10^{th} **International Physics Olympiad**
1977, Hradec Králové, Czechoslovakia

Problem 1. The compression ratio of a four-stroke internal combustion engine is $\varepsilon = 9.5$. The engine draws in air and gaseous fuel at a temperature $27\,^{\circ}\mathrm{C}$ at a pressure 1 atm = 100 kPa. Compression follows an adiabatic process from point 1 to point 2, see Fig. 1. The pressure in the cylinder is doubled during the mixture ignition (2–3). The hot exhaust gas expands adiabatically to the volume V_2 pushing the piston downwards (3–4). Then the exhaust valve opens and the pressure gets back to the initial value of 1 atm. All processes in the cylinder are supposed to be ideal. The Poisson constant (i.e. the ratio of specific heats C_p/C_V) for the mixture and exhaust gas is $\kappa = 1.40$. (The compression ratio is the ratio of the volume of the cylinder when the piston is at the bottom to the volume when the piston is at the top.)

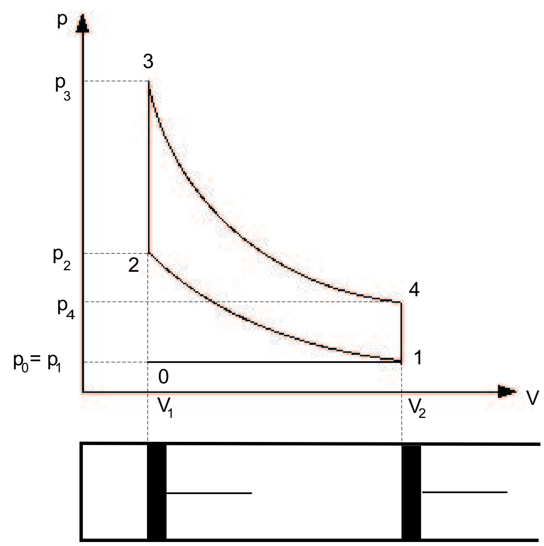

Figure 1:

a) Which processes run between the points 0–1, 2–3, 4–1, 1–0?

b) Determine the pressure and the temperature in the states 1, 2, 3 and 4.

c) Find the thermal efficiency of the cycle.

d) Discuss obtained results. Are they realistic?

Solution: a) The description of the processes between particular points is the following:

0–1 :	intake stroke	isobaric and isothermal process
1–2 :	compression of the mixture	adiabatic process
2–3 :	mixture ignition	isochoric process
3–4 :	expansion of the exhaust gas	adiabatic process
4–1 :	exhaust	isochoric process
1–0 :	exhaust	isobaric process

Let us denote the initial volume of the cylinder before induction at the point 0 by V_1, after induction at the point 1 by V_2 and the temperatures at the particular points by T_0, T_1, T_2, T_3 and T_4.

b) The equations for particular processes are as follows.

0–1 : The fuel-air mixture is drawn into the cylinder at the temperature of $T_0 = T_1 = 300$ K and a pressure of $p_0 = p_1 = 0.10$ MPa.

1–2 : Since the compression is very fast, one can suppose the process to be adiabatic. Hence:

$$p_1 V_2^\kappa = p_2 V_1^\kappa \quad \text{and} \quad \frac{p_1 V_2}{T_1} = \frac{p_2 V_1}{T_2} \, .$$

From the first equation one obtains

$$p_2 = p_1 \left(\frac{V_2}{V_1} \right)^\kappa = p_1 \varepsilon^\kappa$$

and by the dividing of both equations we arrive after a straightforward calculation at

$$T_1 V_2^{\kappa - 1} = T_2 V_1^{\kappa - 1} \, , \quad T_2 = T_1 \left(\frac{V_2}{V_1} \right)^{\kappa - 1} = T_1 \varepsilon^{\kappa - 1} \, .$$

For given values $\kappa = 1.40$, $\varepsilon = 9.5$, $p_1 = 0.10$ MPa, $T_1 = 300$ K we have $p_2 = 2.34$ MPa and $T_2 = 738$ K ($t_2 = 465\,^\circ$C).

2–3 : Because the process is isochoric and $p_3 = 2p_2$ holds true, we can write

$$\frac{p_3}{p_2} = \frac{T_3}{T_2}\,, \quad \text{which implies} \quad T_3 = T_2\frac{p_3}{p_2} = 2T_2\,.$$

Numerically, $p_3 = 4.68$ MPa, $T_3 = 1476$ K ($t_3 = 1203\,^\circ$C).

