International Physics Olympiad Switzerland Liechtenstein



E-1: Electrical conductivity in two dimensions

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Motivation

Devices: electric properties of materials used are important. Key parameter: Electric resistivity (related to carrier mobility, conductivity).

Modern devices (processors, integrated circuits, microelectronics) reach dimensions of nanometers:

Depending on fabrication procedure, properties vary significantly.



iht.rwth-aachen.de

Has a very thin metal film the same properties as a thick metal wire?





www.bridgetec.com

Textbook physics: resistance and resistivity



Assuming linear dependence between current and voltage, use Ohm's law:

resistance
$$R = \frac{V}{I}$$
 voltage current

Textbook physics: resistance and resistivity

Material property resistivity: measurements on a long conductor



resistance
$$R = \rho \cdot \frac{l}{t \cdot w}$$
 $\rho = \text{resistivity}$

Resistance and resistivity in 2D

Formal transition from 3D to 2D:



Resistance for a stripe and homogeneous current distribution:

$$\rightarrow R = \frac{\rho}{t} \cdot \frac{l}{w} = \rho_{\Box} \cdot \frac{l}{w} \qquad \rho_{\Box} \left[\Omega\right] = \text{ sheet resistivity}$$

Resistance for point-like contacts of *small size and distance*:

$$\rho_{\Box} = \frac{\pi}{\ln 2} \cdot R = \underbrace{\frac{V}{I}}_{\text{measurement}}$$

Measurement depends on geometry

4-Point-Probe (4PP): Separation of current-carrying contacts and voltage probes.

Realization:

- Small point-like contacts on surface of thin film.
- Non-uniform current distribution.



Measurement depends on geometry

4-Point-Probe (4PP): Separation of current-carrying contacts and voltage probes.

Realization:

- Small point-like contacts on surface of thin film.
- Non-uniform current distribution.
- Measurement depends on geometry and sample dimensions:

(b)





Same voltage but less current!



Experimental equipment

Samples: conductive paper, metal coated silicon wafer



Task A (1.1 marks): basic circuit

- Assembly of electric circuit (A1)
- Measurement of resistance of Ohmic resistor (A2-A3)
- Comparison with color coded specifications (A4-A6)



Task B (1.1 marks): 4-point-probe technique

- Introduction of 4-point-probe technique.
- Measurement on full sheet of paper (B1).
- Determination of resistance and uncertainty (B2-B3).



Task C (0.1 marks): sheet resistivity

- Introduction of sheet resistivity.
- Calculation of resistivity of conducting paper for large dimensions (C1)



Tasks D (3.2 marks) and E (1.2 marks): geometrical correction factor

Investigate the influence of finite sample dimensions on 4PP measurement.

- Cut stripes of different width w.
- Measure resistance as function of *w/s* (*s*=distance between probes). (D1)
- Normalize resistance values to resistance of large sheet R_∞. (D2)

$$R(w/s) = R_{\infty} \cdot f(w/s)$$
$$f(w/s) = 1.0 + a \cdot \left(\frac{w}{s}\right)^{b}$$

- Assume power law dependence and plot the data on log-log scale. (E1)
- Fit the curve by a line and deduce the power law parameters. (E2)



Task F1-F4 (1.0 mark): measurements on Cr/Si Investigation of 8 nm Cr/Si by 4PP technique.

- Measure resistance of the Cr/Si wafer by 4PP technique as before (F1-F2).
- Calculate geometrical correction due to finite wafer size (100 mm diameter) using fit results of Part E (F3):

$$R(w/s) = R_{\infty} \cdot f(w/s)$$
$$f(w/s) = 1.0 + a \cdot \left(\frac{w}{s}\right)^{b}$$

• Determine sheet resistivity of Cr film (F4).





Task F5-F10 (2.0 marks): van der Pauw geometry (Leon J. van der Pauw, Philips Research Reports 1958)

- Measure resistance in van der Pauw geometry for
 two different orientations (F5-F7).
- Solve van der Pauw equation and calculate sheet resistivity (F8):

$$e^{-\pi R_{21,34}/\rho_{\Box}} + e^{-\pi R_{14,23}/\rho_{\Box}} \equiv$$

- Compare to 4PP technique (F9).
- Calculate bulk resistivity if thickness = 8 nm (F10).





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What the students are being tested for

Knowledge

- Basic concepts of electro-magnetic fields (Syllabus 2.3.1)
- Circuits (2.3.4)
- Interaction of matter with electric and magnetic fields (2.4.5)
- *Mathematical functions and scaling laws (4.2)*
- Calculus (Linearization, transformation of an equation) (4.7)

Skills

- Measurement techniques and apparatus (3.3) (A, B, D, F)
- Accuracy (in data acquisition) (3.4)
- Plotting data points (3.6) (A, B, E, F)
- Data analysis; fitting and determination of uncertainties (3.6) (A, B, E, F)

Modularity	
A (1.1 marks) Setup/circuit	A1 A2 A3 A4 A5 A6
B (1.1 marks) 4PP	B1 B2 B3
C (0.1 marks) Sheet resistivity	C1
D (3.2 marks) Sample dimensions	D1 D2
E (1.6 marks) Correction factor	E1 = E2
F (3.0 marks) Wafer, van der Pauw	F1 F2 F3 F4 F5 F6 F7 F8 F9 F10

Note: Dependencies between sub-tasks are indicated by arrows.

Summary

Part A (1 mark):

- Assembly of the electrical circuit (A1).
- Measure resistance of Ohmic resistor (A2-A3) and compare to color code (A4-A6).

Part B (1.1 marks) and Part C (0.1 marks): conducting paper

- Determine resistance and uncertainty by 4-point-probe (4PP) technique (B1-B3).
- Calculate sheet resistivity (C1).

Part D (3.2 marks) and Part E (1.6 marks): conducting paper

- Measure resistance of paper stripes as function of stripe width (D1-D2).
- Plot data of Part D and fit data with a power law dependence (E1-E2).

Part F (3 marks): 8 nm Cr film on Si substrate

- Determine sheet resistivity of Cr by 4PP incl. geometrical correction (F1-F4).
- Determine sheet resistivity of Cr by using the van der Pauw technique (F5-F10).

