Problems of the 13th International Physics Olympiad

(Malente, 1982)

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Abstract

The 13th International Physics Olympiad took place in 1982 in the Federal Republic of Germany. This article contains the competition problems, their solutions and a grading scheme.

Introduction

In 1982 the Federal Republic of Germany was the first host of the Physics Olympiad outside the so-called Eastern bloc. The 13th International Physics Olympiad took place in Malente, Schleswig-Holstein. The competition was funded by the German Federal Ministry of Science and Education. The organisational guidelines were laid down by the work group "Olympiads for pupils" of the conference of ministers of education of the German federal states. The Institute for Science Education (IPN) at the University of Kiel was responsible for the realisation of the event. A commission of professors, whose chairman was appointed by the German Physical Society, were concerned with the formulation of the competition problems. All other members of the commission came from physics department of the university of Kiel or from the college of education at Kiel.

The problems as usual covered different fields of classical physics. In 1982 the pupils had to deal with three theoretical and two experimental problems, whereas at the previous Olympiads only one experimental task was given. However, it seemed to be reasonable to put more stress on experimental work. The degree of difficulty was well balanced. One of the theoretical problems could be considered as quite simple (problem 3: "hot-air balloon"). Another theoretical problem (problem 1: "fluorescent lamp") had a mean degree of difficulty and the distribution of the points was a normal distribution with only a few

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excellent and only a few unsatisfying solutions. The third problem (problem 2: “oscillation coat hanger”) turned out to be the most difficult problem. This problem was generally considered as quite interesting because different ways of solving were possible. About one third of the pupils did not find an adequate start to the problem, but nearly one third of the pupils was able to solve the substantial part of the problem. That means, this problem polarized between the pupils. The two experimental tasks were quite different in respect of the input for the experimental setup and the time required for dealing with the problems, whereas they were quite similar in the degree of difficulty. Both required demandingly theoretical considerations and experimental skills. Both experimental problems turned out to be rather difficult. The tasks were composed in a way that on the one hand almost every pupil had the possibility to come to certain partial results and that there were some difficulties on the other hand which could only be solved by very few pupils. The difficulty in the second experimental problem (problem5: “motion of a rolling cylinder”) was the explanation of the experimental results, which were initially quite surprising. The difficulty in the other task (problem 4: “lens experiment") was the revealing of an observation method with a high accuracy (parallax). The five hours provided for solving the two experimental problems were slightly too short. According to that, in both experiments only a few pupils came up with excellent solutions. In problem 5 nobody got the full points.

The problems presented here are based on the original German and English versions of the competition problems. The solutions are complete but in some parts condensed to the essentials. Almost all of the original hand-made figures are published here.

Theoretical Problems

Problem 1: Fluorescent lamp

An alternating voltage of 50 Hz frequency is applied to the fluorescent lamp shown in the accompanying circuit diagram.

The following quantities are measured:

overall voltage (main voltage)	$U = 228.5\ V$
electric current	$I = 0.6\ A$
partial voltage across the fluorescent lamp	$U' = 84\ V$
ohmic resistance of the series reactor	$R_d = 26.3 \Omega$

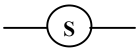
The fluorescent lamp itself may be considered as an ohmic resistor in the calculations.

a) What is the inductance L of the series reactor?

b) What is the phase shift φ between voltage and current?

c) What is the active power P_w transformed by the apparatus?

d) Apart from limiting the current the series reactor has another important function. Name and explain this function!

Hint: The starter 

includes a contact which closes shortly after switching on the lamp, opens up again and stays open.

e) In a diagram with a quantitative time scale sketch the time sequence of the luminous flux emitted by the lamp.

f) Why has the lamp to be ignited only once although the applied alternating voltage goes through zero in regular intervals?

g) According to the statement of the manufacturer, for a fluorescent lamp of the described type a capacitor of about 4.7 μF can be switched in series with the series reactor. How does this affect the operation of the lamp and to what intent is this possibility provided for?

h) Examine both halves of the displayed demonstrator lamp with the added spectroscope. Explain the differences between the two spectra. You may walk up to the lamp and you may keep the spectroscope as a souvenir.

Solution of problem 1:

a) The total resistance of the apparatus is $Z = \frac{228.5\,\text{V}}{0.6\,\text{A}} = 380.8\ \Omega$,

the ohmic resistance of the tube is $R_R = \frac{84\,\text{V}}{0.6\,\text{A}} = 140\ \Omega$.

Hence the total ohmic resistance is $R = 140\,\Omega + 26.3\,\Omega = 166.3\,\Omega$.

