

Solution

Part 1

Theory:

Consider a small mass m of the liquid at the surface (Figure 4).
At dynamic equilibrium

$$N \cos \theta = mg$$

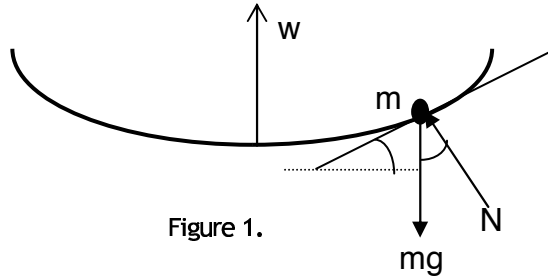
and

$$N \sin \theta = m\omega^2 x$$

Therefore:

$$\tan \theta = \frac{\omega^2 x}{g}.$$

Figure 1.



The profile of the liquid surface can be found as follows:

$$\tan \theta = \frac{dy}{dx}, \quad \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

so that

$$y = \frac{\omega^2 x^2}{2g} + y_0$$

where y_0 is the height at $x = 0$.

At a certain point $x = x_0$, height of the liquid h_0 would be the same as if it not rotating.

In this case,

$$h_0 = y_0 + \frac{\omega^2 x_0^2}{2g} \quad (1)$$

and,

$$x_0^2 = \frac{2g(h_0 - y_0)}{\omega^2}.$$

Since the volume of the liquid is constant,

$$\pi R^2 h_0 = \int y(2\pi x dx) = 2\pi \int (y_0 + \frac{\omega^2 x^2}{2g}) x dx,$$

$$y_0 = h_0 - \frac{\omega^2 R^2}{4g} \quad (2)$$

From Eq.1 and Eq.2 one obtains

$$x_0 = \frac{R}{\sqrt{2}}.$$

Experiment:

$2R(\text{mm})$	$x_0(\text{mm})$	$h_0(\text{mm})$	$H(\text{mm})$
145	51	30	160

$H-h_0=130 \text{ mm}$

Measure 10T at small speeds and measure 15T-20T at high speeds.

Use $\tan(2\theta) = \frac{x}{H-h_0}$ and $w = \frac{2\pi}{T}$.

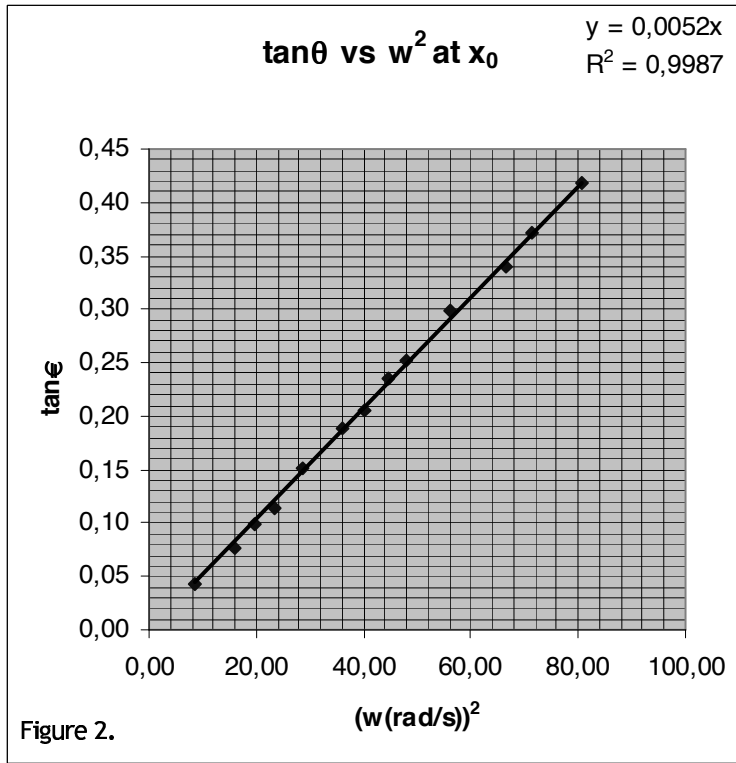
$2R(\text{mm})$	$x_0(\text{mm})$	$h_0(\text{mm})$	$H(\text{mm})$	$H-h_0(\text{mm})$
145	51	30	160	130

