17th Asian Physics Olympiad 1-9 May 2016

Experimental Problem – E1

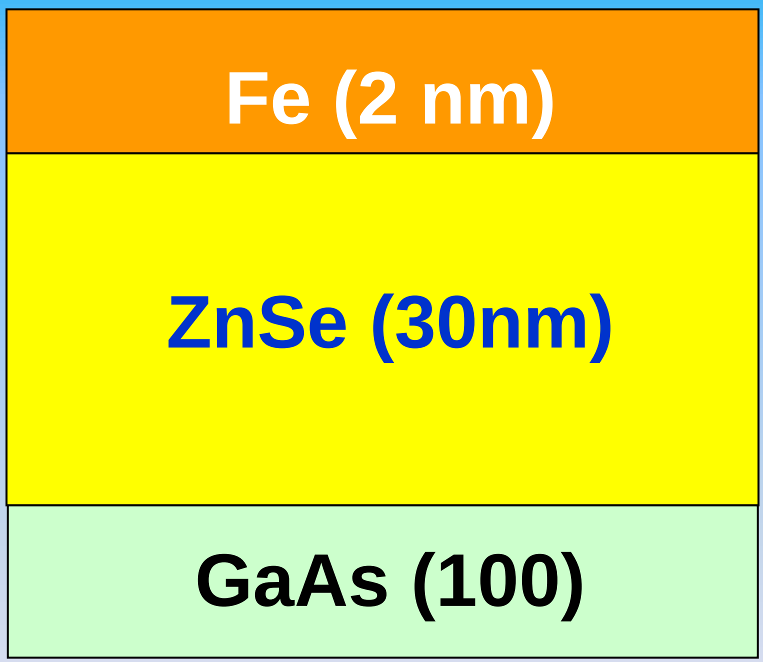
Reflected Optical Diffraction Patterns from One-Dimensional Structures

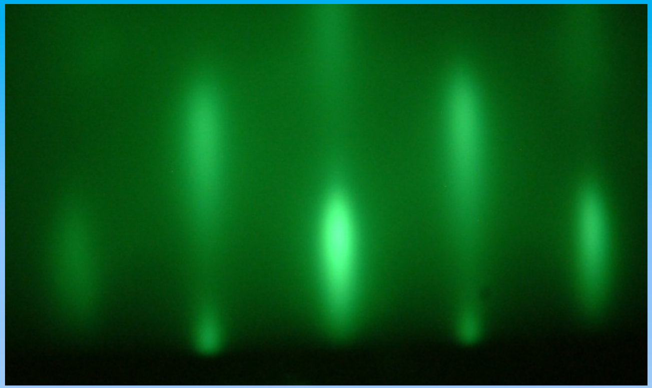
Philip I K Sou (蘇蔭強)

Our Discovery: self-assembled ZnSe nanograting

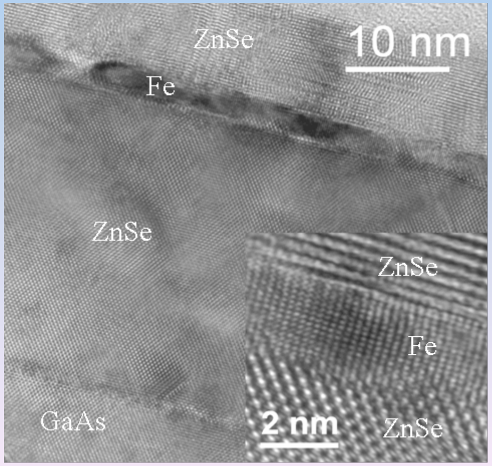
G.Wang,……I.K SOU, Nanotechnology, 20, 215607 (2009)

Sample: Fe/ZnSe bilayer





The RHEED pattern taken right after the growth of an Fe layer on top of a ZnSe layer.



