Theoretical Question 3

Part A

Neutrino Mass and Neutron Decay

A free neutron of mass m_n decays at rest in the laboratory frame of reference into three non-interacting particles: a proton, an electron, and an anti-neutrino. The rest mass of the proton is m_p, while the rest mass of the anti-neutrino m_ν is assumed to be nonzero and much smaller than the rest mass of the electron m_e. Denote the speed of light in vacuum by c. The measured values of mass are as follows:

$$m_n = 939.56563 \text{ MeV}/c^2,\ m_p = 938.27231 \text{ MeV}/c^2,\ m_e = 0.5109907 \text{ MeV}/c^2$$

In the following, all energies and velocities are referred to the laboratory frame. Let E be the total energy of the electron coming out of the decay.

(a) Find the maximum possible value $E_{\max}$ of E and the speed v_{m} of the anti-neutrino when $E = E_{\max}$. Both answers must be expressed in terms of the rest masses of the particles and the speed of light. Given that $m_\nu < 7.3 \text{ eV}/c^2$, compute $E_{\max}$ and the ratio v_{m}/c to 3 significant digits. (4.0 points)

Part B

Light Levitation

A transparent glass hemisphere with radius R and mass m has an index of refraction n. In the medium outside the hemisphere, the index of refraction is equal to one. A parallel beam of monochromatic laser light is incident uniformly and normally onto the central portion of its planar surface, as shown in Figure 3. The acceleration of gravity $\vec{g}$ is vertically downwards. The radius δ of the circular cross-section of the laser beam is much smaller than R. Both the glass hemisphere and the laser beam are axially symmetric with respect to the z-axis.

The glass hemisphere does not absorb any laser light. Its surface has been coated with a thin layer of transparent material so that reflections are negligible when light enters and leaves the glass hemisphere. The optical path traversed by laser light passing through the non-reflecting surface layer is also negligible.

(b) Neglecting terms of the order $(\delta/R)^3$ or higher, find the laser power P needed to balance the weight of the glass hemisphere. (4.0 points)

Hint: $\cos\theta \approx 1 - \theta^2/2$ when θ is much smaller than one.

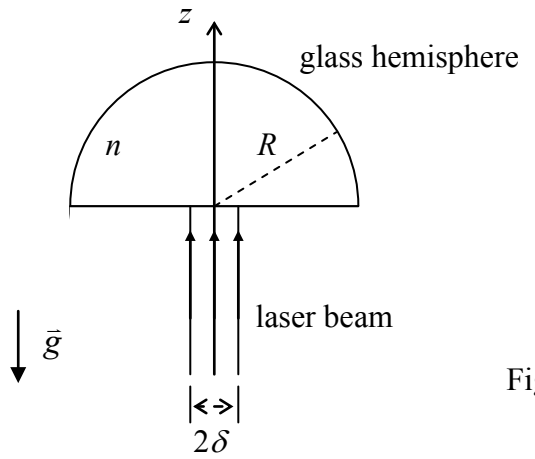

Figure 3

[Answer Sheet] Theoretical Question 3

Wherever requested, give *each answer* as *analytical expressions* followed by *numerical values* and *units*. *For example:* area of a circle $A = \pi r^2 = 1.23\ \mathrm{m}^2$.

Neutrino Mass and Neutron Decay

(a) (Give expressions in terms of rest masses of the particles and the speed of light)

The maximum energy of the electron is (*expression and value*)

E_{max} =

The ratio of anti-neutrino's speed at $E = E_{max}$ to c is (*expression and value*)

v_m / c =

Light Levitation

(b) The laser power needed to balance the weight of the glass hemisphere is

P =