3–4 : The expansion is adiabatic, therefore

$$p_3V_1^\kappa = p_4V_2^\kappa\,, \quad \frac{p_3V_1}{T_3} = \frac{p_4V_2}{T_4}\,.$$

The first equation gives

$$p_4 = p_3\left(\frac{V_1}{V_2}\right)^\kappa = 2p_2\varepsilon^{-\kappa} = 2p_1$$

and by dividing we get

$$T_3V_1^{\kappa-1} = T_4V_2^{\kappa-1}\,.$$

Consequently,

$$T_4 = T_3\varepsilon^{1-\kappa} = 2T_2\varepsilon^{1-\kappa} = 2T_1\,.$$

Numerical results: $p_4 = 0.20$ MPa, $T_3 = 600$ K ($t_3 = 327\,^\circ$C).

4–1 : The process is isochoric. Denoting the temperature by T_1' we can write

$$\frac{p_4}{p_1} = \frac{T_4}{T_1'}\,,$$

which yields

$$T_1' = T_4\frac{p_1}{p_4} = \frac{T_4}{2} = T_1\,.$$

We have thus obtained the correct result $T_1' = T_1$. Numerically, $p_1 = 0.10$ MPa, $T_1' = 300$ K.

c) Thermal efficiency of the engine is defined as the proportion of the heat supplied that is converted to net work. The exhaust gas does work on the piston during the expansion 3–4, on the other hand, the work is done on the mixture during the compression 1–2. No work is done by/on the gas during the processes 2–3 and 4–1. The heat is supplied to the gas during the process 2–3.

The net work done by 1 mol of the gas is

$$W = \frac{R}{\kappa - 1}(T_1 - T_2) + \frac{R}{\kappa - 1}(T_3 - T_4) = \frac{R}{\kappa - 1}(T_1 - T_2 + T_3 - T_4)$$

and the heat supplied to the gas is

$$Q_{23} = C_V(T_3 - T_2)\,.$$

Hence, we have for thermal efficiency

$$\eta = \frac{W}{Q_{23}} = \frac{R}{(\kappa - 1)C_V}\,\frac{T_1 - T_2 + T_3 - T_4}{T_3 - T_2}\,.$$

Since

$$\frac{R}{(\kappa - 1)C_V} = \frac{C_p - C_V}{(\kappa - 1)C_V} = \frac{\kappa - 1}{\kappa - 1} = 1\,,$$

we obtain

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} = 1 - \varepsilon^{1-\kappa}\,.$$

Numerically, $\eta = 1 - 300/738 = 1 - 0.407$, $\eta = 59,3\%$.

d) Actually, the real pV-diagram of the cycle is smooth, without the sharp angles. Since the gas is not ideal, the real efficiency would be lower than the calculated one.

Problem 2. Dipping the frame in a soap solution, the soap forms a rectangle film of length b and height h. White light falls on the film at an angle α (measured with respect to the normal direction). The reflected light displays a green color of wavelength λ_0.

a) Find out if it is possible to determine the mass of the soap film using the laboratory scales which has calibration accuracy of 0.1 mg.

b) What color does the thinnest possible soap film display being seen from the perpendicular direction? Derive the related equations.

Constants and given data: relative refractive index $n = 1.33$, the wavelength of the reflected green light $\lambda_0 = 500$ nm, $\alpha = 30^{\circ}$, $b = 0.020$ m, $h = 0.030$ m, density $\varrho = 1000$ kg m^{-3}.

Solution: The thin layer reflects the monochromatic light of the wavelength λ in the best way, if the following equation holds true

$$2nd\cos\beta = (2k+1)\frac{\lambda}{2}\,, \quad k = 0, 1, 2, \ldots\,, \tag{1}$$

where k denotes an integer and β is the angle of refraction satisfying

$$\frac{\sin\alpha}{\sin\beta} = n\,.$$

Hence,

$$\cos\beta = \sqrt{1-\sin^2\beta} = \frac{1}{n}\sqrt{n^2-\sin^2\alpha}\,.$$

Substituting to (1) we obtain

$$2d\sqrt{n^2-\sin^2\alpha} = (2k+1)\frac{\lambda}{2}\,. \tag{2}$$

If the white light falls on a layer, the colors of wavelengths obeying (2) are reinforced in the reflected light. If the wavelength of the reflected light is λ_0, the thickness of the layer satisfies for the kth order interference

$$d_k = \frac{(2k+1)\lambda_0}{4\sqrt{n^2-\sin^2\alpha}} = (2k+1)d_0\,.$$

For given values and $k = 0$ we obtain $d_0 = 1.01 \cdot 10^{-7}$ m.

a) The mass of the soap film is $m_k = \varrho_k b\, h\, d_k$. Substituting the given values, we get $m_0 = 6.06 \cdot 10^{-2}$ mg, $m_1 = 18.2 \cdot 10^{-2}$ mg, $m_2 = 30.3 \cdot 10^{-8}$ mg, etc. The mass of the thinnest film thus cannot be determined by given laboratory scales.

b) If the light falls at the angle of 30° then the film seen from the perpendicular direction cannot be colored. It would appear dark.