Therefore the inductance of the series reactor is: $\omega \cdot L = \sqrt{Z^2 - R^2} = 342.6\,\Omega$.

This yields $L = \frac{342.6\,\Omega}{100\,\pi\,\text{s}^{-1}} = 1.09\ \text{H}$.

b) The impedance angle is obtained from $\tan\varphi = \frac{\omega \cdot L}{R} = \frac{342.6\,\Omega}{166.3\,\Omega} = 2.06$.

Thus $\varphi = 64.1^\circ$.

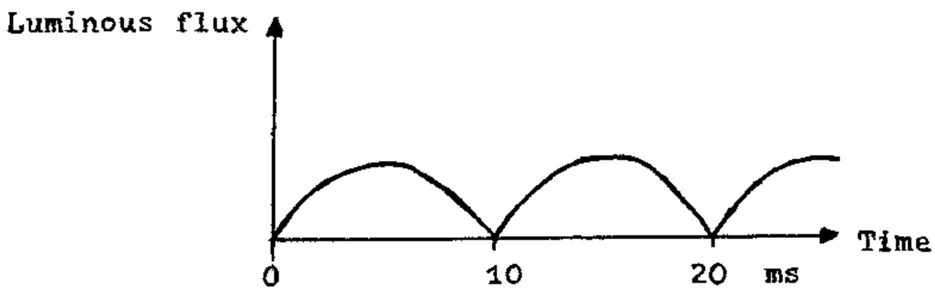
c) The active power can be calculated in different ways:

1) $P_w = U \cdot I \cdot \cos\varphi = 228.5\,\text{V} \cdot 0.6\ \text{A} \cdot \cos 64.1^\circ = 59.88\ \text{W}$

2) $P_w = R \cdot I^2 = 166.3\,\Omega \cdot (0.6\ \text{A})^2 = 59.87\ \text{W}$

d) By opening the contact in the starter a high induction voltage is produced across the series reactor (provided the contact does not open exactly the same moment, when the current goes through zero). This voltage is sufficient to ignite the lamp. The main voltage itself, however, is smaller than the ignition voltage of the fluorescent tube.

e)



f) The recombination time of the ions and electrons in the gaseous discharge is sufficiently large.

g) The capacitive resistance of a capacitor of 4.7 μF is

$$\frac{1}{\omega \cdot C} = (100 \cdot \pi \cdot 4.7 \cdot 10^{-6})^{-1}\ \Omega = 677.3\ \Omega .$$

The two reactances subtract and there remains a reactance of 334.7 Ω acting as a capacitor.

The total resistance of the arrangement is now

$$Z' = \sqrt{(334.7)^2 + (166.3)^2}\ \Omega = 373.7\ \Omega ,$$

which is very close to the total resistance without capacitor, if you assume the capacitor to be loss-free (cf. a)). Thus the lamp has the same operating qualities, ignites the same way, and a difference is found only in the impedance angle φ', which is opposite to the angle φ calculated in b):

$$\tan \varphi' = \frac{\omega \cdot L - (\omega \cdot C)^{-1}}{R} = -\frac{334.7}{166.3} = -2.01$$

$$\varphi' = -63.6^{\circ} .$$

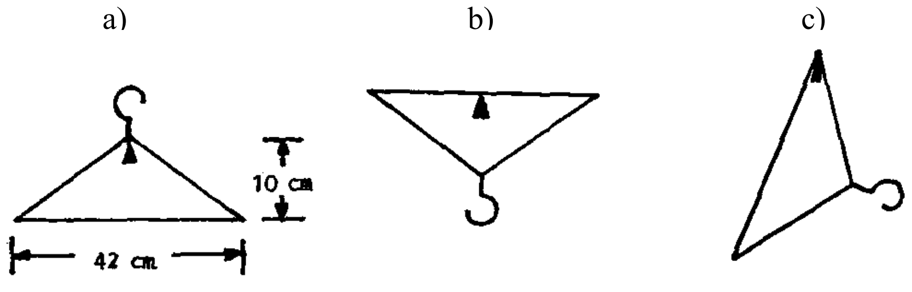
Such additional capacitors are used for compensation of reactive currents in buildings with a high number of fluorescent lamps, frequently they are prescribed by the electricity supply companies. That is, a high portion of reactive current is unwelcome, because the power generators have to be layed out much bigger than would be really necessary and transport losses also have to be added which are not payed for by the customer, if pure active current meters are used.

h) The uncoated part of the demonstrator lamp reveals the line spectrum of mercury, the coated part shows the same line spectrum over a continuous background. The continuous spectrum results from the ultraviolet part of the mercury light, which is absorbed by the fluorescence and re-emitted with smaller frequency (energy loss of the photons) or larger wavelength respectively.