Cross-sectional HRTEM image

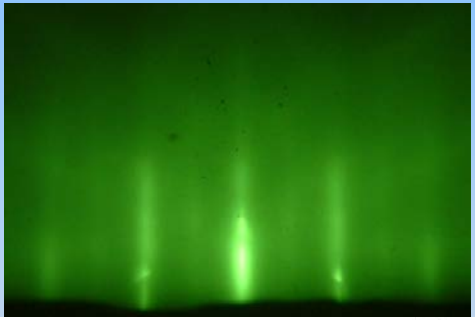
RHEED observation of an annealed Fe/ZnSe bilayer

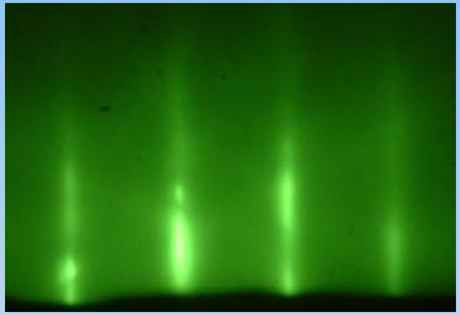
Fe (2nm)/ZnSe(30nm) bilayer annealed at 460^0C for 10min

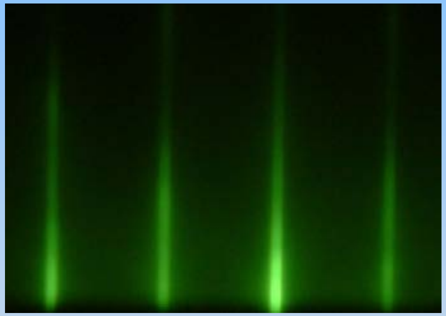
Standard RHEED Pattern for smooth thin films

