 marks)
Coefficient of restitution
D1
D2
D3
D4