Problem 2: Oscillating coat hanger

A (suitably made) wire coat hanger can perform small amplitude oscillations in the plane of the figure around the equilibrium positions shown. In positions a) and b) the long side is horizontal. The other two sides have equal length. The period of oscillation is the same in all cases.

What is the location of the center of mass, and how long is the period?

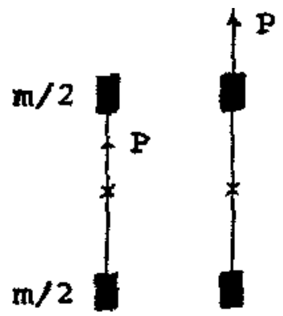


The figure does not contain any information beyond the dimensions given. Nothing is known, e.g., concerning the detailed distribution of mass.

Solution of problem 2

First method:

The motions of a rigid body in a plane correspond to the motion of two equal point masses connected by a rigid massless rod. The moment of inertia then determines their distance. Because of the equilibrium position a) the center of mass is on the perpendicular bipartition of the long side of the coat hanger. If one imagines the equivalent masses and the supporting point P being arranged in a straight line in each case, only two positions of P yield the same period of oscillation (see sketch). One can understand this by considering the limiting cases: 1. both supporting points in the upper mass and 2. one point in the center of mass and the other infinitely high above. Between these extremes the period of oscillation grows continuously. The supporting point placed in the corner of the long side c) has the largest distance from the center of mass, and therefore this point lies outside the two point masses. The two other supporting points a), b) then have to be placed symmetrically to the center of mass between the two point masses, i.e., the center of mass bisects the perpendicular bipartition.



One knows of the reversible pendulum that for every supporting point of the physical pendulum it generally has a second supporting point of the pendulum rotated by 180°, with the same period of oscillation but at a different distance from the center of mass. The section between the two supporting points equals the length of the corresponding mathematical pendulum. Therefore the period of oscillation is obtained through the corresponding length of the pendulum $s_b + s_c$, where $s_b = 5$ cm and $s_c = \sqrt{5^2 + 21^2}$ cm, to be $T = 1.03$ s .

Second method:

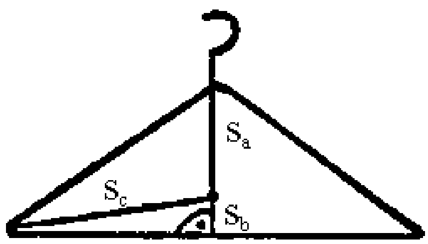
Let s denote the distance between the supporting point and the center of mass, m the mass itself and θ the moment of inertia referring to the supporting point. Then we have the period of oscillation T :

$$T = 2\pi\sqrt{\frac{\theta}{m \cdot g \cdot s}} \quad , \tag{1}$$

where g is the acceleration of gravity, $g = 9.81$ m/s^2. Here θ can be obtained from the moment of inertia θ_o related to the center of mass:

$$\theta = \theta_0 + m \cdot s^2 \tag{2}$$

Because of the symmetrical position in case a) the center of mass is to be found on the perpendicular bisection above the long side. Now (1) and (2) yield



$$\theta_0 + m \cdot s^2 = \left(\frac{T}{2 \cdot \pi}\right)^2 \cdot m \cdot g \cdot s \quad \text{for} \quad s = s_a,\ s_b \ \text{and}\ s_c. \tag{3}$$

because all periods of oscillation are the same. This quadratic equation has only two different solutions at most. Therefore at least two of the three distances are equal. Because of $s_c > 21$ cm $> s_a + s_b$, only s_a and s_b can equal each other. Thus we have

$$s_a = 5 \text{ cm} \tag{4}$$

The moment of inertia θ_0 is eliminated through (3):

$$m \cdot (s_c^{\ 2} - s_a^{\ 2}) = \left(\frac{T}{2 \cdot \pi}\right)^2 \cdot m \cdot g \cdot (s_c - s_a)$$

and we have $$T = 2 \cdot \pi\sqrt{\frac{s_c + s_a}{g}} \tag{5}$$

with the numerical value $T = 1.03\,\mathrm{s}$,

which has been rounded off after two decimals because of the accuracy of g.

Third method:

This solution is identical to the previous one up to equation (2).