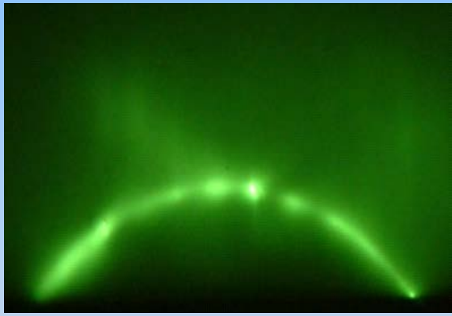
Unusual RHEED observation

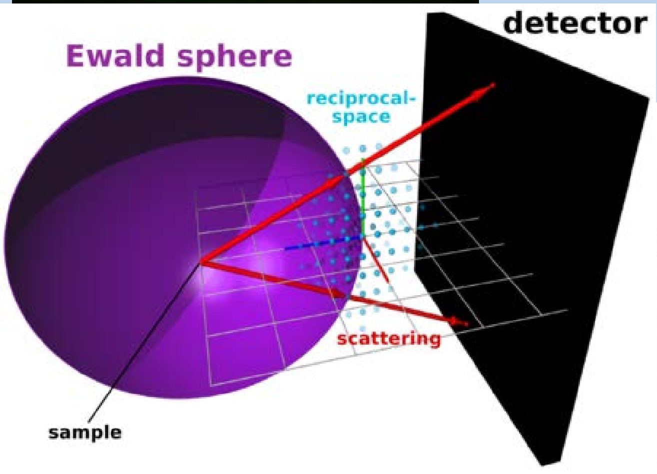
0^0 90^0 0^0 90^0

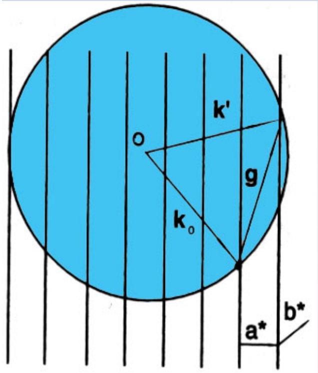






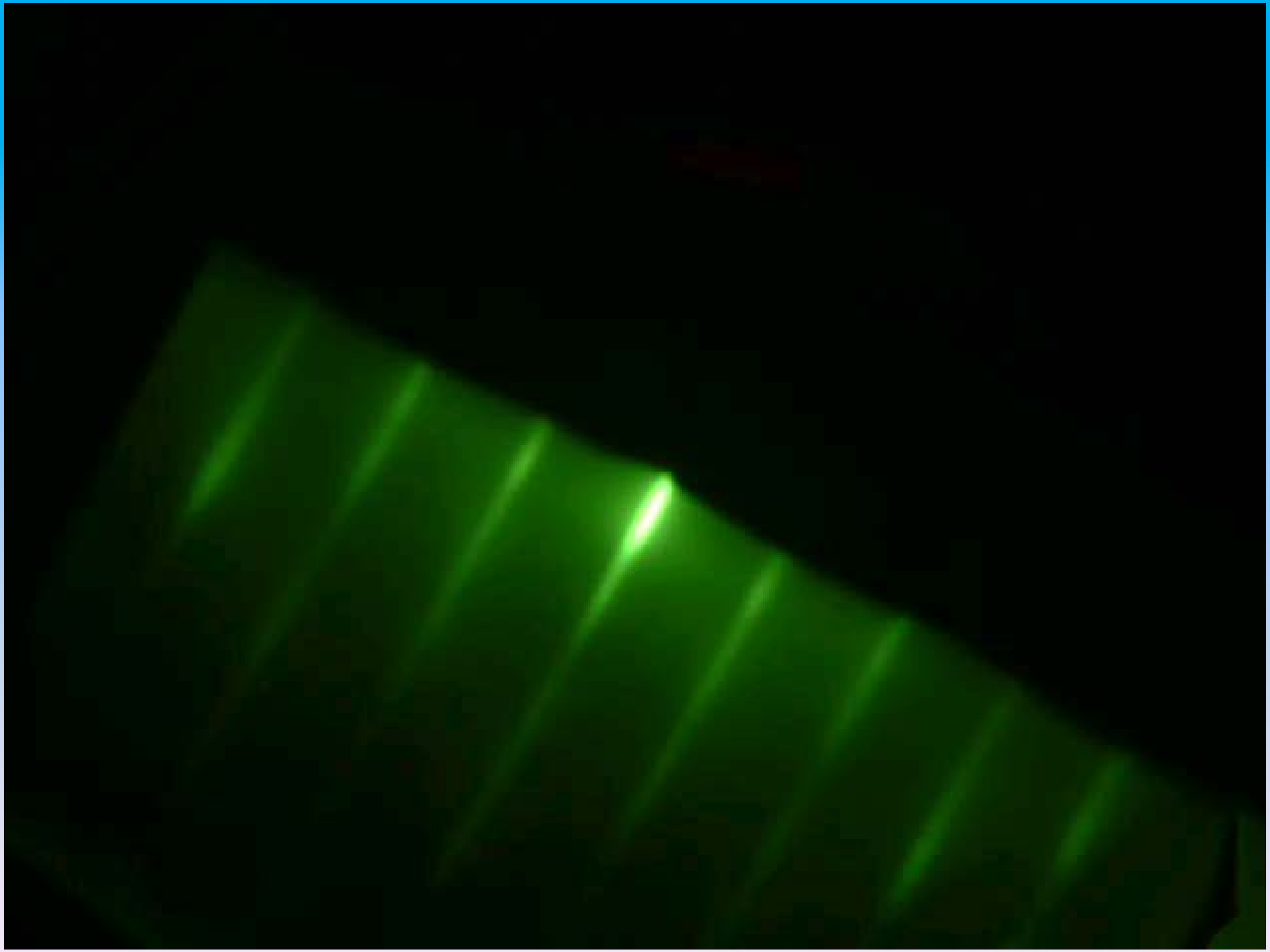






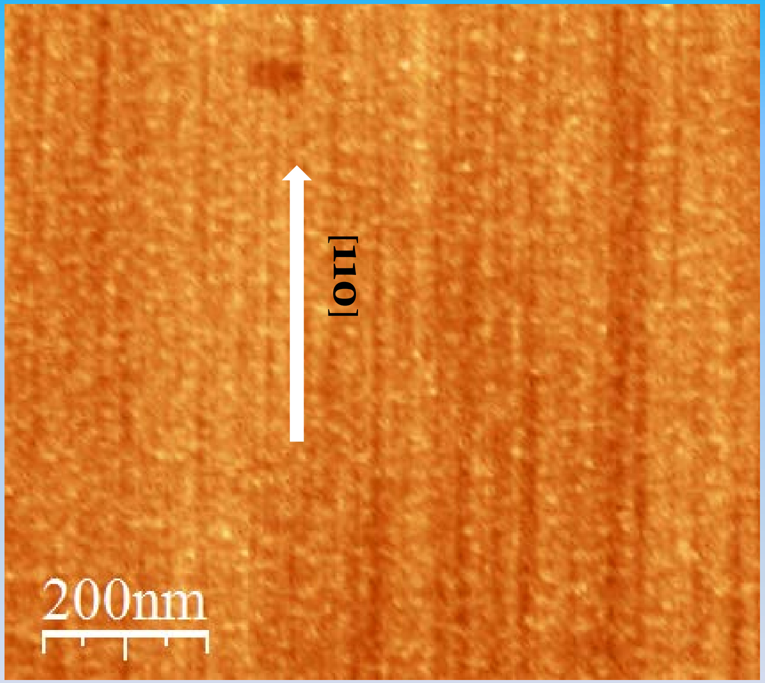
What are the corresponding reciprocal lattice of this bilayer?