$x(\text{mm})$	10T(s)	$w(\text{rad/s})$	$\tan(2\theta)$	$\theta(\text{rad})$	$\theta(\text{deg})$	$\tan(\theta)$	$w^2(\text{rad/s})^2$
11	21.34	2.94	0.08	0.04	2.4	0.04	8.67
20	15.80	3.98	0.15	0.08	4.4	0.08	15.81
26	14.22	4.42	0.20	0.10	5.7	0.10	19.52
30	12.99	4.84	0.23	0.11	6.5	0.11	23.40
40	11.74	5.35	0.31	0.15	8.6	0.15	28.64
51	10.45	6.01	0.39	0.19	10.7	0.19	36.15
56	9.90	6.35	0.43	0.20	11.7	0.21	40.28
65	9.40	6.68	0.50	0.23	13.3	0.24	44.68
70	9.08	6.92	0.54	0.25	14.2	0.25	47.88
85	8.39	7.49	0.65	0.29	16.6	0.30	56.08
100	7.71	8.15	0.77	0.33	18.8	0.34	66.41
112	7.43	8.46	0.86	0.36	20.4	0.37	71.51
132	7.00	8.98	1.02	0.40	22.7	0.42	80.57
61.4	11.19	6.20	0.47	0.21	11.98	0.21	41.51
							Ave.

The last line is for error calculation only.

The slope of the Figure 5 is 0.0052 (s/rad)^2 which gives

$$g = \frac{x_0}{\text{slope}} = \frac{5.1}{0.0052} = 980 \text{ cm/s}^2.$$



Error Calculation (possible methods):

$$g = \frac{w^2 x_0}{\tan \theta}$$

$$\frac{\Delta g}{g} = \sqrt{4 \left(\frac{\Delta w}{w} \right)^2 + \left(\frac{\Delta x_0}{x_0} \right)^2 + \left(\frac{\Delta(\tan \theta)}{\tan \theta} \right)^2} \quad \frac{\Delta w}{w} = \frac{\Delta T}{T}$$

$$\frac{\Delta(\tan \theta)}{\tan \theta} \approx \frac{\Delta \theta}{\theta}$$

(since from the table $\tan \theta \approx \theta$)

$$\theta \approx \frac{x}{H - h_0}, \quad \frac{\Delta \theta}{\theta} = \sqrt{\left(\frac{\Delta x}{x} \right)^2 + \left(\frac{\Delta H + \Delta h_0}{H - h_0} \right)^2}$$

$$\frac{\Delta g}{g} = \sqrt{4 \left(\frac{\Delta T}{T} \right)^2 + \left(\frac{\Delta x_0}{x_0} \right)^2 + \left(\frac{\Delta x}{x} \right)^2 + \left(\frac{\Delta H + \Delta h_0}{H - h_0} \right)^2}$$

Using the values $H=160$ mm, $\Delta H=1$ mm, $h_0=30$ mm, $\Delta h_0=1$ mm, $x_{av}=61.4$ mm, $\Delta x_{av}=1$ mm, $T_{av}=1.1$ s, $\Delta T=0.01$ s, $x_0=51$ mm, $\Delta x_0=1$ mm one obtains

$$g = 980 \pm 34 \text{ cm/s}^2$$

- Note that from the method of least squares one obtains the following results:

$$g = 982 \text{ cm/s}^2 \text{ with a standard deviation of } \sigma = 33 \text{ cm/s}^2$$

