

Solutions to Experimental Problems

Part A: Optical Properties of Laser Diode

Question A-(1) (Total 1.5 point)

Measure, tabulate, and plot the \mathcal{J} vs. I curve.

a. Data (0.3 pts.) : Proper data table marked with variables and units.

Table A-(1): Data for \mathcal{J} vs. I .

I (mA)	9.2	15.2	19.5	21.6	22.2	22.7	23.0	23.4	23.8
\mathcal{J} (V)	0.00	0.01	0.02	0.03	0.05	0.06	0.09	0.12	0.30
I (mA)	24.2	24.6	25.0	25.4	25.8	26.2	26.6	27.0	27.4
\mathcal{J} (V)	0.66	1.02	1.41	1.88	2.23	2.64	3.04	3.36	3.78
I (mA)	27.8	28.2	28.6	29.0	29.4	29.8	30.2	30.5	31.0
\mathcal{J} (V)	4.12	4.48	4.79	5.13	5.44	5.72	6.05	6.25	6.55
I (mA)	31.4	31.8	32.2	32.6	33.0	33.4	33.8	34.2	34.6
\mathcal{J} (V)	6.75	6.99	7.22	7.40	7.60	7.78	7.93	8.07	8.14
I (mA)	35.0	35.5	36.0	36.5	37.0	37.6	38.0	38.6	
\mathcal{J} (V)	8.18	8.20	8.22	8.24	8.24	8.25	8.26	8.27	

Current error : ± 0.1 mA; Voltage error : ± 0.01 V

- b. Plotting (0.3 pts.): Proper sizes of scales, and units for abscissa and ordinate that bear relation to the accuracy and range of the experiment.
- c. Curve (0.9 pts.): Proper data and adequate line shape
- As shown in Fig. A-1. Start $\sim 0 \rightarrow$ Threshold \rightarrow Linear \rightarrow Saturate.

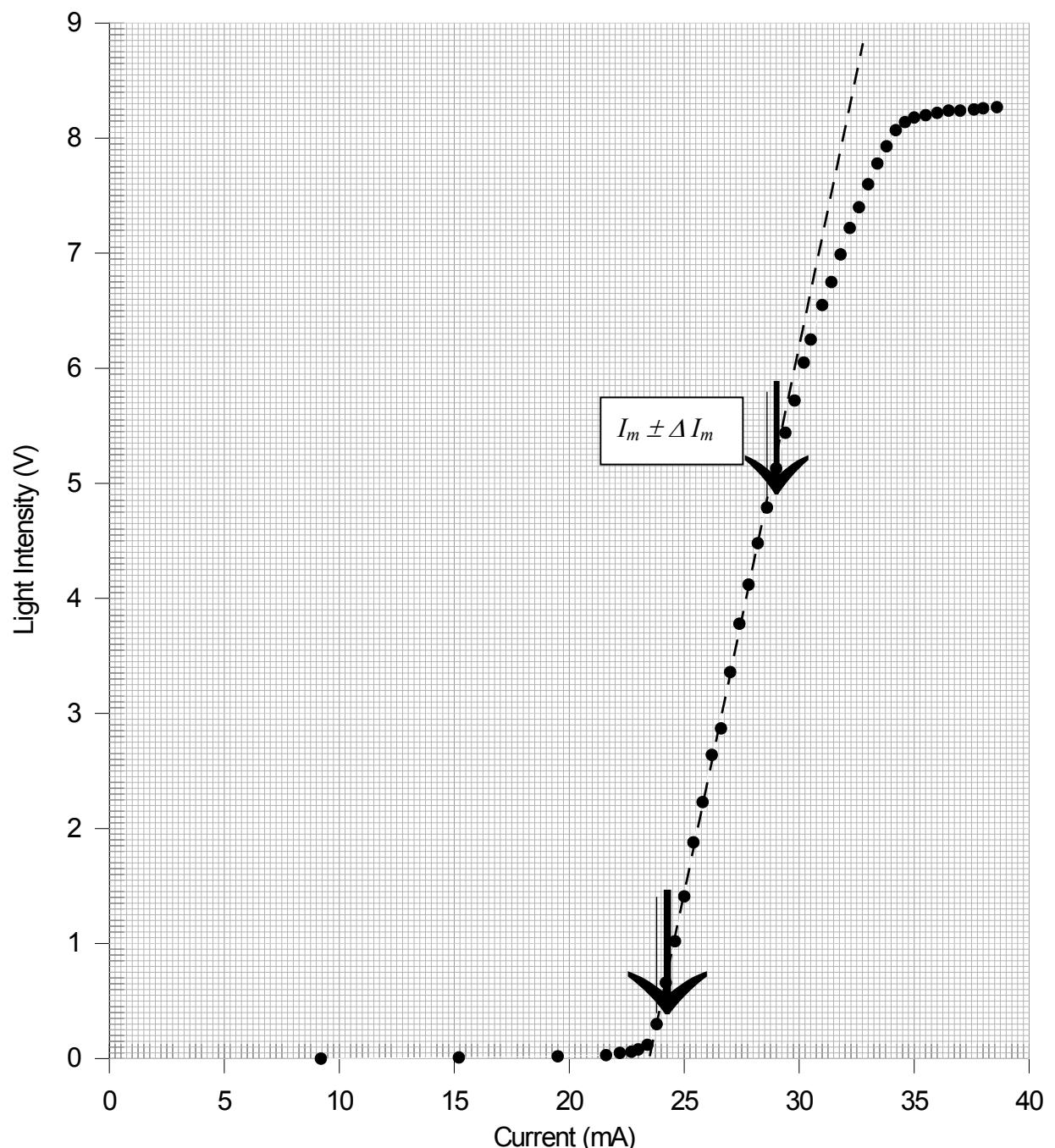


Fig. A-1 Graph of light intensity J versus current I

Question A-(2) (Total 3.5 points)