From (1) and (2) we generally have for equal periods of oscillation $T_1 = T_2$:

$$\frac{\theta_0 + m \cdot s_1^2}{m \cdot g \cdot s_1} = \frac{\theta_0 + m \cdot s_2^2}{m \cdot g \cdot s_2}$$

and therefore $s_2 \cdot \left(\theta_0 + m \cdot s_1^2\right) = s_1 \cdot \left(\theta_0 + m \cdot s_2^2\right)$

or
$$\left(s_2 - s_1\right) \cdot \left(\theta_0 - m \cdot s_1\, s_2\right) = 0 \qquad (6)$$

The solution of (6) includes two possibilities: $s_1 = s_2$ or $s_1 \cdot s_2 = \frac{\theta_0}{m}$

Let $2 \cdot a$ be the length of the long side and b the height of the coat hanger. Because of $T_b = T_c$ we then have either $s_b = s_c$ or $s_b \cdot s_c = \frac{\theta_0}{m}$, where $s_c = \sqrt{s_b^2 + a^2}$,

which excludes the first possibility. Thus
$$s_b \cdot s_c = \frac{\theta_0}{m}. \qquad (7)$$

For $T_a = T_b$ the case $s_a \cdot s_b = \frac{\theta_0}{m}$ is excluded because of eq. (7), for we have

$$s_a \cdot s_b < s_c \cdot s_b = \frac{\theta_0}{m}.$$

Hence $s_a = s_b = \frac{1}{2} b$, $s_c = \sqrt{\frac{1}{4} b^2 + a^2}$

and
$$T = 2 \cdot \pi \sqrt{\frac{\frac{\theta_0}{m} + s_b^2}{g \cdot s_b}} = 2\pi \sqrt{\frac{s_b \cdot s_c + s_b^2}{g \cdot s_b}}$$

The numerical calculation yields the value $T = 1.03\,\mathrm{s}$.

Problem 3: Hot-air-balloon

Consider a hot-air balloon with fixed volume $V_B = 1.1\ m^3$. The mass of the balloon-envelope, whose volume is to be neglected in comparison to V_B, is $m_H = 0.187$ kg.

The balloon shall be started, where the external air temperature is $\vartheta_1 = 20\ ^oC$ and the normal external air pressure is $p_o = 1.013 \cdot 10^5$ Pa. Under these conditions the density of air is $\rho_1 = 1.2\ kg/m^3$.

a) What temperature ϑ_2 must the warmed air inside the balloon have to make the balloon just float?

b) First the balloon is held fast to the ground and the internal air is heated to a steady-state temperature of $\vartheta_3 = 110\ ^oC$. The balloon is fastened with a rope.

   Calculate the force on the rope.

c) Consider the balloon being tied up at the bottom (the density of the internal air stays constant). With a steady-state temperature $\vartheta_3 = 110\ ^oC$ of the internal air the balloon rises in an isothermal atmosphere of 20 °C and a ground pressure of $p_0 = 1.013 \cdot 10^5$ Pa. Which height h can be gained by the balloon under these conditions?

d) At the height h the balloon (question c)) is pulled out of its equilibrium position by 10 m and then is released again.
   Find out by qualitative reasoning what kind of motion it is going to perform!

Solution of problem 3:

a) Floating condition:

   The total mass of the balloon, consisting of the mass of the envelope m_H and the mass of the air quantity of temperature ϑ_2 must equal the mass of the displaced air quantity with temperature $\vartheta_1 = 20\ ^oC$.

$$V_B \cdot \rho_2 + m_H = V_B \cdot \rho_1$$

$$\rho_2 = \rho_1 - \frac{m_H}{V_B} \qquad (1)$$

Then the temperature may by obtained from

$$\frac{\rho_1}{\rho_2} = \frac{T_2}{T_1},$$

$$T_2 = \frac{\rho_1}{\rho_2} \cdot T_1 = 341.53\ \text{K} = 68.38\ °\text{C} \qquad (2)$$

b) The force F_B acting on the rope is the difference between the buoyant force F_A and the weight force F_G:

$$F_B = V_B \cdot \rho_1 \cdot g \ - \ (V_B \cdot \rho_3 + m_H) \cdot g \qquad (3)$$