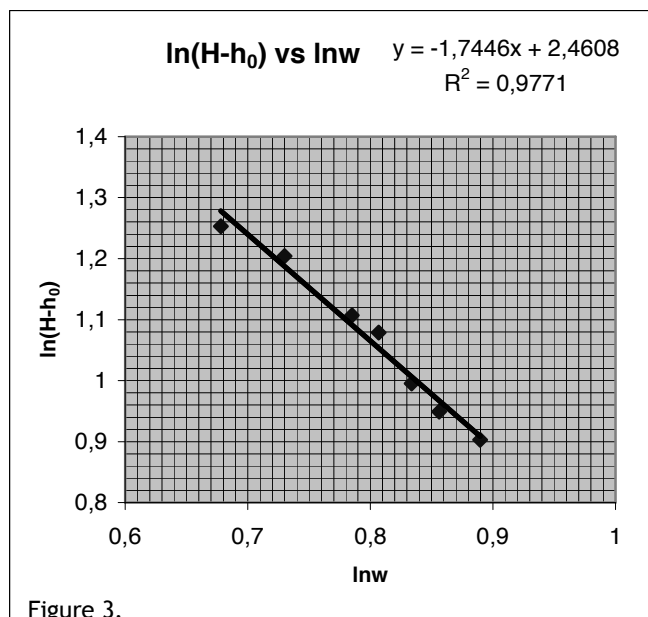
- From the linear regression of the data slope $\tan\theta$ vs w^2 is found to be 0.052 with a standard error of 5.14×10^{-5} , therefore:

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta(\text{slope})}{\text{slope}}\right)^2 + \left(\frac{\Delta x_0}{x_0}\right)^2} = 0.02$$

$$g = 980 \pm 20 \text{ cm/s}^2$$

Part 2a

H(mm)	10T(s)	w(rad/s)	lnw	H-h ₀ (mm)	ln(H-h ₀)
158	10.31	6.09	0.784921	128	2.107
209	13.19	4.76	0.677935	179	2.253
190	11.70	5.37	0.729994	160	2.204
150	9.80	6.41	0.806954	120	2.079
129	9.21	6.82	0.83392	99	1.996
119	8.75	7.18	0.856172	89	1.949
110	8.10	7.76	0.889695	80	1.903



Thus the focal length depends on w as

$$f = Aw^n,$$

and

$$n \sim -1.7.$$

The plot of $H-h_0$ vs. $1/w^2$ is also acceptable as a correct plot.

Part 2b

ω Range(rad/s)	Orientation	Variation of the size	Image
$\omega=0$	ER		V
$0<\omega<8.2^*$ $0<\omega<6.3^{**}$	ER	D	V
$8.2<\omega<14.6^*$ $6.3<\omega<14.0^{**}$	INV	I	R
$14.6<\omega<\omega_{\max}^*$ $14.0<\omega<\omega_{\max}^{**}$	ER	NC	V

* for $H=110$ mm

** for $H=240$ mm

ω values depend on the initial values of H , h_0 , etc.

Note that measurements only at one H value are required from the students.

Part 3

Measurement of wavelength

Both the grating and the screen are in air. Normal incidence.

Screen to grating distance : L
Distance between the diffraction spots seen on the screen : x
Order of diffraction : m

- L= 225 mm, $x_{av}=77$ mm for m=±1 d=1/500 mm
$$\tan \alpha = \frac{x_{av}}{L} = \frac{77}{225}$$
$$\lambda = \frac{1}{500} \sin \alpha = 647 \text{ nm}$$
- L= 128 mm, $x_{av}=44$ mm for m=±1, d=1/500 mm
$$\tan \alpha = \frac{44}{128}$$
$$\lambda = \frac{1}{500} \sin \alpha = 650 \text{ nm}$$
- L= 128 mm, $x_{av}=111$ mm for m=±2, d=1/500 mm
$$\tan \alpha = \frac{111}{128}$$
$$\lambda = \frac{1}{2 \times 500} \sin \alpha = 655 \text{ nm}$$

The average value of λ is $\lambda_{av}=651$ nm.

