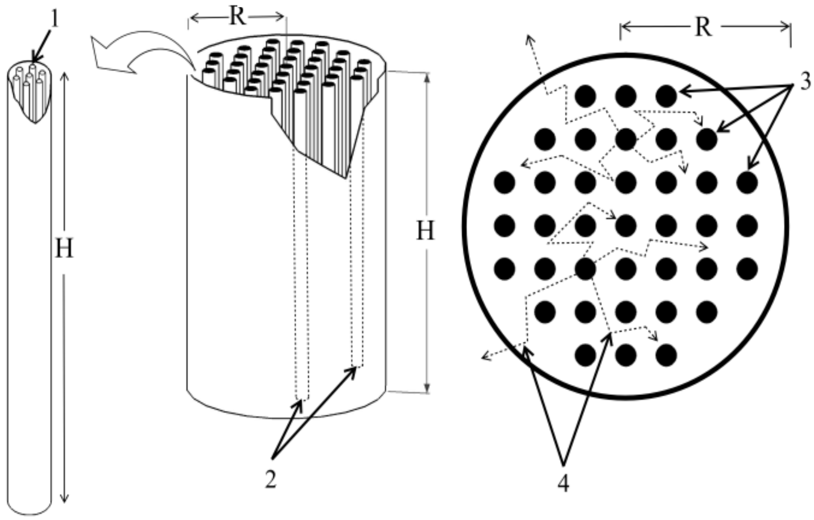
The Design of a Nuclear Reactor[1] **(Total Marks: 10)**

Uranium occurs in nature as UO_2 with only 0.720% of the uranium atoms being ^{235}U. Neutron induced fission occurs readily in ^{235}U with the emission of 2-3 fission neutrons having high kinetic energy. This fission probability will increase if the neutrons inducing fission have low kinetic energy. So by reducing the kinetic energy of the fission neutrons, one can induce a chain of fissions in other ^{235}U nuclei. This forms the basis of the power generating nuclear reactor (NR).

A typical NR consists of a cylindrical tank of height H and radius R filled with a material called moderator. Cylindrical tubes, called fuel channels, each containing a cluster of cylindrical fuel pins of natural UO_2 in solid form of height H, are kept axially in a square array. Fission neutrons, coming outward from a fuel channel, collide with the moderator, losing energy and reach the surrounding fuel channels with low enough energy to cause fission (Figs I-III). Heat generated from fission in the pin is transmitted to a coolant fluid flowing along its length. In the current problem we shall study some of the physics behind the (A) Fuel Pin, (B) Moderator and (C) NR of cylindrical geometry.

Fig-I Fig-II Fig-III



Schematic sketch of the Nuclear Reactor (NR)
Fig-I: Enlarged view of a fuel channel (1-*Fuel Pins*)
Fig-II: A view of the NR (2-*Fuel Channels*)
Fig-III: Top view of NR (3-*Square Arrangement of Fuel Channels* and 4-*Typical Neutron Paths*).
Only components relevant to the problem are shown (e.g. control rods and coolant are not shown).

A Fuel Pin

Data for UO_2	1. Molecular weight $M_w = 0.270$ kg mol^{-1}	2. Density $\rho = 1.060\times10^4$ kg m^{-3}
	3. Melting point $T_m = 3.138\times10^3$ K	4. Thermal conductivity $\lambda = 3.280$ W m^{-1} K^{-1}

A1	Consider the following fission reaction of a stationary ^{235}U after it absorbs a neutron of negligible kinetic energy. $$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{94}\mathrm{Zr} + {}^{140}\mathrm{Ce} + 2\,{}^{1}\mathrm{n} + \Delta E$$ Estimate ΔE (in MeV) the total fission energy released. The nuclear masses are: $m(^{235}\mathrm{U}) = 235.044$ u; $m(^{94}\mathrm{Zr}) = 93.9063$ u; $m(^{140}\mathrm{Ce}) = 139.905$ u; $m(^{1}\mathrm{n}) = 1.00867$ u and 1 u = 931.502 MeV c^{-2}. Ignore charge imbalance.	**0.8**
A2	Estimate N the number of ^{235}U atoms per unit volume in natural UO_2.	**0.5**
A3	Assume that the neutron flux density, $\varphi = 2.000\times10^{18}$ m^{-2} s^{-1} on the fuel is uniform. The fission cross-section (effective area of the target nucleus) of a ^{235}U nucleus is $\sigma_f = 5.400\times10^{-26}$ m^2. If 80.00% of the fission energy is available as heat, estimate Q (in W m^{-3}), the rate of heat production in the pin per unit volume. 1MeV = 1.602×10^{-13} J	**1.2**
A4	The steady-state temperature difference between the center (T_c) and the surface (T_s) of the pin can be expressed as $T_c - T_s = k\,F(Q,a,\lambda)$, where $k = 1/4$ is a dimensionless constant and a is the radius of the pin. Obtain $F(Q,a,\lambda)$ by dimensional analysis. Note that λ is the thermal conductivity of UO_2.	**0.5**

