17th Asian Physics Olympiad 1-9 May 2016

Theoretical Question – T3

Magnetic field effects on superconductors

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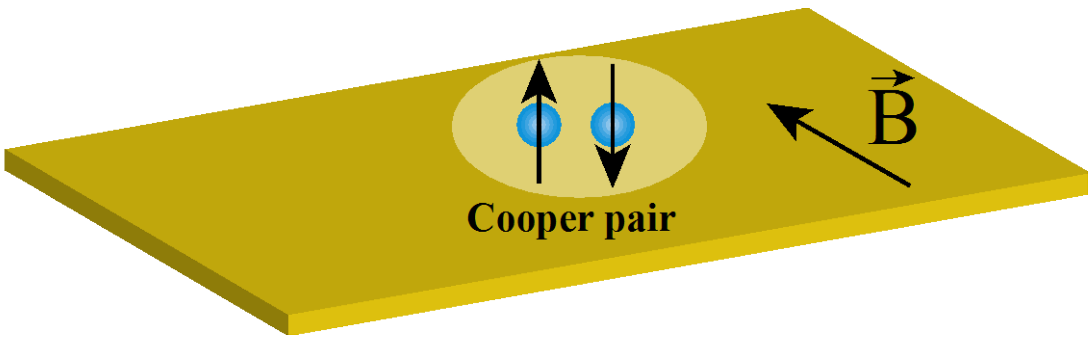
Conventional Superconductors

1. Two electron with opposite spins can form a Cooper pair and lower than energy by 2Δ.

The total energy of the pair of electrons becomes:

$$\frac{p_1^2}{2m_e}+\frac{p_2^2}{2m_e}-2\Delta$$



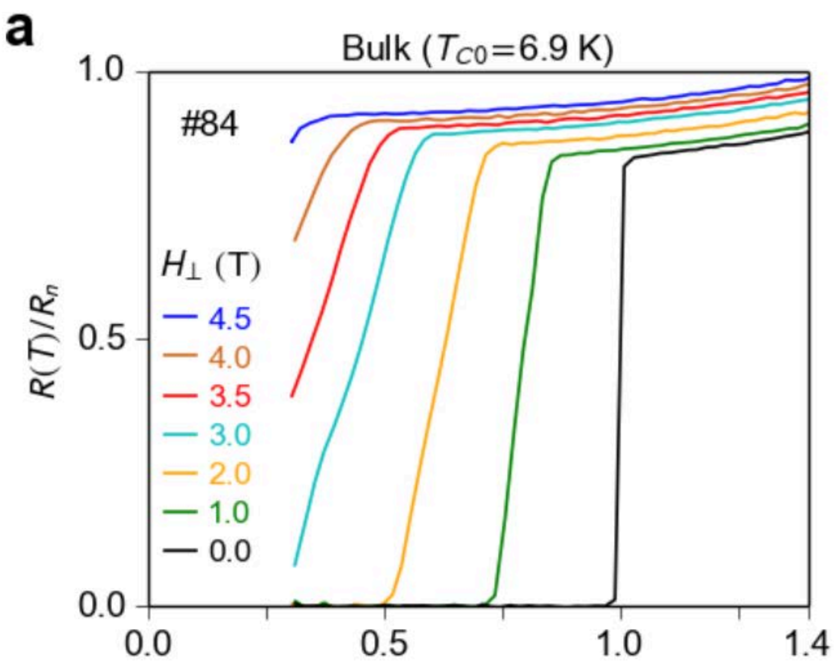
2. The Cooper pairs can flow without resistance and result in superconductivity

Ising Superconductors

- Usually, magnetic field can destroy superconductivity, which we will explain how later.
- Recently, our experimental colleagues discovered a new type of superconductor which we called Ising superconductors. They are very robust against external magnetic fields.

