Problems of the 8th International Physics Olympiad (Güstrow, 1975)

Gunnar Friege[1] & Gunter Lind

Introduction

The 8th International Physics Olympiad took place from the 7.7. to the 12.7. 1975 in Güstrow, in the German Democratic Republic (GDR). Altogether, 9 countries with 45 pupils participated. The teams came from Bulgaria, the German Democratic Republic, the Federal Republic of Germany (FRG), France, Poland, Rumania, Tchechoslowakia, Hungary and the USSR. The entire event took place in the pedagogic academy of Güstrow. Pupils and leaders were accommodated inside the university academy complex. On the schedule there was the competition and receptions as well as excursions to Schwerin, Rostock, and Berlin were offered. The delegation of the FRG reported of a very good organisation of the olympiad.

The problems and solutions of the 8th International Physics Olympiad were created by a commission of university physics professors and lecturers. The same commission set marking schemes and conducted the correction of the tests. The correction was carried out very quickly and was considered as righteous and, in cases of doubt, as very generous.

The main competition consisted of a 5 hour test in theory and a 4.5 hour experimental test. The time for the theoretical part was rather short and for the experimental part rather long. The problems originated from central areas of classical physics. The theoretical problems were relatively difficult, although solvable with good physics knowledge taught at school. The level of difficulty of the experimental problem was adequate. There were no additional devices necessary for the solution of the problems. Only basic formula knowledge was requested, and could be demanded from all pupils. Critics were only uttered concerning the second theoretical problem (thick lens). This problem requested relatively little physical understanding, but tested the mathematical skills and the routine in approaching problems (e.g. correct distinction of cases). However, it is also difficult to find substantial physics problems in the area of geometrical optics.

---

[1] Remark: This article was written due to the special request to us by Dr. W. Gorzkowski, in order to close one of the last few gaps in the IPhO-report collection.

Contact: Dr. Gunnar Friege, Leibniz-Institute for Science Education (IPN) at the University of Kiel, Olshausenstr. 62, 24098 Kiel, Germany, friege@ipn.uni-kiel.de

Altogether 50 points were the maximum to achieve; 30 in the theoretical test and 20 in the experimental test. The best contestant came from the USSR and had 43 points. The first prize (gold medal) was awarded with 39 points, the second prize (silver medal) with 34 points, the third prize (bronze medal) with 28 points and the fourth prize (honourable mention) with 22 points. Among the 45 contestants, 7 I. prizes, 9 II. prizes, 12 III. prizes and 8 IV. prizes were awarded, meaning that 80 % of all contestants were awarded.

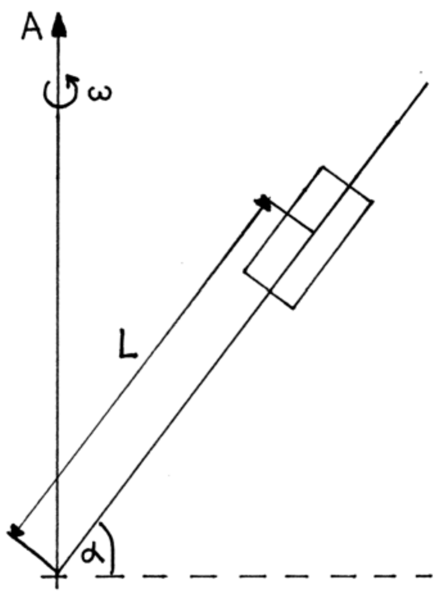
The following problem descriptions and solution are based mainly on a translation of the original German version from 1975. Because the original drafts are not well preserved, some new sketches were drawn. We also gave the problems headlines and the solutions are in more detail.

Theoretical problem 1: “Rotating rod”

A rod revolves with a constant angular velocity ω around a vertical axis A. The rod includes a fixed angle of $\pi/2-\alpha$ with the axis. A body of mass m can glide along the rod. The coefficient of friction is $\mu = \tan\beta$. The angle β is called „friction angle“.

a) Determine the angles α under which the body remains at rest and under which the body is in motion if the rod is not rotating (i.e. $\omega = 0$).

b) The rod rotates with constant angular velocity $\omega > 0$. The angle α does not change during rotation. Find the condition for the body to remain at rest relative to the rod.



You can use the following relations:

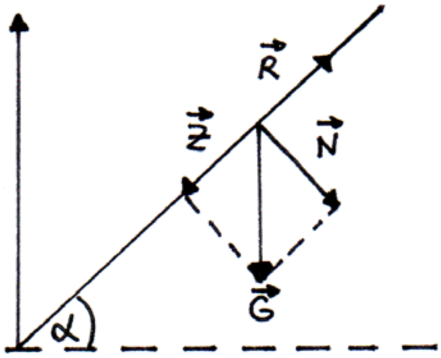
$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$

Solution of problem 1:

a) $\omega = 0$:

The forces in this case are (see figure):



$\vec{G} = \vec{Z} + \vec{N} = m \cdot \vec{g}$ (1),

$\left|\vec{Z}\right| = m \cdot g \cdot \sin\alpha = Z$ (2),

$\left|\vec{N}\right| = m \cdot g \cdot \cos\alpha = N$ (3),

$\left|\vec{R}\right| = \mu \cdot N = \mu \cdot m \cdot g \cdot \cos\alpha = R$ (4).