Estimate the maximum current I_m with uncertainty in the linear region of the \mathcal{J} - I . Mark the linear region on the \mathcal{J} - I curve figure by using arrows (\downarrow) and determine the threshold current I_{th} with detailed error analysis.

- a. Linear region marking (0.5 pts.) in Fig. A-1.
- b. Least-square method or eye-ball fitting with ruler and error analysis (1.5 pt.)

Least-square fitting	Eye-ball fitting with ruler
Error bar in graph 0.0x mA (0.5 pts)	Error bar in graph 0.x mA (0.5 pts)
Least-square method (0.5 pts)	Expanded scale graph (0.5 pts)
Error analysis (0.5 pts)	draw three lines for error analysis(0.5 pts)

- c. $I_m \pm \Delta I_m$ (0.5 pts.): Adequate value of I_m (0.3 pts.) and error($\pm \Delta I_m$) (0.2 pts.) from the linear region of \mathcal{J} -I curve.
- d. Adequate value of I_{th} with error (1.0 pts.)

$$I_{th} = (21\text{--}26) \pm (0.01 \text{ or } 0.2 \text{ for single value}) \text{ mA}$$

Adequate value of I_{th} (0.5 pts.) and error ($\pm \Delta I_{th}$) (0.5 pts.)

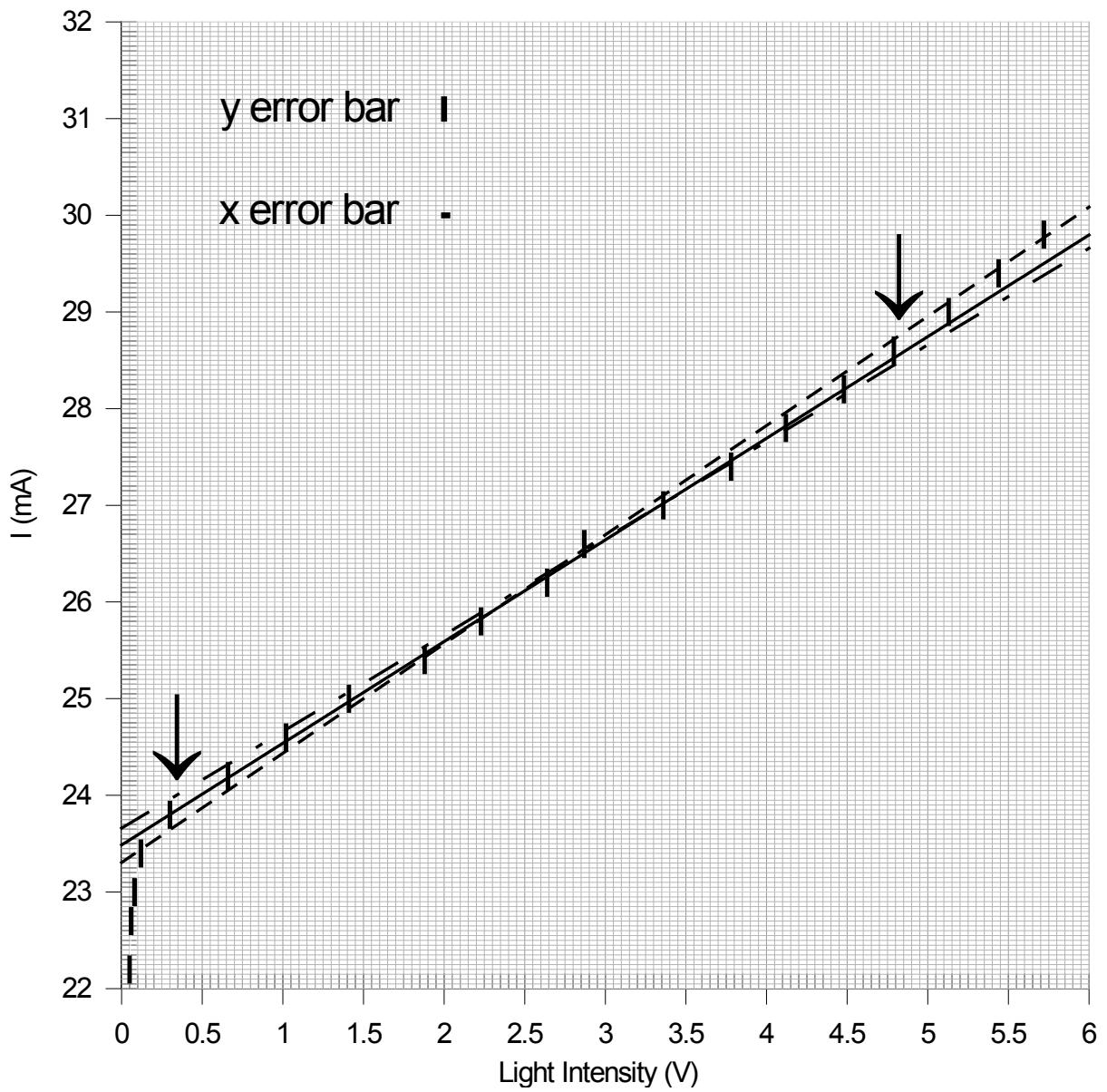


Fig. A-2 Straight lines and extrapolations

Appendix :

◎A1-1

- Least-Square Method :

$$I = mJ + b \rightarrow b = I_{\text{th}}$$

For $y = mx + b$

	$y: I(\text{mA})$	$x: J$	xy	x^2	$y(x) = mx + b$	$(y - y(x))^2$
1	23.8	0.30	7.14	0.090	23.7937	3.969E-05
2	24.2	0.66	15.972	0.4356	24.17134	0.000821
3	24.6	1.02	25.092	1.0404	24.54898	0.00260