It follows with $\rho_3 \cdot T_3 = \rho_1 \cdot T_1$

$$F_B = V_B \cdot \rho_1 \cdot g \cdot \left(1 - \frac{T_1}{T_3}\right) - m_H \cdot g = 1{,}21\ \text{N} \qquad (4)$$

c) The balloon rises to the height h, where the density of the external air ρ_h has the same value as the effective density ρ_{eff}, which is evaluated from the mass of the air of temperature $\vartheta_3 = 110\ ^{o}\text{C}$ (inside the balloon) and the mass of the envelope m_H:

$$\rho_{eff} = \frac{m_2}{V_B} = \frac{\rho_3 \cdot V_B + m_H}{V_B} = \rho_h = \rho_1 \cdot e^{-\frac{\rho_1 \cdot g \cdot h}{p_0}} \qquad (5)$$

Resolving eq. (5) for h gives: $h = \frac{p_o}{\rho_1 \cdot g} \cdot \ln \frac{\rho_1}{\rho_{eff}} = 843\,\text{m}$ (6).

d) For *small* height differences (10 m in comparison to 843 m) the exponential pressure drop (or density drop respectively) with height can be approximated by a linear function of height. Therefore the driving force is proportional to the elongation out of the equilibrium position.

This is the condition in which harmonic oscillations result, which of course are damped by the air resistance.

Experimental Problems

Problem 4: Lens experiment

The apparatus consists of a symmetric biconvex lens, a plane mirror, water, a meter stick, an optical object (pencil), a supporting base and a right angle clamp. Only these parts may be used in the experiment.

a) Determine the focal length of the lens with a maximum error of $\pm$ 1 %.

b) Determine the index of refraction of the glass from which the lens is made.

The index of refraction of water is $n_w = 1.33$. The focal length of a thin lens is given by

$$\frac{1}{f} = (n-1)\cdot\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where n is the index of refraction of the lens material and r_1 and r_2 are the curvature radii of the refracting surfaces. For a symmetric biconvex lens we have $r_1 = - r_2 = r$, for a symmetric biconcave lens $r_1 = - r_2 = - r$.

Solution of problem 4:

a) For the determination of f_L , place the lens on the mirror and with the clamp fix the pencil to the supporting base. Lens and mirror are then moved around until the vertically downward looking eye sees the pencil and its image side by side.

   In order to have object and image in focus at the same time, they must be placed at an equal distance to the eye. In this case object distance and image distance are the same and the magnification factor is 1 .

   It may be proved quite accurately, whether magnification 1 has in fact been obtained, if one concentrates on parallatical shifts between object and image when moving the eye: only when the distances are equal do the pencil-tips point at each other all the time.

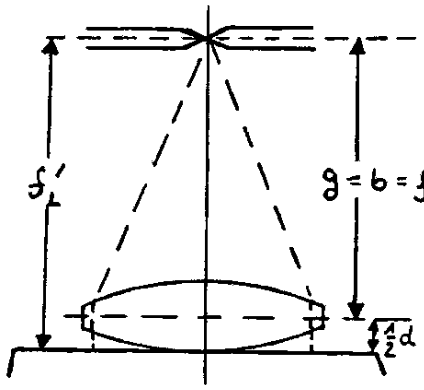
   The light rays pass the lens twice because they are reflected by the mirror. Therefore the optical mapping under consideration corresponds to a mapping with two lenses placed directly one after another:

$$\frac{1}{g}+\frac{1}{b}=\frac{1}{f}, \quad \text{where} \quad \frac{1}{f}=\frac{1}{f_L}+\frac{1}{f_L}$$

i.e. the effective focal length f is just half the focal length of the lens. Thus we find for magnification 1:

$$g=b \quad \text{and} \quad \frac{2}{g}=\frac{2}{f_L} \quad \text{i.e.} \quad f_L = g.$$

A different derivation of $f_L = g = b$: For a mapping of magnification 1 the light rays emerging from a point on the optical axis are reflected into themselves. Therefore these rays have to hit the mirror at right angle and so the object distance g equals the focal length f_L of the lens in this case.



The distance between pencil point and mirror has to be determined with an accuracy, which enables one to state f_L with a maximum error of $\pm 1\,\%$. This is accomplished either by averaging several measurements or by stating an uncertainty interval, which is found through the appearance of parallaxe.

Half the thickness of the lens has to be subtracted from the distance between pencil-point and mirror.

$$f_L = f_L{}' - \frac{1}{2}d, \quad d = 3.0 \pm 0.5\text{ mm}$$

The nominal value of the focal length of the lens is f_L = 30 cm. However, the actual focal length of the single lenses spread considerably. Each lens was measured separately, so the individual result of the student can be compared with the exact value.

b) The refractive index n of the lens material can be evaluated from the equation

$$\frac{1}{f_L} = (n-1)\cdot\frac{2}{r}$$

if the focal length f_L and the curvature radius r of the symmetric biconvex lens are known. f_L was determined in part a) of this problem.

