33rd
INTERNATIONAL
PHYSICS OLYMPIAD
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BALI - INDONESIA

THEORETICAL COMPETITION

Tuesday, July 23rd, 2002

Solution I: Ground-Penetrating Radar

1. Speed of radar signal in the material v_m:

$$\omega t - \beta z = \text{constant} \rightarrow \beta z = \text{-constant} + \omega t \quad \text{(0.2 pts)}$$

$$v_m = \frac{\omega}{\beta}$$

$$v_m = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[(1+\frac{\sigma^2}{\varepsilon^2\omega^2})^{1/2}+1\right]\right\}^{1/2}} \quad \text{(0.4 pts)}$$

$$v_m = \frac{1}{\left\{\frac{\mu\varepsilon}{2}(1+1)\right\}^{1/2}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(0.4 pts)}$$

2. The maximum depth of detection (skin depth, δ) of an object in the ground is inversely proportional to the attenuation constant:

(0.5 pts) **(0.3 pts)** **(0.2 pts)**

$$\delta = \frac{1}{a} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)^{1/2}-1\right]\right\}^{1/2}} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega^2}\right)-1\right]\right\}^{1/2}} = \frac{1}{\omega\left\{\frac{\mu\varepsilon}{2}.\frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega^2}\right\}^{1/2}}$$

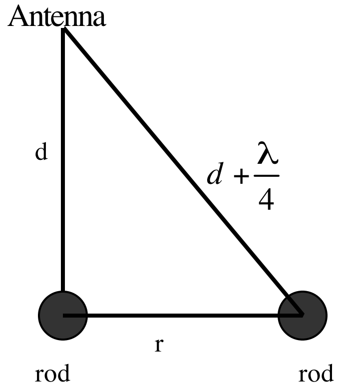
$$\delta = \left(\frac{2}{\sigma}\right)\left(\frac{\varepsilon}{\mu}\right)^{1/2}.$$

Numerically $\delta = \frac{\left(5.31\sqrt{\varepsilon_r}\right)}{\sigma}$ m, where σ is in mS/m. **(0.5 pts)**

For a medium with conductivity of 1.0 mS/m and relative permittivity of 9, the skin depth

$$\delta = \frac{\left(5.31\sqrt{9}\right)}{1.0} = 15.93 \text{ m}$$ **(0.3 pts) + (0.2 pts)**

3. Lateral resolution:



$$r^2 + d^2 = (d + \frac{\lambda}{4})^2$$

$$r = \left(\frac{\lambda d}{2} + \frac{\lambda^2}{16}\right)^{1/2}$$

(1.0 pts)

r =0.5 m, d =4 m: $\frac{1}{2} = \left(\frac{4\lambda}{2} + \frac{\lambda^2}{16}\right)^{1/2}$, $\lambda^2 + 32\lambda - 4 = 0$ **(0.5 pts)**

The wavelength is λ=0.125 m. **(0.3 pts) + (0.2 pts)**

The propagation speed of the signal in medium is

$$v_m = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$v_m = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{0.3}{\sqrt{\varepsilon_r}} \text{ m/ns} \quad \text{, where } c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \text{ and } \mu_r = 1$$

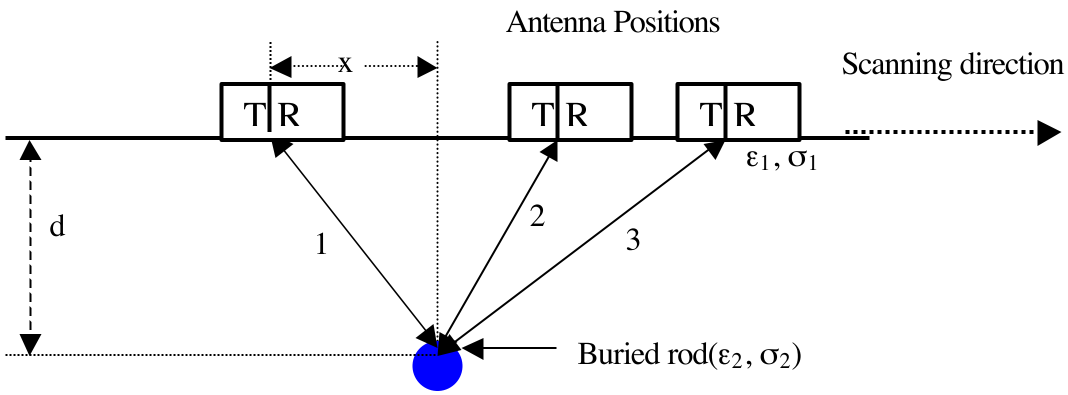
$$v_m = 0.1 \text{ m/ns} = 10^8 \text{ m/s}$$ **(0.5 pts)**

The minimum frequency need to distinguish the two rods as two separate objects is

$$f_{\min} = \frac{v}{\lambda}$$ **(0.5 pts)**

$$f_{\min} = \frac{\frac{0.3}{\sqrt{9}}}{0.125} x10^9 \text{ Hz} = 800 \text{ MHz}$$ **(0.3 pts) + (0.20 pts)**

4. Path of EM waves for some positions on the ground surface

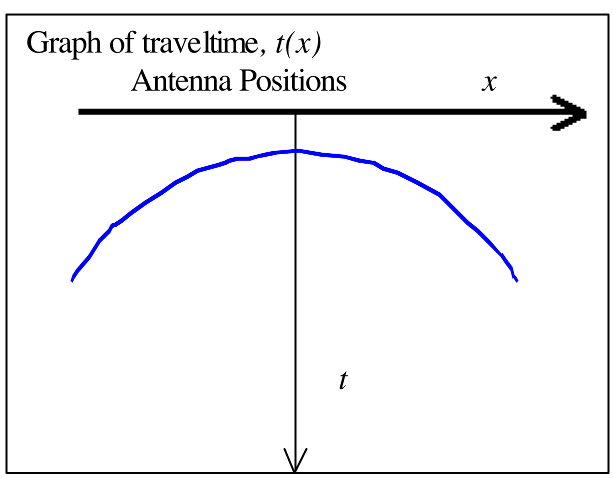


The traveltime as function of x is

$$\left(\frac{t\ v}{2}\right)^2 = d^2 + x^2\ ,$$ **(1.0 pts)**

$$t(x) = \sqrt{\frac{4d^2 + 4x^2}{v}}$$ **(1.0 pts)**

$$t(x) = \frac{2\sqrt{\varepsilon_{1r}}}{0.3}\sqrt{d^2 + x^2}$$



For x =0 **(1.0 pts)**

100 = 2×(3/0.3) d

d = 5 m **(0.5 pts)**