4	25.0	1.41	35.25	1.9881	24.95809	0.00176
5	25.4	1.88	47.752	3.5344	25.45112	0.00261
6	25.8	2.23	57.534	4.9729	25.81827	0.000334
7	26.2	2.64	69.168	6.9696	26.24836	0.00234
8	26.6	3.04	80.864	9.2416	26.66796	0.00462
9	27.0	3.36	90.72	11.2896	27.00364	1.325E-05
10	27.4	3.78	103.572	14.2884	27.44422	0.00196
11	27.8	4.12	114.536	16.9744	27.80088	7.744E-07
12	28.2	4.48	126.336	20.0704	28.17852	0.000461
13	28.6	4.79	136.994	22.9441	28.50371	0.00927
	$\Sigma y =$ 340.6	$\Sigma x =$ 33.71	$\Sigma xy =$ 910.93	$\Sigma x^2 =$ 113.840		$\Sigma (y - y(x))^2 =$ 0.0268

$$\Delta = N\Sigma x^2 - (\Sigma x)^2 = 13(113.840) - (33.71)^2 = 343.556$$

$$m = \frac{1}{\Delta} (\Sigma xy - \Sigma x \Sigma y) = \frac{13(910.93) - (33.71)(340.6)}{343.556} = 1.049$$

$$b = \frac{1}{\Delta} (\Sigma x^2 \Sigma y - \Sigma x \Sigma xy) = \frac{(113.840)(340.6) - (33.71)(910.93)}{343.556} = 23.479$$

$$\sigma_y = \sqrt{\frac{\sum(y - y(x))^2}{N-2}} = \sqrt{\frac{0.0268}{13-2}} = 0.049$$

$$\sigma = \sqrt{(\sigma_y)^2 + \left(\frac{dy}{dx}\sigma_x\right)^2} = \sqrt{(0.049)^2 + (1.049 \times 0.005)^2} = 0.049$$

$$\sigma_m = \sqrt{\frac{N\sigma^2}{\Delta}} = \sqrt{\frac{13 \times 0.049^2}{343.556}} = 0.0095$$

$$\sigma_b = \sqrt{\frac{\sigma^2}{\Delta} \sum x^2} = 0.049 \times \sqrt{\frac{113.840}{343.556}} = 0.028$$

$$I_{th} = 23.48 \pm 0.03 \text{ mA}$$

◎A1-2

- Eye-balling Method :