[$\vec{R}$: force of friction]

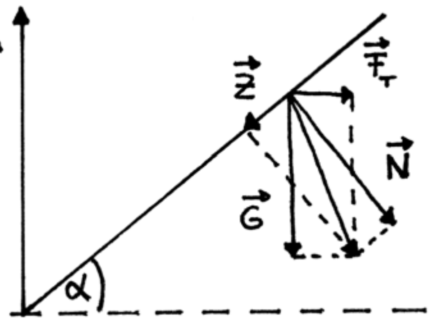
The body is at rest relative to the rod, if $Z \leq R$. According to equations (2) and (4) this is equivalent to $\tan\alpha \leq \tan\beta$. That means, the body is at rest relative to the rod for $\alpha \leq \beta$ and the body moves along the rod for $\alpha > \beta$.

b) $\omega > 0$:

Two different situations have to be considered: 1. $\alpha > \beta$ and 2. $\alpha \leq \beta$.

If the rod is moving ($\omega \neq 0$) the forces are $\vec{G} = m \cdot \vec{g}$ and $\left|\vec{F}_r\right| = m \cdot r \cdot \vec{\omega}^2$.

From the parallelogramm of forces (see figure):



$\vec{Z} + \vec{N} = \vec{G} + \vec{F}_r$ (5).

The condition of equilibrium is:

$\left|\vec{Z}\right| = \mu\left|\vec{N}\right|$ (6).

Case 1: $\vec{Z}$ is oriented downwards, i.e. $g \cdot \sin\alpha > r \cdot \omega^2 \cdot \cos\alpha$.

$$\left|\vec{Z}\right| = m \cdot g \cdot \sin\alpha - m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad \left|\vec{N}\right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

Case 2: $\vec{Z}$ is oriented upwards, i.e. $g \cdot \sin\alpha < r \cdot \omega^2 \cdot \cos\alpha$.

$$\left|\vec{Z}\right| = -m \cdot g \cdot \sin\alpha + m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad \left|\vec{N}\right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

It follows from the condition of equilibrium equation (6) that

$$\pm\left(g \cdot \sin\alpha - r \cdot \omega^2 \cdot \cos\alpha\right) = \tan\beta \cdot \left(g \cdot \cos\alpha + r \cdot \omega^2 \cdot \sin\alpha\right) \qquad (7).$$

Algebraic manipulation of equation (7) leads to:

$$g \cdot \sin(\alpha - \beta) = r \cdot \omega^2 \cdot \cos(\alpha - \beta) \qquad (8),$$

$$g \cdot \sin(\alpha + \beta) = r \cdot \omega^2 \cdot \cos(\alpha + \beta) \qquad (9).$$

That means,

$$r_{1,2} = \frac{g}{\omega^2} \cdot \tan(\alpha \mp \beta) \qquad (10).$$

The body is at rest relative to the rotating rod in the case $\alpha > \beta$ if the following inequalities hold:

$r_1 \le r \le r_2$ with $r_1, r_2 > 0$ (11)

or

$L_1 \le L \le L_2$ with $L_1 = r_1 / \cos\alpha$ and $L_2 = r_2 / \cos\alpha$ (12).

The body is at rest relative to the rotating rod in the case $\alpha \le \beta$ if the following inequalities hold:

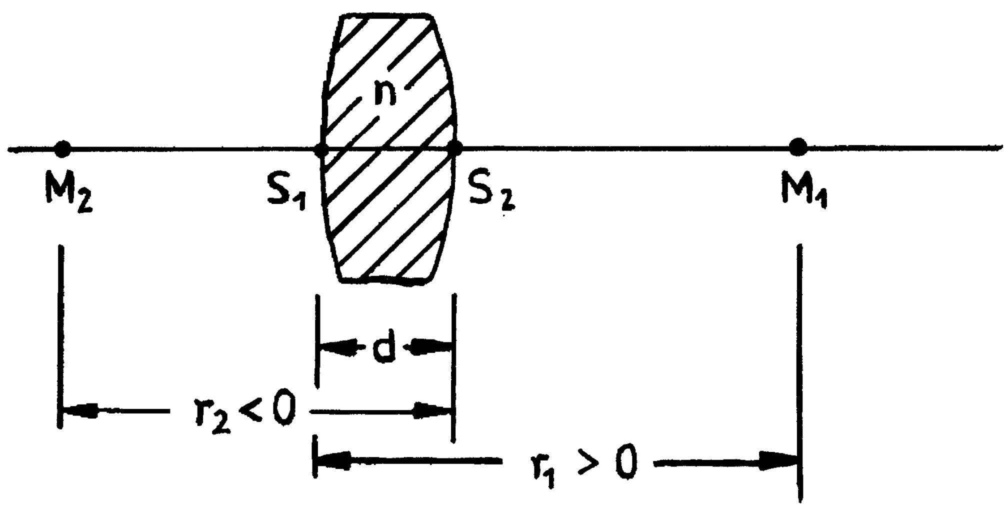
$0 \le r \le r_2$ with $r_1 = 0$ (since $r_1 < 0$ is not a physical solution), $r_2 > 0$ (13).