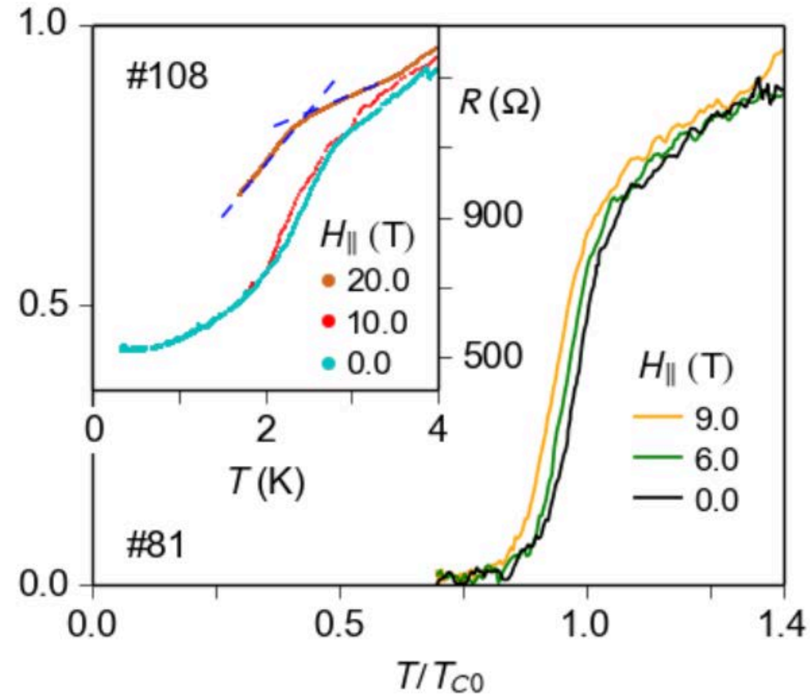
Magnetic field effects on resistance

Conventional superconductors



T/Tc

Ising Superconductor



Ising Superconductors

1. **Two Dimensional Ising Superconductivity in Gated MoS_2**

J. M. Lu, O. Zeliuk, I. Leermakers, Noah F. Q. Yuan, U. Zeitler, ***K. T. Law****, J. T. Ye*

Science 350, 1353 (2015).

2. Evidence of Ising pairing in superconducting $NbSe_2$ atomic layers

Xiaoxiang Xi, Zefang Wang, Weiwei Zhao, Ju-Hyun Park, ***K. T. Law****, Helmuth Berger, László Forró, Jie Shan, Kin Fai Mak*

Nature Physics 12, 139-143 (2016).

3. Superconductivity protected by spin-valley locking in gate-tuned MoS_2

Yu Saito et al.

Nature Physics **12**, 144–149 (2016).

Paramagnetic Effects 1

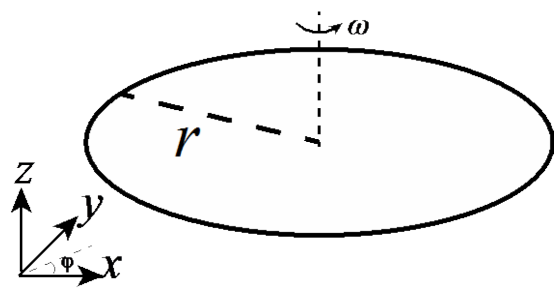
However, in the presence of a magnetic field, forming Cooper pairs may not be the best way to lower the energy of the system because:

1. Forming a Cooper pair requires two electrons with opposite spin.
2. Magnetic field try to align electron spins parallel to the magnetic field direction.

Which one will win? Depending on the magnetic field strength. Students will work out the so-called Pauli limit at which the magnetic field will win and destroy superconductivity.

Part A: Electron Model 1

Problem A1: Assume an electron is a ring with charge -e and mass m_e



Work out the spin angular momentum of the electron.

Part A: Electron Model 2

Problem A2: Magnetic moment magnitude is *M = IA.* Here, *I* is current, *A* is area of the ring.

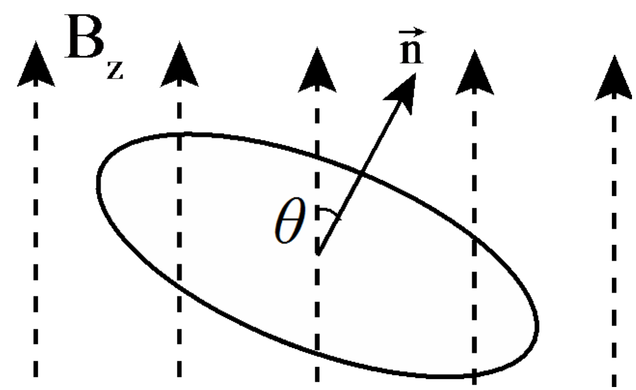
Magnetic moment direction is parallel to angular momentum.

Work out the relationship between magnetic moment and angular momentum: $\vec{M} = -\frac{e\vec{L}}{2m}$

Part A: Electron Model 3

Problem A3: Potential energy of a magnetic moment in a magnetic field.

$$U = -M \bullet B$$



It tells you that the magnetic field tries to align the magnetic moments of electrons so that they are parallel to the magnetic field.