$$I = mJ + b \rightarrow b = I_{th}$$

For $y = mx + b$

Line 1: $y = 1.00 x + 23.66$

Line 2: $y = 1.05 x + 23.48$

Line 3: $y = 1.13 x + 23.31$

$$I_{th}(\text{av.}) = 23.48$$

$$I_{th}(\text{std.}) = 0.18$$

$$I_{th} = 23.5 \pm 0.2 \text{ mA}$$

Part B: Optical Properties of Nematic Liquid Crystal

Electro-optical switching characteristic of 90° TN LC cell

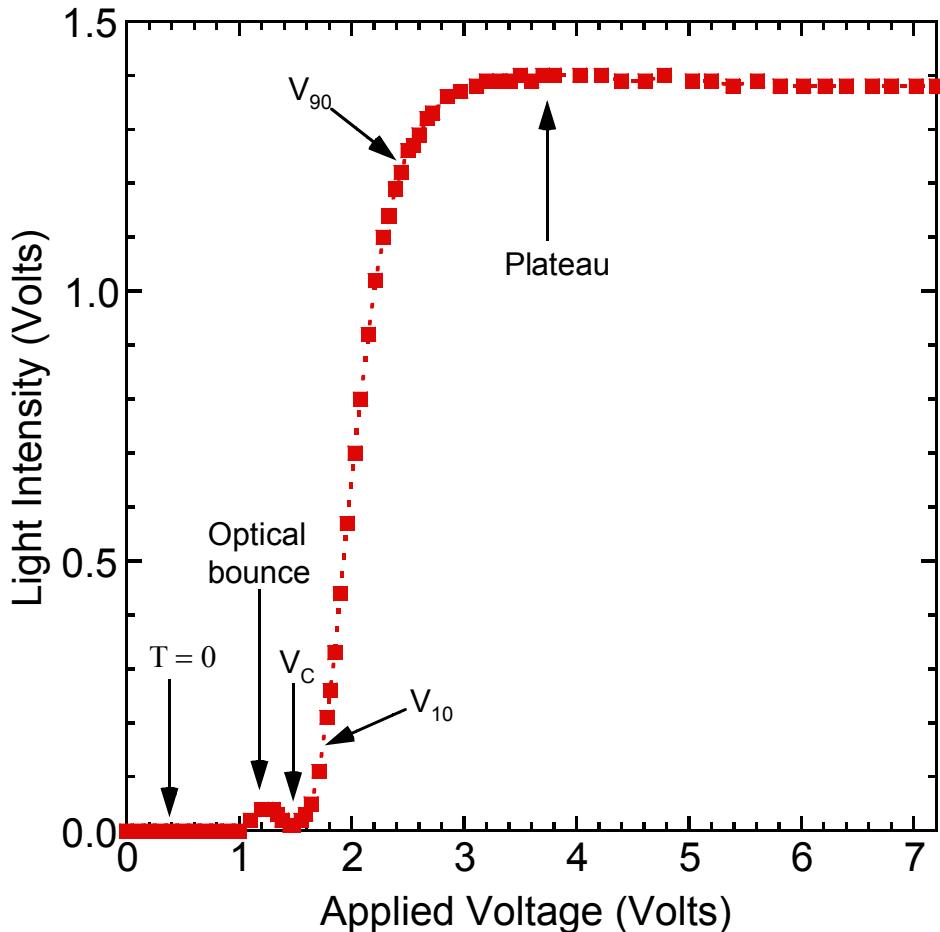
Question B-(1) (5.0 points)

Measure, tabulate, and plot the electro-optical switching curve (I vs. V_{rms} curve) of the NB 90° TN LC, and find its switching slope γ , where γ is defined as $(V_{90} - V_{10})/V_{10}$.

- a. Proper data table marked with variables and units. (0.3 pts)

Applied voltage (Volts)	Light intensity (Volts)	Applied voltage (Volts)	Light intensity (Volts)
0.00	0.00	2.44	1.22
0.10	0.00	2.50	1.26
0.20	0.00	2.55	1.27
0.30	0.00	2.60	1.29
0.40	0.00	2.67	1.32
0.50	0.00	2.72	1.33
0.60	0.00	2.85	1.36
0.70	0.00	2.97	1.37
0.80	0.00	3.11	1.38
0.90	0.00	3.20	1.39
1.00	0.00	3.32	1.39
1.10	0.02	3.41	1.39
1.20	0.04	3.50	1.40
1.24	0.04	3.60	1.39
1.30	0.04	3.70	1.40
1.34	0.03	3.80	1.40
1.38	0.02	4.03	1.40
1.45	0.01	4.22	1.40
1.48	0.01	4.40	1.39
1.55	0.02	4.61	1.39
1.59	0.03	4.78	1.40
1.64	0.05	5.03	1.39
1.71	0.11	5.20	1.39
1.78	0.21	5.39	1.38
1.81	0.26	5.61	1.39
1.85	0.33	5.81	1.38
1.90	0.44	6.02	1.38
1.96	0.57	6.21	1.38
2.03	0.70	6.40	1.38
2.08	0.80	6.63	1.38
2.15	0.92	6.80	1.38
2.21	1.02	7.02	1.38
2.28	1.10	7.20	1.38
2.33	1.14		
2.39	1.19		

- b. Properly choose the size of scales and units for abscissa and ordinate that bears the relation to the accuracy and range of the experiment. (0.3 pts)
- c. Correct measurement of the light intensity (\mathcal{I}) as a function of the applied voltage (V_{rms}) and adequate \mathcal{I} - V_{rms} curve plot.
 - The intensity of the transmission light is smaller than 0.05 Volts in the normally black mode. (0.4 pts)
 - There is a small optical bounce before the external applied voltage reaches the critical voltage. (0.8 pts)
 - The intensity of the transmission light increases rapidly and abruptly when the external applied voltage exceeds the critical voltage. (0.4 pts)
 - The intensity of the transmission light displays the plateau behavior as the external applied voltage exceeds 3.0 Volts. (0.4 pts)