The still unknown curvature radius r of the symmetric biconvex lens is found in the following way: If one pours some water onto the mirror and places the lens in the water, one gets a plane-concave water lens, which has one curvature radius equalling the glass lens' radius and the other radius is ∞.

Because the refractive index of water is known in this case, one can evaluate the curvature radius through the formula above, where $r_1 = -r$ and $r_2 = \infty$:

$$-\frac{1}{f_w} = (n_w - 1)\cdot\frac{1}{r}.$$

Only the focal length f' of the combination of lenses is directly measured, for which we have

$$\frac{1}{f'} = \frac{1}{f_L} + \frac{1}{f_w}.$$

This focal length is to be determined by a mapping of magnification 1 as above.

Then the focal length of the water lens is $\frac{1}{f_w} = \frac{1}{f'} - \frac{1}{f_L}$

and one has the curvature radius $r = -(n_w - 1)\cdot f_w$.

Now the refractive index of the lens is determined by $n = \frac{r}{2\cdot f_L} + 1$

with the known values of f_L and r, or, if one wants to express n explicitly through the measured quantities: $n = \frac{f'\cdot(n_w - 1)}{2\cdot(f' - f_L)} + 1.$

The nominal values are: $f' = 43.9$ cm, $f_w = -94.5$ cm, $r = 31.2$ cm, $n = 1.52$.

Problem 5: Motion of a rolling cylinder

The rolling motion of a cylinder may be decomposed into rotation about its axis and horizontal translation of the center of gravity. In the present experiment only the translatory acceleration and the forces causing it are determined directly.

Given a cylinder of mass M , radius R , which is placed on a horizontal plane board. At a distance r_i (i = 1 … 6) from the cylinder axis a force acts on it (see sketch). After letting the cylinder go, it rolls with constant acceleration.

a) Determine the linear accelerations a_i (i = 1 … 6) of the cylinder axis experimentally for several distances r_i (i = 1 … 6).

b) From the accelerations a_i and given quantities, compute the forces F_i which act in horizontal direction between cylinder and plane board.

c) Plot the experimental values F_i as functions of r_i. Discuss the results.

Before starting the measurements, adjust the plane board horizontally. For present purposes it suffices to realize the horizontal position with an uncertainty of ± 1 mm of height difference on 1 m of length; this corresponds to the distance between adjacent markings on the level. What would be the result of a not horizontal position of the plane board?

Describe the determination of auxiliary quantities and possible further adjustments; indicate the extent to which misadjustments would influence the results.

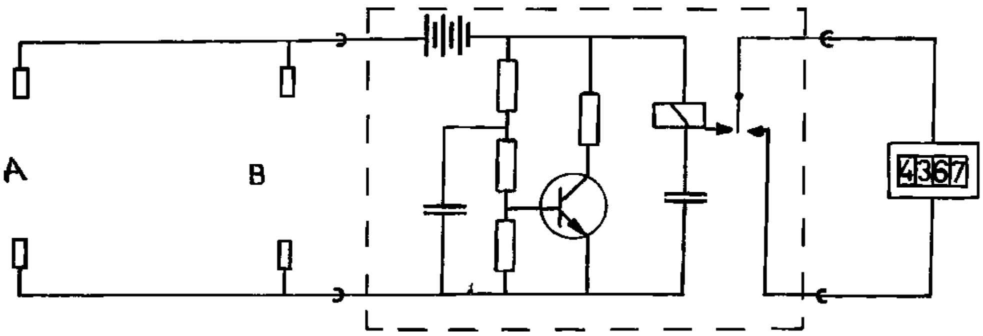
The following quantities are given:

R	=	5 cm	r_1	=	0.75 cm
M	=	3.275 kg	r_2	=	1.50 cm
m	=	2 x 50 g	r_3	=	2.25 cm
D	=	1.50 cm	r_4	=	3.00 cm
d	=	0.1 mm	r_5	=	3.75 cm
			r_6	=	4.50 cm

Mass and friction of the pulleys c may be neglected in the evaluation of the data.

By means of knots, the strings are put into slots at the cylinder. They should be inserted as deeply as possible. You may use the attached paper clip to help in this job.

The stop watch should be connected, as shown in the sketch, with electrical contacts at A and B via an electronic circuit box. The stop watch starts running as soon as the contact at A is opened, and it stops when the contact at B is closed.



The purpose of the transistor circuit is to keep the relay position after closing of the contact at B, even if this contact is opened afterwards for a few milliseconds by a jump or chatter of the cylinder.