Inequality (13) is equivalent to

$0 \le L \le L_2$ with $L_2 = r_2 / \cos\alpha > 0$ (14).

Theoretical problem 2: "Thick lens"

The focal length f of a thick glass lens in air with refractive index n, radius curvatures r_1, r_2 and vertex distance d (see figure) is given by: $f = \dfrac{n\, r_1 r_2}{(n-1)\left[n(r_2 - r_1) + d(n-1)\right]}$



Remark: $r_i > 0$ means that the central curvature point M_i is on the right side of the aerial vertex S_i, $r_i < 0$ means that the central curvature point M_i is on the left side of the aerial vertex S_i (i = 1,2).

For some special applications it is required, that the focal length is independent from the wavelength.

a) For how many different wavelengths can the same focal length be achieved?

b) Describe a relation between r_i (i = 1,2), d and the refractive index n for which the required wavelength independence can be fulfilled and discuss this relation.
Sketch possible shapes of lenses and mark the central curvature points M_1 and M_2.

c) Prove that for a given planconvex lens a specific focal length can be achieved by only one wavelength.

d) State possible parameters of the thick lens for two further cases in which a certain focal length can be realized for one wavelength only. Take into account the physical and the geometrical circumstances.

Solution of problem 2:

a) The refractive index n is a function of the wavelength λ, i.e. $n = n(\lambda)$. According to the given formula for the focal length f (see above) which for a given f yields to an equation quadratic in n there are at most two different wavelengths (indices of refraction) for the same focal length.

b) If the focal length is the same for two different wavelengths, then the equation

$$f(\lambda_1) = f(\lambda_2) \quad or \quad f(n_1) = f(n_2) \tag{1}$$

holds. Using the given equation for the focal length it follows from equation (1):

$$\frac{n_1 r_1 r_2}{(n_1 - 1)\left[n_1(r_2 - r_1) + d(n_1 - 1)\right]} = \frac{n_2 r_1 r_2}{(n_2 - 1)\left[n_2(r_2 - r_1) + d(n_2 - 1)\right]}$$

Algebraic calculations lead to:

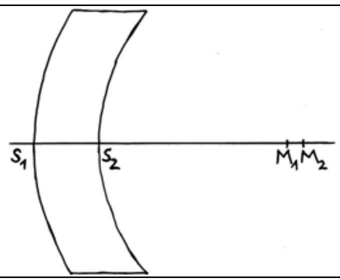
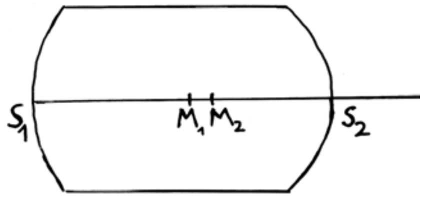
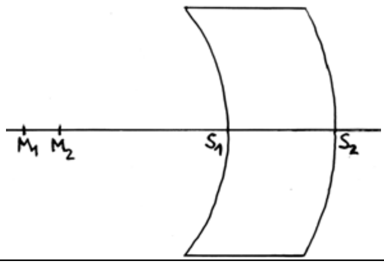
$$\mathrm{r}_1 - \mathrm{r}_2 = \mathrm{d} \cdot \left(1 - \frac{1}{\mathrm{n}_1 \mathrm{n}_2}\right) \tag{2}$$

If the values of the radii r_1, r_2 and the thickness satisfy this condition the focal length will be the same for two wavelengths (indices of refraction). The parameters in this equation are subject to some physical restrictions: The indices of refraction are greater than 1 and the thickness of the lens is greater than 0 m. Therefore, from equation (2) the relation

$$d > r_1 - r_2 > 0 \tag{3}$$

is obtained.

The following table shows a discussion of different cases:

r_1	r_2	***condition***	***shape of the lens***	***centre of curvature***
$r_1 > 0$	$r_2 > 0$	$0 < r_1 - r_2 < d$ or $r_2 < r_1 < d + r_2$		*M_2 is always right of M_1.* $\overline{M_1M_2} < \overline{S_1S_2}$
$r_1 > 0$	$r_2 < 0$	$r_1 + \lvert r_2 \rvert < \mathrm{d}$		*Order of points:* $S_1M_1M_2S_2$
$r_1 < 0$	$r_2 > 0$	*never fulfilled*		
$r_1 < 0$	$r_2 < 0$	$0 < \lvert r_2 \rvert - \lvert r_1 \rvert < d$ or $\lvert r_1 \rvert < \lvert r_2 \rvert < d + \lvert r_1 \rvert$		*M_2 is always right of M_1.* $\overline{M_1M_2} < \overline{S_1S_2}$

c) The radius r_1 or the radius r_2 is infinite in the case of the planconvex lens. In the following it is assumed that r_1 is infinite and r_2 is finite.