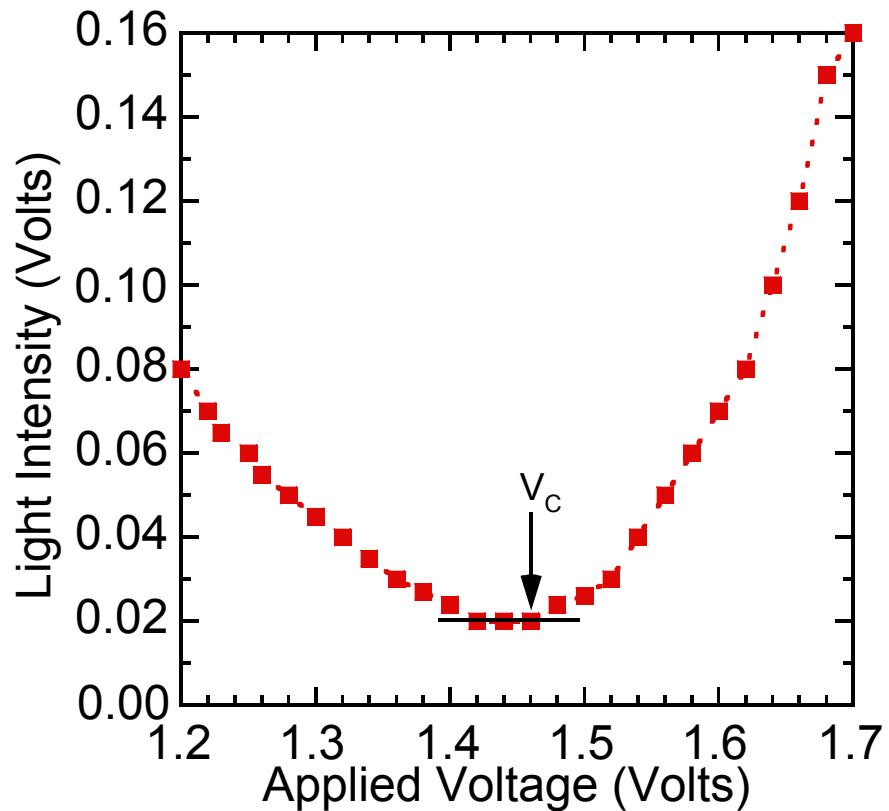
- d. Adequate value of γ with error.
 - Find the maximum value of the light intensity in the region of the applied voltage between 3.0 and 7.2 Volts (0.6 pts)
 - Determine the value of 90 % of the maximum light intensity. Obtain the value of the applied voltage V_{90} by interpolation. (0.6 pts)

- Determine the value of 10 % of the maximum light intensity. Obtain the value of the applied voltage V_{10} by interpolation. (0.6 pts)
- Correct $\gamma \pm \Delta\gamma$ value, $(0.42 \sim 0.44) \pm 0.02$. (0.4+0.2 pts)

Question B-(2) (Total 2.5 points)

Determine the critical voltage V_c of this NB 90° TN LC cell. Show explicitly with graph how you determine the value V_c .

- Adequate value of V_C with error, $V_C \pm \Delta V_C$.
 - Make the expanded scale plot and take more data points in the region of V_C . (0.8 pts)
 - Determine the value of V_C when the intensity of the transmission light increases rapidly and abruptly. (0.7 pts)
 - Correct $V_C \pm \Delta V_C$ value, $(1.20 \sim 1.50) \pm 0.01$ Volts. (0.8+0.2 pts)



(The data shown in this graph do not correspond to the data shown on the previous page. This graph only shows how to obtain V_c .)

**Part C: Optical Properties of Nematic Liquid Crystal :
Electro-optical switching characteristic of parallel aligned LC cell**

Question C-(1) (2.5 points)

Assume that the wavelength of laser light 650 nm, LC layer thickness 7.7 μm , and approximate value of $\Delta n \approx 0.25$ are known. From the experimental data T_{\perp} and T_{\parallel} obtained above, calculate the accurate value of the phase retardation δ and accurate value of birefringence Δn of this LC cell at V=0.

a. Adequate value of δ and Δn with error.

- Take and average the values of T_{\parallel} . (0.3 pts)
- Take and average the values of T_{\perp} . (0.3 pts)
- Determine the value of order m. (0.9 pts)
- Correct δ value, 15.7 ~ 18.2. (0.5 pts)
- Correct Δn value, 0.20 ~ 0.24 (0.5 pts)

$$T_{\parallel} = \frac{0.31 + 0.31 + 0.31}{3} = 0.31 \pm 0.01 \text{ Volts}$$

$$T_{\perp} = \frac{1.04 + 1.03 + 1.04}{3} = 1.04 \pm 0.01 \text{ Volts}$$

$$\tan \frac{\delta}{2} = \pm \frac{\sqrt{T_{\perp}}}{\sqrt{T_{\parallel}}} = -1.83^* \quad \therefore \delta = 4.14 + 2m\pi \quad (\text{or } -2.14 + 2m\pi)$$

$$\delta = \frac{2\pi d \Delta n}{\lambda} = \frac{2\pi \times 7.7 \times 0.25}{0.65} = 18.61$$

$$\text{Take } m = 2(\text{or}3) \quad \therefore \delta = 16.70(5.32\pi)$$

$$\text{From } \delta = \frac{2\pi d \Delta n}{\lambda} \quad \therefore \Delta n = \frac{\delta \lambda}{2\pi d} = 0.22$$

$$\text{Accepted value for } \therefore \Delta n = (0.20 \sim 0.24)$$

*If $\tan \frac{\delta}{2} = 1.83$, the value for δ will be either 4.68π or 6.68π , which is not consistent with data figure of problem C-(2).