RHEED Movie of the annealed Fe/ZnSe bilayer

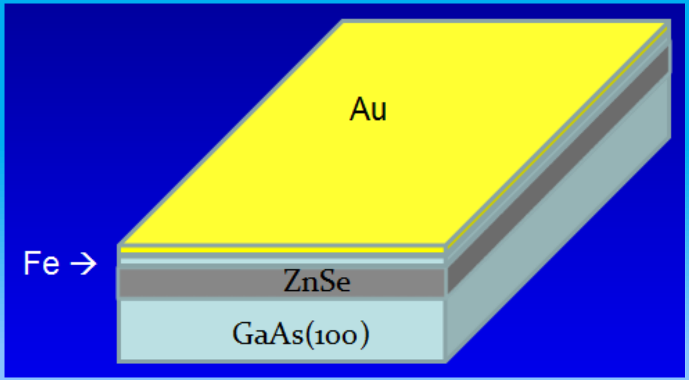


Ex-situ AFM imaging

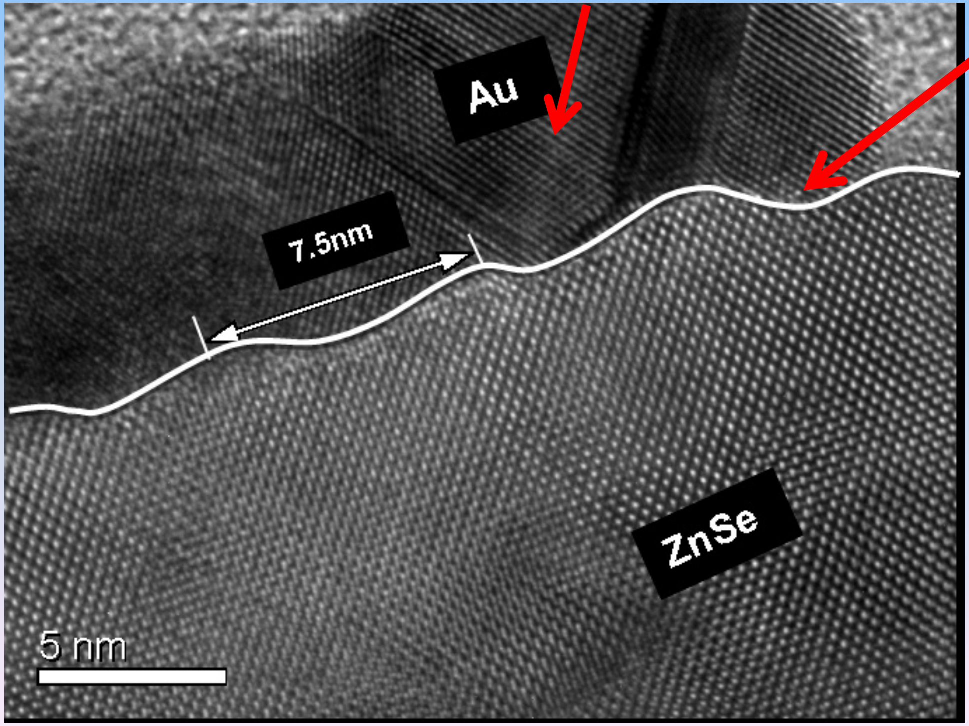
Its *ex-situ* AFM image clearly shows a 1D nanostructure aligned along the [110] direction though the resolution is poor due to air contamination.