$$\lim_{r_1\to\infty} f = \lim_{r_1\to\infty} \frac{n\,r_2}{(n-1)\left[n\left(\frac{r_2}{r_1}-1\right)+(n-1)\frac{d}{r_1}\right]} = \frac{r_2}{1-n} \tag{4}$$

Equation (4) means, that for each wavelength (refractive index) there exists a different value of the focal length.

d) From the given formula for the focal length (see problem formulation) one obtains the following quadratic equation in n:

$$A\cdot n^2 + B\cdot n + C = 0 \tag{5}$$

with $\quad A = (r_2 - r_1 + d)\cdot f\ ,\ B = -\left[f\cdot(r_2 - r_1) + 2\cdot f\cdot d + r_1\cdot r_2\right]$ and $C = f\cdot d$.

Solutions of equation (5) are:

$$n_{1,2} = -\frac{B}{2 \cdot A} \pm \sqrt{\frac{B^2}{4 \cdot A^2} - \frac{C}{A}} \qquad (6).$$

Equation (5) has only one physical correct solution, if...

I) A = 0 (i.e., the coefficient of n^2 in equation (5) vanishes)

In this case the following relationships exists:

$$r_1 - r_2 = d \qquad (7),$$

$$n = \frac{f \cdot d}{f \cdot d + r_1 \cdot r_2} > 1 \qquad (8).$$

II) B = 0 (i.e. the coefficient of n in equation (5) vanishes)

In this case the equation has a positive and a negative solution. Only the positve solution makes sense from the physical point of view. It is:

$$f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2 = 0 \qquad (9),$$

$$n^2 = -\frac{C}{A} = -\frac{d}{(r_2 - r_1 + d)} > 1 \qquad (10),$$

III) $B^2 = 4\,AC$

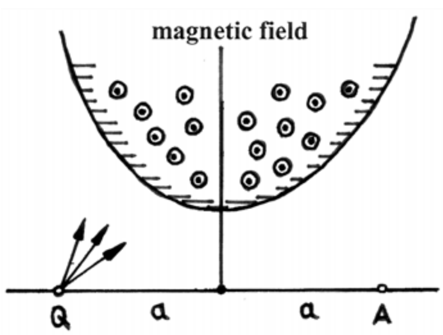
In this case two identical real solutions exist. It is:

$$\left[f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2\right]^2 = 4 \cdot (r_2 - r_1 + d) \cdot f^2 \cdot d \qquad (11),$$

$$n = -\frac{B}{2 \cdot A} = \frac{f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2}{2 \cdot f\,(r_2 - r_1 + d)} > 1 \qquad (12).$$

Theoretical problem 3: "Ions in a magnetic field"

A beam of positive ions (charge +e) of the same and constant mass *m* spread from point Q in different directions in the plane of paper (see figure[2]). The ions were accelerated by a voltage *U*. They are deflected in a uniform magnetic field *B* that is perpendicular to the plane of paper. The boundaries of the magnetic field are made in a way that the initially diverging ions are focussed in point A ($\overline{QA} = 2 \cdot a$). The trajectories of the ions are symmetric to the middle perpendicular on $\overline{QA}$.



[2] Remark: This illustrative figure was <u>not</u> part of the original problem formulation.

Among different possible boundaries of magnetic fields a specific type shall be considered in which a contiguous magnetic field acts around the middle perpendicular and in which the points Q and A are in the field free area.

a) Describe the radius curvature R of the particle path in the magnetic field as a function of the voltage U and the induction B.
b) Describe the characteristic properties of the particle paths in the setup mentioned above.
c) Obtain the boundaries of the magnetic field boundaries by geometrical constructions for the cases $R < a$, $R = a$ and $R > 0$.
d) Describe the general equation for the boundaries of the magnetic field.

Solution of problem 3:

a) The kinetic energy of the ion after acceleration by a voltage U is:

$$\tfrac{1}{2} mv^2 = eU \qquad (1).$$

From equation (1) the velocity of the ions is calculated:

$$v = \sqrt{\frac{2 \cdot e \cdot U}{m}} \qquad (2).$$

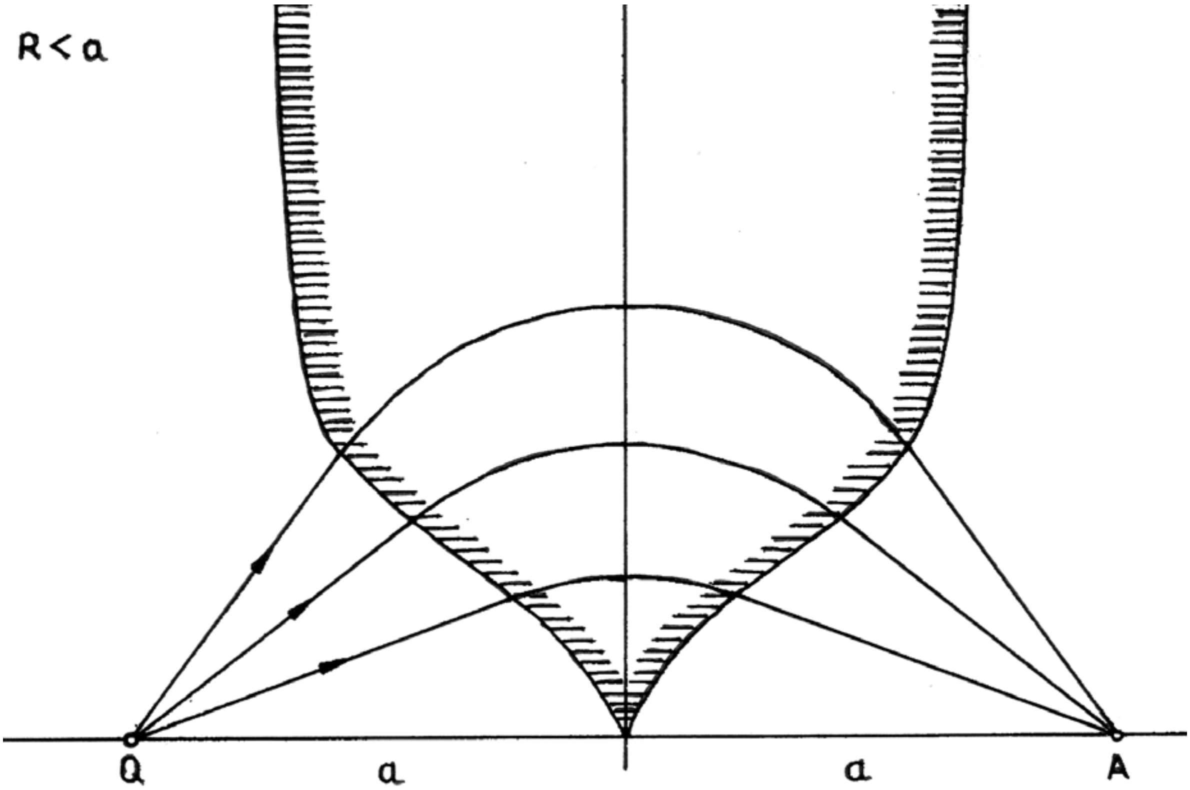
On a moving ion (charge e and velocity v) in a homogenous magnetic field B acts a Lorentz force F. Under the given conditions the velocity is always perpendicular to the magnetic field. Therefore, the paths of the ions are circular with Radius R. Lorentz force and centrifugal force are of the same amount:

$$e \cdot v \cdot B = \frac{m \cdot v^2}{R} \qquad (3).$$

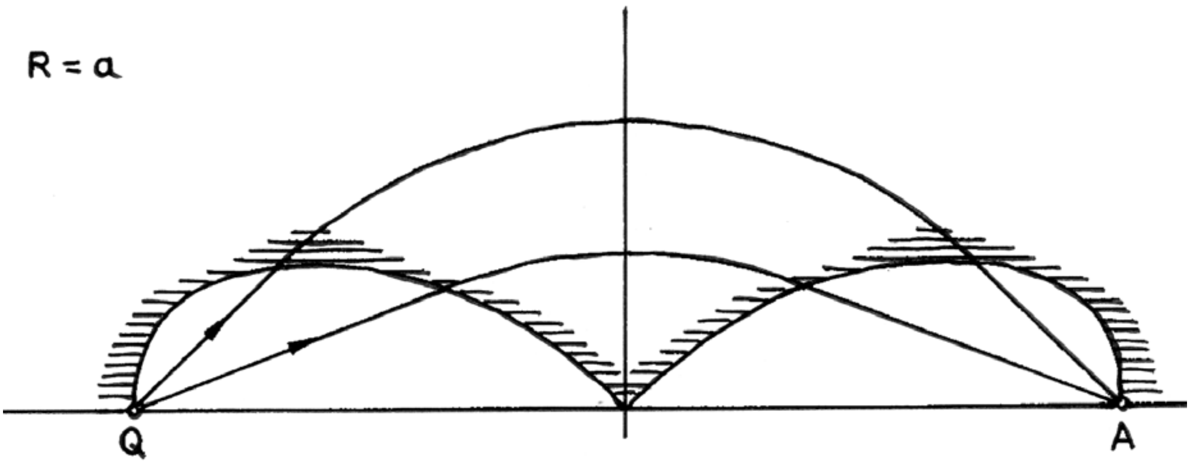
From equation (3) the radius of the ion path is calculated:

$$R = \frac{1}{B}\sqrt{\frac{2 \cdot m \cdot U}{e}} \qquad (4).$$

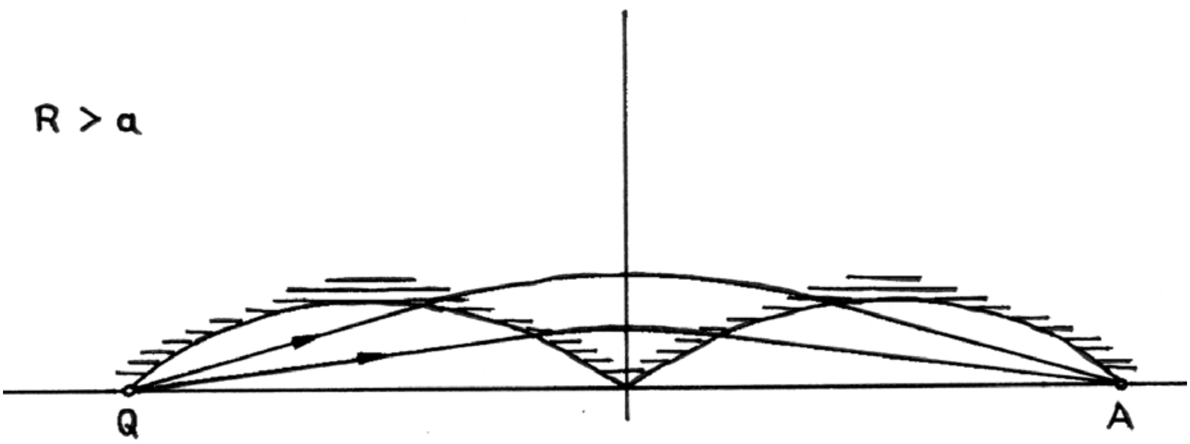
b) All ions of mass m travel on circular paths of radius $R = v{\cdot}m / e{\cdot}B$ inside the magnetic field. Leaving the magnetic field they fly in a straight line along the last tangent. The centres of curvature of the ion paths lie on the middle perpendicular on $\overline{QA}$ since the magnetic field is assumed to be symmetric to the middle perpendicular on $\overline{QA}$. The paths of the focussed ions are above $\overline{QA}$ due to the direction of the magnetic field.



R < a
Q
a
a
A



R = a
Q
A

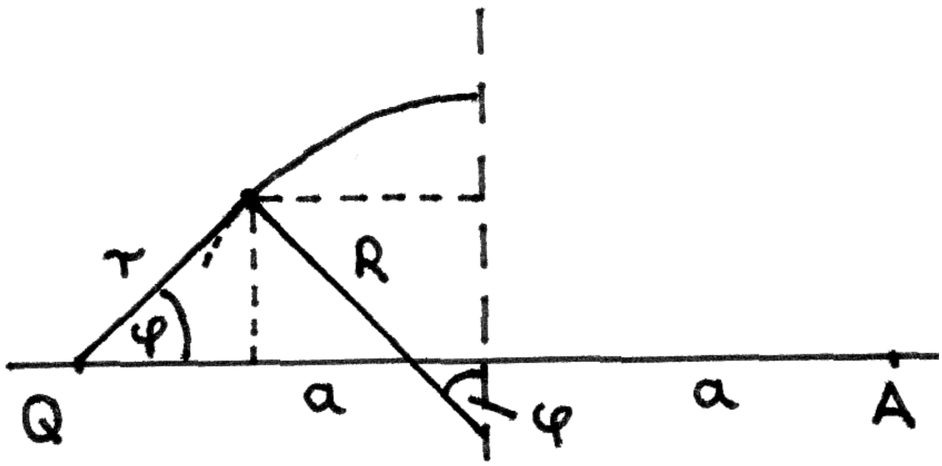


R > a
Q
A

c) The construction method of the boundaries of the magnetic fields is based on the considerations in part b:

- Sketch circles of radius R and different centres of curvature on the middle perpendicular on $\overline{\mathrm{QA}}$.
- Sketch tangents on the circle with either point Q or point A on these straight lines.
- The points of tangency make up the boundaries of the magnetic field. If $R > a$ then not all ions will reach point A. Ions starting at an angle steeper than the tangent at Q, do not arrive in A. The figure on the last page shows the boundaries of the magnetic field for the three cases $R < a$, $R = a$ and $R > a$.

d) It is convenient to deduce a general equation for the boundaries of the magnetic field in polar coordinates (r, φ) instead of using cartesian coordinates (x, y).



The following relation is obtained from the figure:

$$r \cdot \cos\varphi + R\sin\varphi = a \qquad (7).$$

The boundaries of the magnetic field are given by:

$$r = \frac{a}{\cos\varphi}\left(1 - \frac{R}{a}\sin\varphi\right) \qquad (8).$$

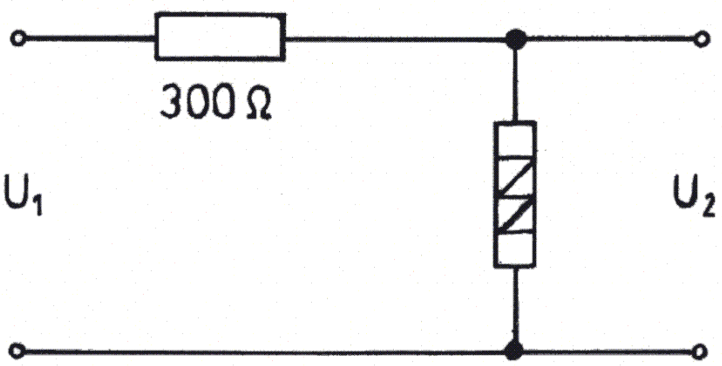
Experimental problem: "Semiconductor element"