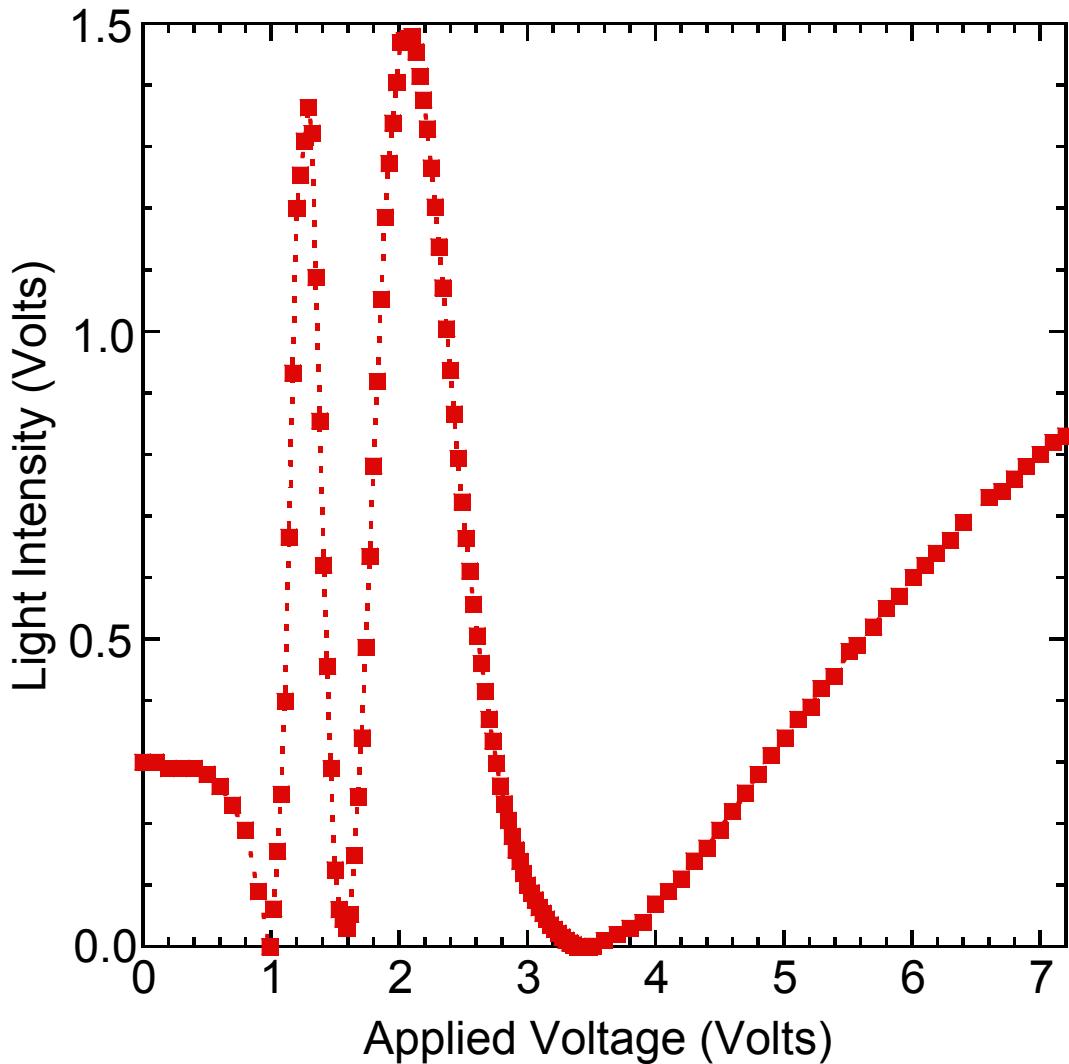
Question C-(2) (Total 3.0 points)

Measure, tabulate, and plot the electro-optical switching curve for T_{\parallel} of this parallel aligned LC cell in the $\theta = 45^{\circ}$ configuration.

a. Proper data table marked with variables and units. (0.3 pts)