[1] *Joseph Amal Nathan (BARC) and Vijay A. Singh (ex-National Coordinator, Science Olympiads) were the principal authors of this problem. The contributions of the Academic Committee, Academic Development Group and the International Board are gratefully acknowledged.*

A5	The desired temperature of the coolant is 5.770×10^2 K. Estimate the upper limit a_u on the radius a of the pin.	**1.0**

B The Moderator

Consider the two dimensional elastic collision between a neutron of mass 1 u and a moderator atom of mass A u. Before collision all the moderator atoms are considered at rest in the laboratory frame (LF). Let $\overrightarrow{v_b}$ and $\overrightarrow{v_a}$ be the velocities of the neutron before and after collision respectively in the LF. Let $\overrightarrow{v_m}$ be the velocity of the center of mass (CM) frame relative to LF and θ be the neutron scattering angle in the CM frame. All the particles involved in collisions are moving at nonrelativistic speeds.

B1	The collision in LF is shown schematically, where θ_L is the scattering angle (Fig-IV). Sketch the collision schematically in CM frame. Label the particle velocities for 1, 2 and 3 in terms of $\overrightarrow{v_b}$, $\overrightarrow{v_a}$ and $\overrightarrow{v_m}$. Indicate the scattering angle θ.	**1.0**

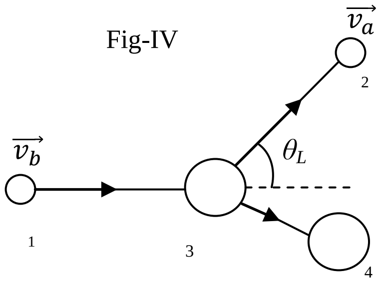


Fig-IV

Collision in the Laboratory Frame
1-*Neutron* before collision
2-*Neutron* after collision
3-*Moderator Atom* before collision
4-*Moderator Atom* after collision

B2	Obtain v and V, the speeds of the neutron and moderator atom in the CM frame after collision, in terms of A and v_b.	**1.0**
B3	Derive an expression for $G(\alpha, \theta) = E_a/E_b$, where E_b and E_a are the kinetic energies of the neutron, in the LF, before and after the collision respectively and $\alpha \equiv [(A-1)\ /\ (A+1)]^2$.	**1.0**
B4	Assume that the above expression holds for D_2O molecule. Calculate the maximum possible fractional energy loss $f_l \equiv \frac{E_b - E_a}{E_b}$ of the neutron for the D_2O (20 u) moderator.	**0.5**

C The Nuclear Reactor

To operate the NR at any constant neutron flux ψ (steady state), the leakage of neutrons has to be compensated by an excess production of neutrons in the reactor. For a reactor in cylindrical geometry the leakage rate is $k_1\,[(2.405/R)^2 + (\pi/H)^2]\,\psi$ and the excess production rate is $k_2\,\psi$. The constants k_1 and k_2 depend on the material properties of the NR.

C1	Consider a NR with $k_1 = 1.021\times10^{-2}$ m and $k_2 = 8.787\times10^{-3}$ m^{-1}. Noting that for a fixed volume the leakage rate is to be minimized for efficient fuel utilization, obtain the dimensions of the NR in the steady state.	**1.5**
C2	The fuel channels are in a square arrangement (Fig-III) with the nearest neighbour distance 0.286 m. The effective radius of a fuel channel (if it were solid) is 3.617×10^{-2} m. Estimate the number of fuel channels F_n in the reactor and the mass M of UO_2 required to operate the NR in steady state.	**1.0**