In this experiment a semiconductor element ( ), an adjustable resistor (up to 140 Ω), a fixed resistor (300 Ω), a 9-V-direct voltage source, cables and two multimeters are at disposal. It is not allowed to use the multimeters as ohmmeters.

a) Determine the current-voltage-characteristics of the semiconductor element taking into account the fact that the maximum load permitted is 250 mW. Write down your data in tabular form and plot your data. Before your measurements consider how an overload of the semiconductor element can surely be avoided and note down your thoughts. Sketch the circuit diagram of the chosen setup and discuss the systematic errors of the circuit.

b) Calculate the resistance (dynamic resistance) of the semiconductor element for a current of 25 mA.

c) Determine the dependence of output voltage U_2 from the input voltage U_1 by using the circuit described below. Write down your data in tabular form and plot your data.



The input voltage U_1 varies between 0 V and 9 V. The semiconductor element is to be placed in the circuit in such a manner, that U_2 is as high as possible. Describe the entire circuit diagram in the protocol and discuss the results of the measurements.

d) How does the output voltage U_2 change, when the input voltage is raised from 7 V to 9 V? Explain qualitatively the ratio $\Delta U_1 / \Delta U_2$.

e) What type of semiconductor element is used in the experiment? What is a practical application of the circuit shown above?

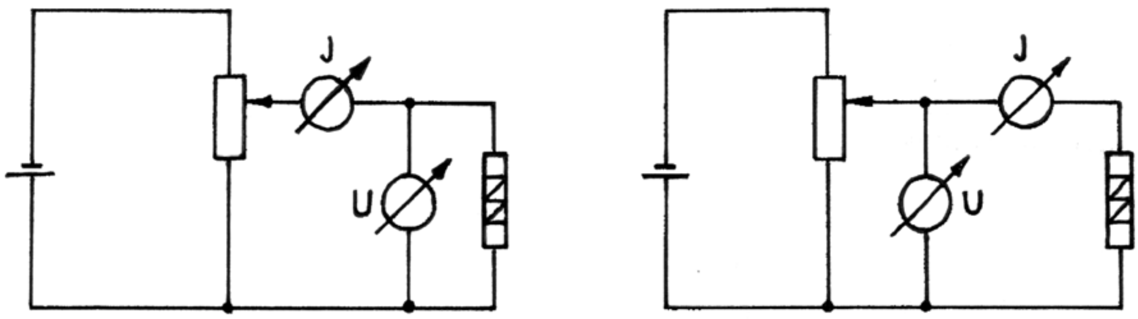
Hints: The multimeters can be used as voltmeter or as ammeter. The precision class of these instruments is 2.5% and they have the following features:

measuring range	50 μA	300 μA	3 mA	30 mA	300 mA	0,3 V	1 V	3 V	10 V
internal resistance	2 kΩ	1 kΩ	100 Ω	10 Ω	1 Ω	6 kΩ	20 kΩ	60 kΩ	200 kΩ

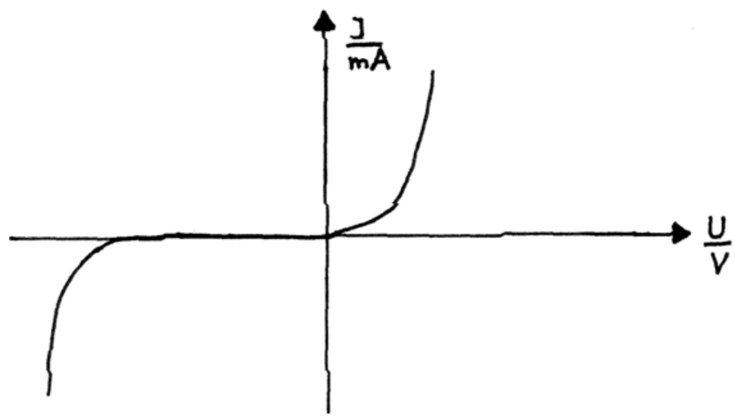
Solution of the experimental problem:

a) Some considerations: the product of the voltage across the semiconductor element U and current I through this element is not allowed to be larger than the maximum permitted load of 250 mW. Therefore the measurements have to be processed in a way, that the product $U \cdot I$ is always smaller than 250 mW.