Applied voltage (Volts)	Light intensity (Volts)	Applied voltage (Volts)	Light intensity (Volts)	Applied voltage (Volts)	Light intensity (Volts)
0.00	0.30	2.01	1.47	3.33	0.00
0.10	0.30	2.04	1.48	3.36	0.00
0.20	0.29	2.07	1.48	3.39	0.00
0.30	0.29	2.10	1.48	3.42	0.00
0.40	0.29	2.13	1.45	3.45	0.00
0.50	0.28	2.16	1.42	3.48	0.00
0.60	0.26	2.19	1.38	3.51	0.00
0.70	0.23	2.22	1.33	3.60	0.01
0.80	0.19	2.25	1.27	3.70	0.02
0.90	0.09	2.28	1.20	3.80	0.03
0.99	0.00	2.31	1.14	3.90	0.04
1.02	0.06	2.34	1.07	4.00	0.07
1.05	0.16	2.37	1.00	4.10	0.09
1.08	0.25	2.40	0.94	4.20	0.11
1.11	0.40	2.43	0.87	4.30	0.14
1.14	0.67	2.46	0.79	4.40	0.16
1.17	0.93	2.49	0.72	4.50	0.19
1.20	1.25	2.52	0.66	4.60	0.22
1.26	1.31	2.55	0.61	4.70	0.25
1.29	1.36	2.58	0.56	4.80	0.28
1.32	1.32	2.61	0.51	4.90	0.31
1.35	1.09	2.64	0.46	5.01	0.34
1.38	0.85	2.67	0.42	5.11	0.37
1.41	0.62	2.70	0.37	5.21	0.39
1.44	0.46	2.73	0.33	5.29	0.42
1.47	0.29	2.76	0.30	5.39	0.44
1.50	0.13	2.79	0.26	5.51	0.48
1.53	0.06	2.82	0.23	5.57	0.49
1.59	0.03	2.85	0.21	5.70	0.52
1.62	0.05	2.88	0.18	5.80	0.55
1.65	0.15	2.91	0.16	5.90	0.57
1.68	0.24	2.94	0.14	6.01	0.60
1.71	0.34	2.97	0.12	6.10	0.62
1.74	0.49	3.00	0.09	6.19	0.64
1.77	0.63	3.06	0.08	6.30	0.66
1.80	0.78	3.09	0.06	6.40	0.69
1.83	0.92	3.12	0.05	6.60	0.73
1.86	1.05	3.18	0.04	6.70	0.74
1.89	1.19	3.21	0.03	6.80	0.76
1.92	1.27	3.24	0.02	7.00	0.80
1.95	1.34	3.27	0.02	7.20	0.83
1.98	1.40	3.30	0.01		

- b. Properly choose the size of scales and units for abscissa and ordinate that bears the relation to the accuracy and range of the experiment. (0.3 pts)
- c. Correct measurement of the T_{\parallel} as a function of the applied voltage (V_{rms}) and adequate T_{\parallel} - V_{rms} curve plot.
 - Three minima and two sharp maxima. (1.5 pts)
 - Maxima values within 15% from each other. (0.5 pts)
 - Minima are less than the values of 0.1 Volts. (0.4 pts)

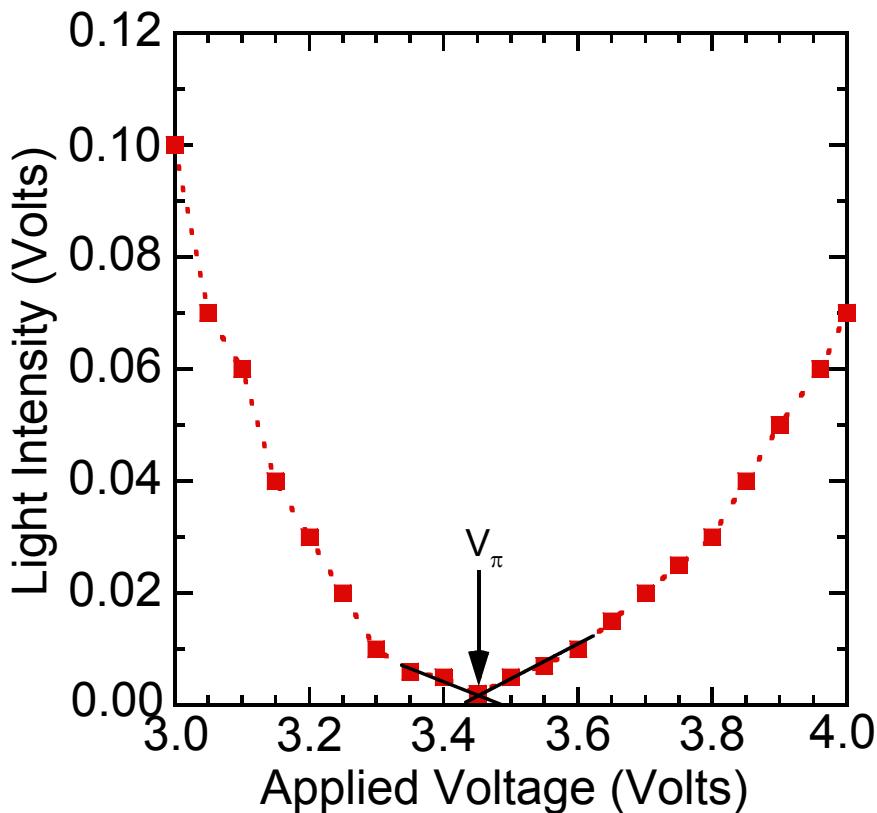


Question C-(3) (Total 2.0 points)

From the electro-optical switching data, find the value of the external applied voltage V_π .

a. Adequate value of V_π with error.