Part A: Electron Model 4

Problem A4: An electron carries an intrinsic spin angular momentum

$$M_z = \frac{-e}{2m_e} S_z \qquad S_z = \frac{1}{2}\hbar$$

The potential energies for spin up and spin down electrons:

$$U_{\text{up}} = \frac{1}{2}\mu_B B \qquad U_{\text{down}} = -\frac{1}{2}\mu_B B$$

Part A: Electron Model 5

Problem A5: The potential energy according to quantum mechanics is twice the values found in A4. Calculate the potential energy of an electron in a 1 Tesla magnetic field:

$$\tilde{U}_{\text{up}} = 5.788 \times 10^{-5}\,\text{eV} \cdot \text{T}^{-1}$$

$$\tilde{U}_{\text{down}} = -5.788 \times 10^{-5}\,\text{eV} \cdot \text{T}^{-1}$$

Part B: Paramagnetic Effects of Magnetic fields 1

- Problem B1:

Assuming that the effect of the external magnetic field is only on the spin of the electrons but not on the orbital motion of the electrons. What is the energy of the Cooper pair under a uniform magnetic field $\vec{B} = (B_x, 0, 0)$? Recall that the electrons which form a Cooper pair must have opposite spins.

Answer: $E_S = \frac{p_1^2}{2m_e} + \frac{p_2^2}{2m_e} - 2\Delta$

Part B: Paramagnetic Effects of Magnetic fields 2

Problem B2: In the normal state (non-superconducting state), electrons do not form Cooper pairs. What is the lowest energy for the two electrons under a uniform in-plane magnetic field pointing to the *x*-direction?

Answer:

$$\begin{aligned} E_N &= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - 2\mu_\mathrm{B} B_x \\ &= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{e\hbar}{m_\mathrm{e}} B_x \end{aligned}$$

Part B: Paramagnetic Effects of Magnetic fields 3

Problem B3: Find the critical magnetic field B_p at which

$$E_N = E_S$$

Answer: $$B_P = \frac{\Delta}{\mu_B} = \frac{2m_e\Delta}{e\hbar}$$

For B > B_p, the non-superconducting state will have lower energy than the superconducting state and superconductivity is destroyed.

B_p is called the Pauli limit.

Part C: Diamagnetic effects 1

Problem C1: Magnetic field can also increase the energy of the Cooper pairs by changing its orbital motions because the Cooper pairs are charged. The superconducting state energy minus the non-superconducting state energy is:

$$F = \int_{-\infty}^{+\infty} \psi \left(-\alpha\psi - \frac{\hbar^2}{4m_\mathrm{e}} \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{e^2 B_z^2 x^2}{m_\mathrm{e}} \psi \right) \mathrm{d}x$$

Given $\psi(x) = \left(\frac{2\lambda}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\lambda x^2}$

Find: λ

Part C: Diamagnetic effects 2

Problem C2: Work out the critical value of B_z at which the superconducting state is no longer energetically favorable.

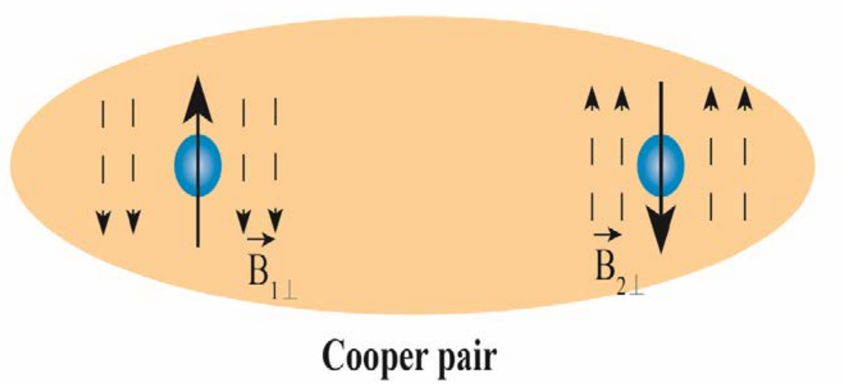
Answer: First minimize the energy difference

$$F_{\min}(\psi) = \frac{\hbar e B_z}{2m_{\mathrm{e}}} - \alpha$$

Second: $$B_z = \frac{2m_e\alpha}{e\hbar}$$

Part D: Ising Superconductors 1

Problem D1: Electrons experience internal magnetic fields.



The energy of the Cooper pair becomes:

$$E_{\mathrm{I}} = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - 2\Delta - \frac{e\hbar}{m_{\mathrm{e}}} B_z$$

Part D: Ising Superconductors 2

Problem D2: Energy of electrons in the non-superconducting states.

$$E_{||} = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{e\hbar}{m_e}\sqrt{B_x^2 + B_z^2}$$

Part D: Ising Superconductors 3

Problem D3: To destroy superconductivity, we need:

$$B_x > \frac{\sqrt{\Delta^2 + 2\Delta\mu_{\mathrm{B}}B_z}}{\mu_{\mathrm{B}}}$$