The figure shows two different circuit diagram that can be used in this experiment:



The complete current-voltage-characteristics look like this:



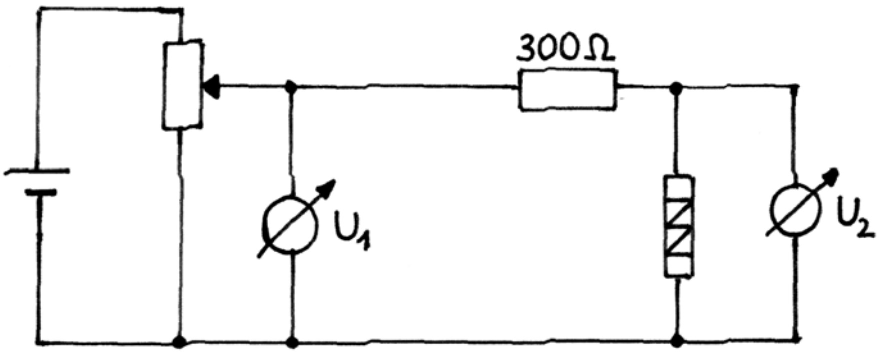
The systematic error is produced by the measuring instruments. Concerning the circuit diagram on the left ("Stromfehlerschaltung"), the ammeter also measures the current running through the voltmeter. The current must therefore be corrected. Concerning the circuit diagram on the right ("Spannungsfehlerschaltung") the voltmeter also measures the voltage across the ammeter. This error must also be corrected. To this end, the given internal resistances of the measuring instruments can be used. Another systematic error is produced by the uncontrolled temperature increase of the semiconductor element, whereby the electric conductivity rises.

b) The dynamic resistance is obtained as ratio of small differences by

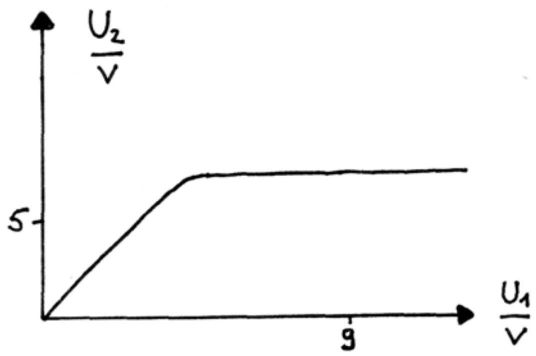
$$R_i = \frac{\Delta U}{\Delta I} \qquad (1).$$

The dynamic resistance is different for the two directions of the current. The order of magnitude in one direction (backward direction) is 10 Ω ± 50% and the order of magnitude in the other direction (flux direction) is 1 Ω ± 50%.

c) The complete circuit diagram contains a potentiometer and two voltmeters.



The graph of the function $U_2 = f(U_1)$ has generally the same form for both directions of the current, but the absolute values are different. By requesting that the semiconductor element has to be placed in such a way, that the output voltage U_2 is as high as possible, a backward direction should be used.



Comment: After exceeding a specific input voltage U_1 the output voltage increases only a little, because with the alteration of U_1 the current I increases (breakdown of the diode) and therefore also the voltage drop at the resistance.

d) The output voltages belonging to U_1 = 7 V and U_1 = 9 V are measured and their difference ΔU_2 is calculated:

$\Delta U_2 = 0.1$ V ± 50% (2).

Comment: The circuit is a voltage divider circuit. Its special behaviour results from the different resistances. The resistance of the semiconductor element is much smaller than the resistance. It changes nonlinear with the voltage across the element. From $R_i << R_V$ follows $\Delta U_2 < \Delta U_1$ in the case of $U_1 > U_2$.

e) The semiconductor element is a Z-diode (Zener diode); also correct: diode and rectifier. The circuit diagram can be used for stabilisation of voltages.

Marking scheme

Problem 1: “Rotating rod” (10 points)

Part a	1 point
Part b – cases 1. and 2.	1 point
– forces and condition of equilibrium	1 point
– case Z downwards	2 points
– case Z upwards	2 points
– calculation of $r_{1,2}$	1 point
– case $\alpha > \beta$	1 point
– case $\alpha \leq \beta$	1 point

Problem 2: “Thick lens” (10 points)

Part a	1 point
Part b – equation (1), equation (2)	2 points
– physical restrictions, equation (3)	1 point
– discussion of different cases	2 points
– shapes of lenses	1 point
Part c – discussion and equation (4)	1 point
Part d	2 point

Problem 3: “Ions in a magnetic field” (10 points)

Part a – derivation of equations (1) and (2)	1 point
– derivation of equation (4)	1 point
Part b – characteristics properties of the particle paths	3 points
Part c – boundaries of the magnetic field for the three cases	3 points
Part d	2 points

Experimental problem: “Semiconductor element” (20 points)

Part a – considerations concerning overload, circuit diagram, experiment and measurements, complete current-voltage- -characteristics discussion of the systematic errors	6 points
Part b – equation (1) dynamic resistance for both directions correct results within ±50%	3 points
Part c – complete circuit diagram, measurements, graph of the function $U_2 = f(U_1)$, correct comment	5 points
Part d – correct ΔU_2 within ±50%, correct comment	3 points
Part e – Zener-diode (diode, rectifier) and stabilisation of voltages	3 points

Remarks: If the diode is destroyed two points are deducted.

If a multimeter is destroyed five points are deducted.