Solution of problem 5:

Theoretical considerations:

a) The acceleration of the center of mass of the cylinder is $a = \frac{2 \cdot s}{t^2}$ (1)

b) Let a_m be the acceleration of the masses m and T the sum of the tensions in the two strings, then

$$T = m \cdot g - m \cdot a_m \qquad (2)$$

The acceleration a of the center of mass of the cylinder is determined by the resultant force of the string-tension T and the force of interaction F between cylinder and the horizontal plane.

$$M \cdot a = T - F \qquad (3)$$

If the cylinder rotates through an angle θ the mass m moves a distance x_m.
It holds

$$x_m = (R + r) \cdot \theta$$

$$a_m = (R + r) \cdot \frac{a}{R} \qquad (4)$$

From (2), (3) and (4) follows $F = mg - \left[M + m \cdot \left(1 + \frac{r}{R} \right) \right] \cdot a$. (5)

c) From the experimental data we see that for small r_i the forces $M \cdot a$ and T are in opposite direction and that they are in the same direction for large r_i.

For small values of r the torque produced by the string-tensions is not large enough to provide the angular acceleration required to prevent slipping. The interaction force between cylinder and plane acts into the direction opposite to the motion of the center of mass and thereby delivers an additional torque.

For large values of r the torque produced by string-tension is too large and the interaction force has such a direction that an opposed torque is produced.

From the rotary-impulse theorem we find

$$T \cdot r + F \cdot R = I \cdot \ddot{\theta} = I \cdot \frac{a}{R},$$

where I is the moment of inertia of the cylinder.

With (3) and (5) you may eliminate T and a from this equation. If the moment of inertia of the cylinder is taken as $I = \frac{1}{2} \cdot M \cdot R^2$ (neglecting the step-up cones) we find after some arithmetical transformations

$$F = mg \cdot \frac{1 - 2 \cdot \frac{r}{R}}{3 + 2 \cdot \frac{m}{M} \cdot \left(1 + \frac{r}{R} \right)^2}.$$

For $r = 0 \rightarrow \quad F = \frac{m \cdot g}{3 + 2 \cdot \frac{m}{M}} > 0$.

For $r = R \Rightarrow \quad F = \frac{-m \cdot g}{3 + 8 \cdot \frac{m}{M}} < 0$.

Because $\frac{m}{M} \ll 1$ it is approximately $F = \frac{1}{3} m \cdot g - \frac{2}{3} \cdot \frac{r}{R}$.

That means: the dependence of F from r is approximately linear. F will be zero if