- Make the expanded scale plot and take more data points in the region of V_π . (0.3 pts)
- Indicate the correct minimum of V_π . (0.8 pts)
- Obtain the value of V_π by interpolation or rounding. (0.5 pts)
- Correct V_π value : $(3.2 \sim 3.5) \pm 0.01$ Volts. (0.2+0.2 pts)



Marking Scheme

Part A: Optical Properties of Laser Diode

No.	Contents	Sub Scores	Total Scores
A(1)	Measure, tabulate, and plot the \mathcal{J} vs. I curve.		1.5 pts.
a	Proper data table marked with variables and units.	0.3	
b	Proper sizes of scales, and units for abscissa and ordinate that bear relation to the accuracy and range of the experiment.	0.3	
c	Proper data and adequate curve plotting (Fig. A-1)	0.9	
A(2)	Estimate the maximum current I_m with uncertainty in the linear region of the \mathcal{J} vs. I curve. Mark the linear region on the \mathcal{J} - I curve figure by using arrows (\downarrow) and determine the threshold current I_{th} with uncertainty.		3.5 pts.
a	Mark the linear region.	0.5	
b	Least-square fit or eye-balling with ruler and error analysis	1.5	
c	Obtain $I_m \pm \Delta I_m$ properly	0.5	
d	Adequate value of $I_{th} \pm \Delta I_{th}$	1.0	

Part B: Optical Properties of Nematic Liquid Crystal

Electro-optical switching characteristic of 90° TN LC cell

No.	Contents	Sub Scores	Total Scores
B-(1)	Measure, tabulate, and plot the electro-optical switching curve (\mathcal{J} vs. V_{rms} curve) of the NB 90° TN LC, and find its switching slope γ , where γ is defined as $(V_{90} - V_{10})/V_{10}$.		5.0 pts.
a	Proper data table marked with variables and units.	0.3	
b	Properly choose the size of scales and units for abscissa and ordinate that bears the relation to the accuracy and range of the experiment.	0.3	
c	Correct measurement of the light intensity (\mathcal{J}) as a function of the applied voltage (V_{rms}) and adequate \mathcal{J} - V_{rms} curve plot.		
	■ The intensity of the transmission light reaches zero value in the normally black mode.	0.4	
	■ There is a small optical bounce before the external applied voltage reaches the critical voltage.	0.8	
	■ The intensity of the transmission light increases rapidly and	0.4	

	abruptly when the external applied voltage exceeds the critical voltage.		
	■ The intensity of the transmission light displays the plateau behavior as the external applied voltage exceeds 3.0 Volts.	0.4	
d	Adequate value of γ with error, $\gamma \pm \Delta\gamma$.		
	■ Correctly analyzing the maximum light intensity.	0.6	
	■ Correctly analyzing the value of V_{90} .	0.6	
	■ Correctly analyzing the value of V_{10} .	0.6	
	■ Correct $\gamma \pm \Delta\gamma$ value, $(0.42 \sim 0.44) \pm 0.02$.	0.6	
B-(2)	Determine the critical voltage V_c of this NB 90° TN LC cell. Show explicitly with graph how you determine the value V_c .		2.5 pts.
	Adequate value of V_c with error, $V_c \pm \Delta V_c$.		
	■ Make the expanded scale plot and take more data points in the region of V_c .	0.8	
	■ Correctly analyzing the value of V_c .	0.7	
	■ Correct $V_c \pm \Delta V_c$ value, $(1.2 \sim 1.5) \pm 0.01$ Volts.	1.0	

Part C: Optical Properties of Nematic Liquid Crystal :
Electro-optical switching characteristic of parallel aligned LC cell

No.	Contents	Sub Scores	Total Scores
C-(1)	Assume that the wavelength of laser light 650 nm, LC layer thickness 7.7 μm , and approximate value of $\Delta n \approx 0.25$ are known. From the experimental data T_\perp and T_\parallel obtained above, calculate the accurate value of the phase retardation δ and accurate value of birefringence Δn of this LC cell at $V=0$.		2.5 pts.
	Adequate value of δ and Δn with error.		
	■ Correctly analyzing the values of T_\parallel .	0.3	
	■ Correctly analyzing the values of T_\perp .	0.3	
	■ Correctly determining the value of order m.	0.9	
	■ Correct δ value, $17.7 \sim 18.2$.	0.5	
	■ Correct Δn value, $0.23 \sim 0.25$.	0.5	
C-(2)	Measure, tabulate, and plot the electro-optical switching curve for T of this parallel aligned LC cell in the $\theta = 45^\circ$ configuration.		3.0 pts.
a	Proper data table marked with variables and units.	0.3	
b	Properly choose the size of scales and units for abscissa and ordinate	0.3	

	that bears the relation to the accuracy and range of the experiment.		
c	Correct measurement of the T_{\parallel} as a function of the applied voltage (V_{rms}) and adequate $T_{\parallel} - V_{rms}$ curve plot.		
	■ Three minima and two sharp maxima.	1.5	
	■ Maxima values within 15 % from each other.	0.5	
	■ Minima are less than the values of 0.1 Volts.	0.4	
C-(3)	From the electro-optical switching data, find the value of the external applied voltage V_{π}		2.0 pts.
	Adequate value of V_{π} with error.		
	■ Make the expanded scale plot and take more data points in the region of V_{π} .	0.3	
	■ Indicate the correct minimum of V_{π} .	0.8	
	■ Correctly analyzing the value of V_{π} .	0.5	
	■ Correct $V_{\pi} \pm \Delta V_{\pi}$ value, $(3.2 \sim 3.5) \pm 0.1$ Volts.	0.4	