Problem 3. An electron gun T emits electrons accelerated by a potential difference U in a vacuum in the direction of the line a as shown in Fig. 2. The target M is placed at a distance d from the electron gun in such a way that the line segment connecting the points T and M and the line a subtend the angle α as shown in Fig. 2. Find the magnetic induction B of the uniform magnetic field

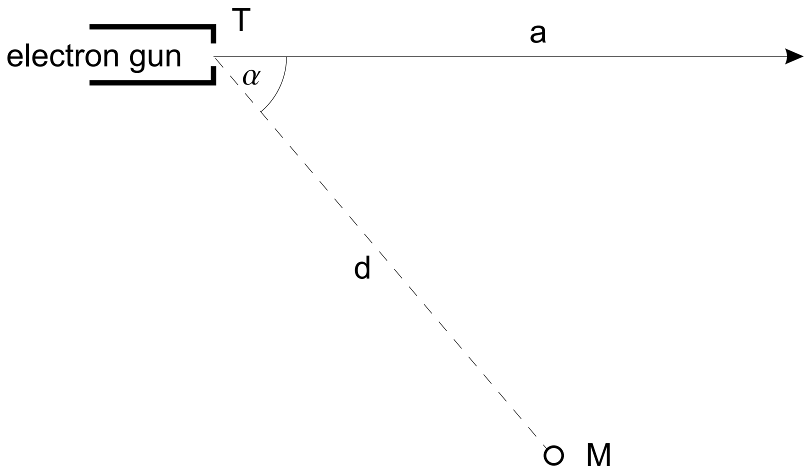

Figure 2:

a) perpendicular to the plane determined by the line a and the point M

b) parallel to the segment TM

in order that the electrons hit the target M. Find first the general solution and then substitute the following values: $U = 1000$ V, $e = 1.60 \cdot 10^{-19}$ C, $m_e = 9.11 \cdot 10^{-31}$ kg, $\alpha = 60^{\mathrm{o}}$, $d = 5.0$ cm, $B < 0.030$ T.

Solution: a) If a uniform magnetic field is perpendicular to the initial direction of motion of an electron beam, the electrons will be deflected by a force that is always perpendicular to their velocity and to the magnetic field. Consequently, the beam will be deflected into a circular trajectory. The origin of the centripetal force is the Lorentz force, so

$$Bev = \frac{m_e v^2}{r}\,. \tag{3}$$

Geometrical considerations yield that the radius of the trajectory obeys (cf. Fig. 3).

$$r = \frac{d}{2\sin\alpha}\,. \tag{4}$$

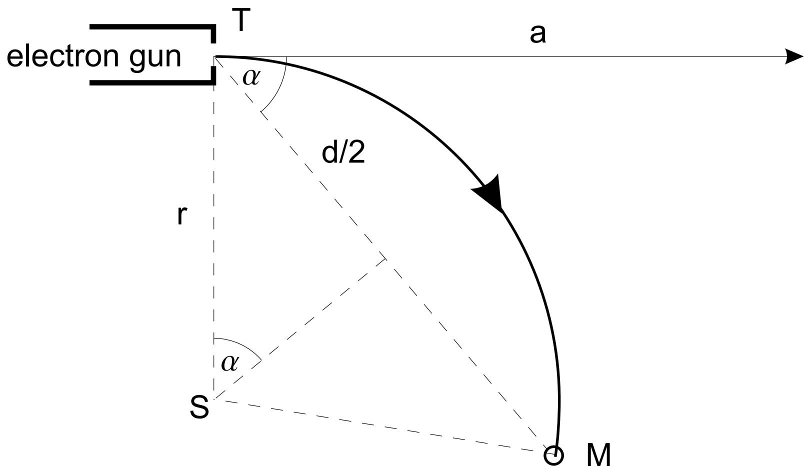

Figure 3:

The velocity of electrons can be determined from the relation between the kinetic energy of an electron and the work done on this electron by the electric field of the voltage U inside the gun,

$$\frac{1}{2} m_e v^2 = eU\,. \tag{5}$$

Using (3), (4) and (5) one obtains

$$B = m_e \sqrt{\frac{2eU}{m_e}} \frac{2\sin\alpha}{ed} = 2\sqrt{\frac{2Um_e}{e}} \frac{\sin\alpha}{d}\,.$$

Substituting the given values we have $B = 3.70 \cdot 10^{-3}$ T.

b) If a uniform magnetic field is neither perpendicular nor parallel to the initial direction of motion of an electron beam, the electrons will be deflected into a helical trajectory. Namely, the motion of electrons will be composed of an uniform motion on a circle in the plane perpendicular to the magnetic field and of an uniform rectilinear motion in the direction of the magnetic field. The component $\vec{v}_1$ of the initial velocity $\vec{v}$, which is perpendicular to the magnetic field (see Fig. 4), will manifest itself at the Lorentz force and during the motion will rotate uniformly around the line parallel to the magnetic field. The component $\vec{v}_2$ parallel to the magnetic field will remain

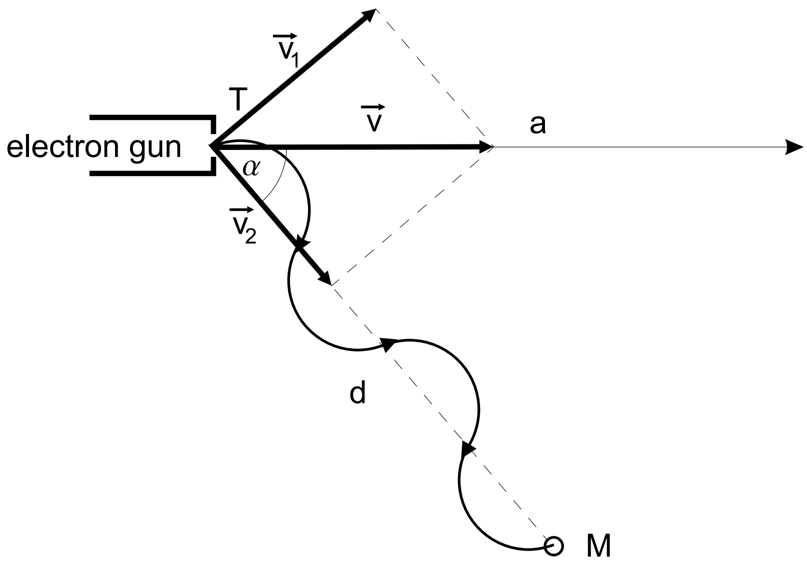

Figure 4:

constant during the motion, it will be the velocity of the uniform rectilinear motion. Magnitudes of the components of the velocity can be expressed as

$$v_1 = v \sin\alpha \quad v_2 = v \cos\alpha \,.$$

Denoting by N the number of screws of the helix we can write for the time of motion of the electron

$$t = \frac{d}{v_2} = \frac{d}{v\cos\alpha} = \frac{2\pi r N}{v_1} = \frac{2\pi r N}{v \sin\alpha} \,.$$

Hence we can calculate the radius of the circular trajectory

$$r = \frac{d \sin\alpha}{2\pi N \cos\alpha} \,.$$

However, the Lorentz force must be equated to the centripetal force

$$Bev \sin\alpha = \frac{m_e v^2 \sin^2\alpha}{r} = \frac{m_e v^2 \sin^2\alpha}{\frac{d\sin\alpha}{2\pi N \cos\alpha}} \,. \tag{6}$$

Consequently,

$$B = \frac{m_e v^2 \sin^2\alpha \, 2\pi N \cos\alpha}{d \sin\alpha \, ev \sin\alpha} = \frac{2\pi N m_e v \cos\alpha}{de} \,.$$

The magnitude of velocity v again satisfies (5), so

$$v = \sqrt{\frac{2Ue}{m_e}} \,.$$

Substituting into (6) one obtains

$$B = \frac{2\pi N \cos\alpha}{d} \sqrt{\frac{2U m_e}{e}} \,.$$

Numerically we get $B = N \cdot 6.70 \cdot 10^{-3}$ T. If $B < 0.030$ T should hold true, we have four possibilities ($N \leq 4$). Namely,

$$\begin{aligned} B_1 &= 6.70 \cdot 10^{-3} \text{ T} \,, \\ B_2 &= 13.4 \cdot 10^{-3} \text{ T} \,, \\ B_3 &= 20.1 \cdot 10^{-3} \text{ T} \,, \\ B_4 &= 26.8 \cdot 10^{-3} \text{ T} \,. \end{aligned}$$