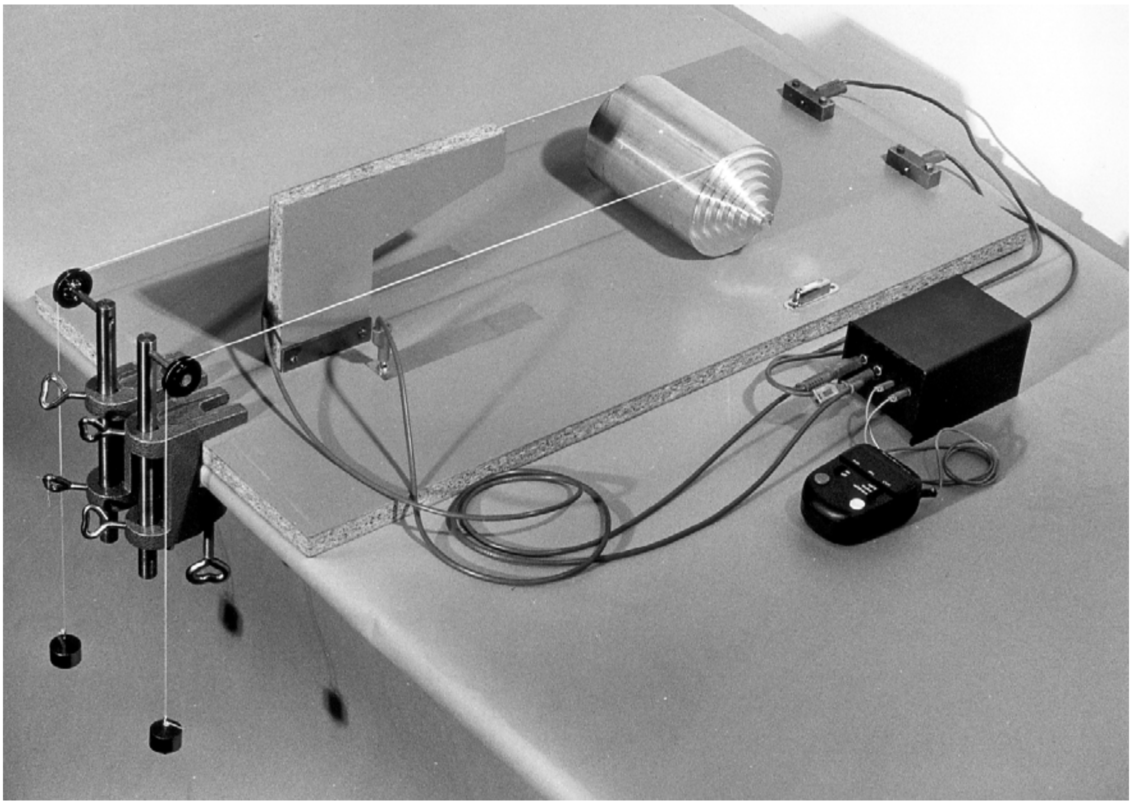
$$\frac{r}{R} = \frac{m \cdot g}{2}.$$

Experimental results:

$$s = L - (2 \cdot R \cdot D + D^2)^{\frac{1}{2}} - (2 \cdot R \cdot d - d^2)^{\frac{1}{2}}$$

$$s = L - 4.5\ \text{cm} = 39.2\ \text{cm} - 4.5\ \text{cm} = 34.7\ \text{cm}$$

r [cm]	t [s]			$\bar{t}$ [s]	a [m/s^2]	F [N]
0.75	1.81	1.82	1.82	1.816	0.211	0.266
1.50	1.71	1.72	1.73	1.720	0.235	0.181
2.25	1.63	1.63	1.64	1.633	0.261	0.090
3.00	1.56	1.56	1.57	1.563	0.284	0.004
3.75	1.51	1.51	1.52	1.513	0.304	- 0.066
4.50	1.46	1.46	1.46	1.456	0.328	- 0.154



Grading schemes

Theoretical problems

Problem 1: Fluorescent lamp	pts.
Part a	2
Part b	1
Part c	1
Part d	1
Part e	1
Part f	1
Part g	2
Part h	1
	10

Problem 2: Oscillating coat hanger	pts.
equation (1)	1,5
equation (2)	1,5
equation (4)	3
equation (5)	2
numerical value for T	1
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Problem 3: Hot-air-balloon	pts.
Part a	3
Part b	2
Part c	3
Part d	2
	10

Experimental problems

Problem 4: Lens experiment	pts.
correct description of experimental prodedure	1
selection of magnification one	0.5
parallaxe for verifying his magnification	1
$f_L = g = b$ with derivation	1
several measurements with suitable averaging or other determination of error interval	1
taking into account the lens thickness and computing f_L, including the error	0.5
idea of water lens	0.5
theory of lens combination	1
measurements of f'	0.5
calculation of n and correct result	1
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Problem 5: Motion of a rolling cylinder	pts.
Adjustment mentioned of strings a) horizontally and b) in direction of motion	0.5
Indication that angle offset of strings enters the formula for the acting force only quadratically, i.e. by its cosine	0.5
Explanation that with non-horizontal position, the force m·g is to be replaced by $m \cdot g \pm M \cdot g \cdot \sin\alpha$	1.0
Determination of the running length according for formula $s = L - \left(2 \cdot R \cdot D + D^2\right)^{1/2} - \left(2 \cdot R \cdot d + d^2\right)^{1/2}$ including correct numerical result	1.0
Reliable data for rolling time	1.0
accompanied by reasonable error estimate	0.5
Numerical evaluation of the F_i	0.5
Correct plot of $F_i(v_i)$	0.5
Qualitative interpretation of the result by intuitive consideration of the limiting cases $r = 0$ and $r = R$	1.0
Indication of a quantitative, theoretical interpretation using the concept of moment of inertia	1.0
Knowledge and application of the formula $a = 2\,s / t^2$	0.5
Force equation for small mass and tension of the string $m \cdot (g - a_m) = T$	1.0
Connection of tension, acceleration of cylinder and reaction force $T - F = M \cdot a$	1.0
Connection between rotary and translatory motion	
$x_m = (R + r) \cdot \theta$	0.5
$a_m = (1 + r/R) \cdot a$	0.5
Final formula for the reaction force $F = m \cdot g - \left(M + m \cdot (1 + r/R)\right) \cdot a$	1.0
If final formulae are given correctly, the knowledge for preceding equations must be assumed and is graded accordingly.	
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