Measurement of refractive index

$$2R=145 \text{ mm}$$

Distance between the spots measured on the curved screen = $R\alpha$

$$R\alpha_{av} = 17 \text{ mm} \quad \text{for } m=\pm 1 \quad \alpha_{av} = 0.234 \text{ rad}$$

using $n = \frac{m\lambda}{d \sin(\alpha)}$, one obtains $n=1.40$

If the curvature of the screen is neglected:

$$\tan \alpha = \frac{17}{72.5}$$

$$\alpha = 13.20^\circ$$

$$n = \frac{\lambda}{d \sin(\alpha)} = \frac{651(\text{nm})}{\frac{1}{500}(\text{mm}) \times 10^6 \sin(\alpha)} = 1.43$$

Grading Scheme for Experimental Competition

Part 1	7.5 pts
• Derivation of Equation 1	<u>1.0 pts</u>
• Calculation of ω using period measurements	<u>1.0 pts</u>
At low speeds 10T is OK	
<i>At high speeds 20T is expected</i>	<i>-0.2 pts</i>
<i>Missing units</i>	<i>-0.2 pts</i>
• Calculation of $\tan 2\theta$, $\tan \theta$ at each ω	<u>1.0 pts</u>
Calculation of $\tan 2\theta$	0.5 pts
Calculation of $\tan \theta$	0.5 pts
• Plot of $\tan \theta$ vs ω^2	<u>1.5 pts</u>
Axes with labels and units	0.4 pts
Drawing best fit line	0.5 pts
At least 6 different data in a wide range of ω	0.6 pts
<i>No. of measurements 5:</i>	<i>-0.2 pts</i>
<i>No of measurements 4:</i>	<i>-0.4 pts</i>
<i>No of measurements 3 or less:</i>	<i>-0.6 pts</i>
• Calculations	<u>2.0 pts</u>
calculation of slope with unit	1.0 pts
calculation of g	1.0 pts
<i>FULL credit for</i> <i>9.3 < g < 10.3 m/s² ($\pm 5\%$ error)</i>	
<i>For g values credits to be subtracted from the total credit of 7.5:</i> <i>10.3 < g < 10.5 m/s², 9.1 < g < 9.3 m/s²</i>	<i>-0.5 pts</i>
<i>8.8 < g < 9.1 m/s², 10.3 < g < 10.8 m/s²</i>	<i>-1.0 pts</i>
<i>outside the above ranges</i>	<i>-1.5 pts</i>
• Error Calculation	<u>1.0 pts</u>

Part 2a

5.5 pts

- Measurements of H vs ω 0.6 pts
Calculation of ω using period measurements 0.4 pts

At low speeds $10T$ is OK
At high speeds $20T$ is expected -0.2 pts
 H - ω table 0.2 pts
- Plot of F vs ω 2.4 pts
Calculation of $F = H - h_0$ 0.5 pts
Plot with axis labels 0.8 pts
Drawing best fit line 0.5 pts
At least 6 different data in a wide range of ω 0.6 pts

No. of measurements 5: -0.2 pts
No of measurements 4: -0.4 pts
No of measurements 3 or less: -0.6 pts
- Calculations 2.5 pts
Calculation of slope with unit 1.0 pts
Dependence $F \propto 1/\omega^2$ 1.5 pts

Part 2b

3.5 pts

- Every correct item in the table 0.25 pts

Part 3

3.5 pts

(At least 3 measurements at different orders are required)

- Wavelength measurement 1.2 pts
Distance measurements and calculation of angle 0.6 pts
Calculation of λ 0.6 pts

Credits to be subtracted from the total credit of 1.2:
If λ is outside the range 600-700 nm -0.4 pts
If less than 3 measurements -0.4 pts
- Measurement of n 2.3 pts
Distance measurements and calculation of angle 0.6 pts
Realizing λ/n 0.8 pts
Calculation of n 0.9 pts

credits to be subtracted from the total credit of 2.3:
If n is outside range 1.3-1.6 -0.4 pts
If less than 3 measurements -0.4 pts