HRTEM-cross-sectional image of an annealed Fe/ZnSe bilayer with a Au cap



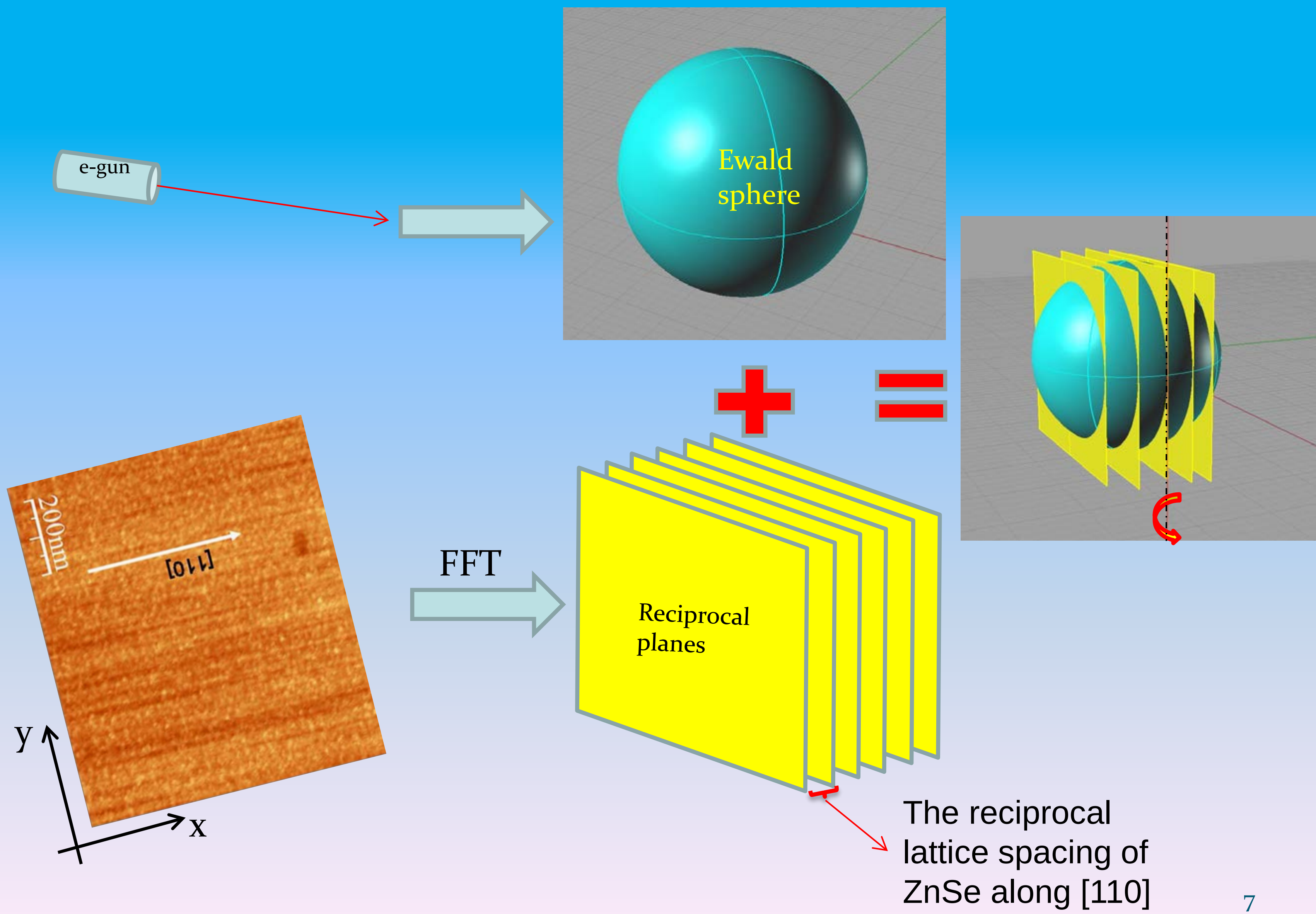
Au film used to protect the surface



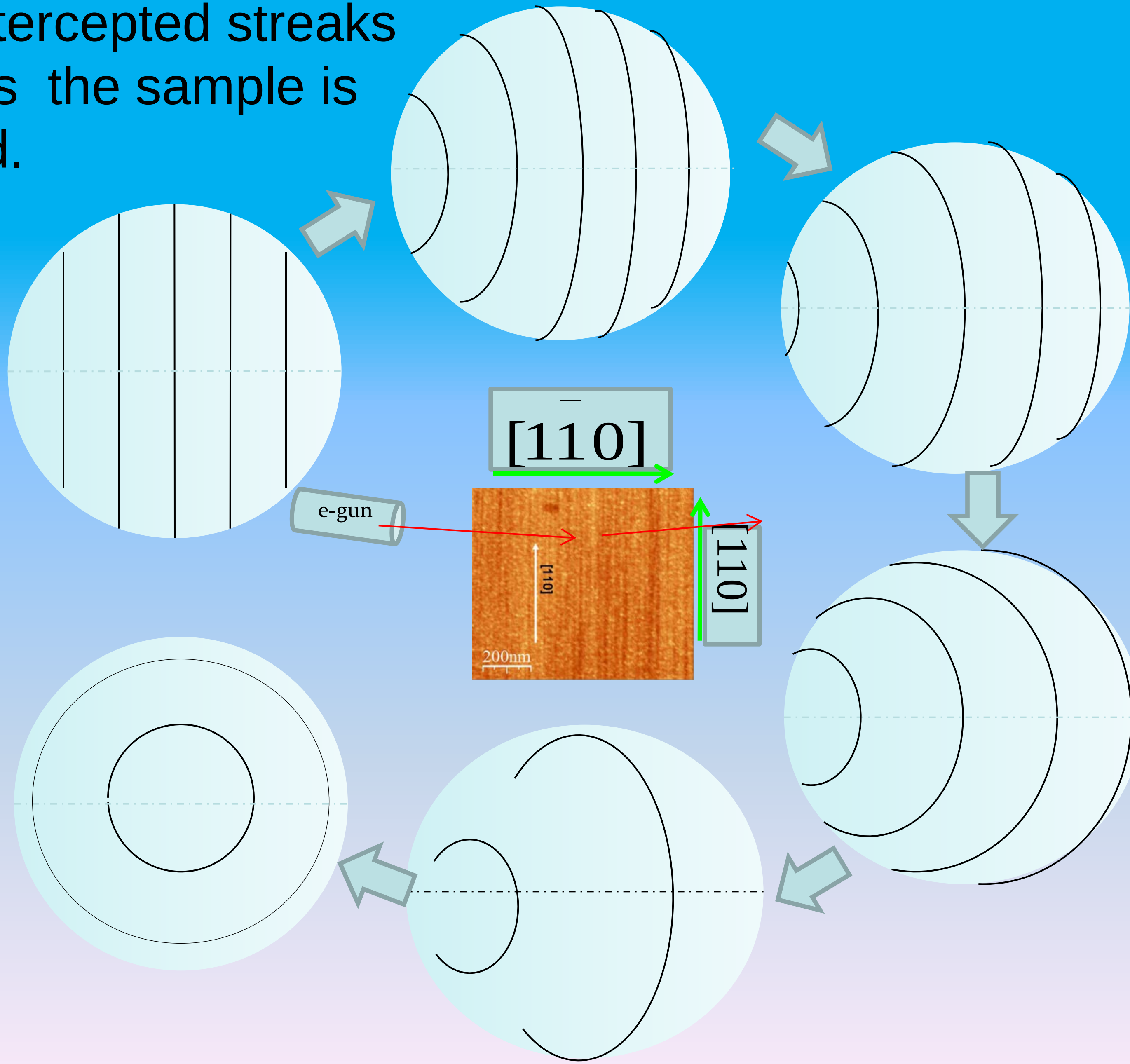
The wavy boundary between the gold layer and ZnSe layer is the cross section view of one-D structure array.

Apparently, the spacing of the 1D structures is 7 to 10 nm , while the height is about 1 to 2nm.

Ewald sphere and Reciprocal lattice



The intercepted streaks bent as the sample is rotated.



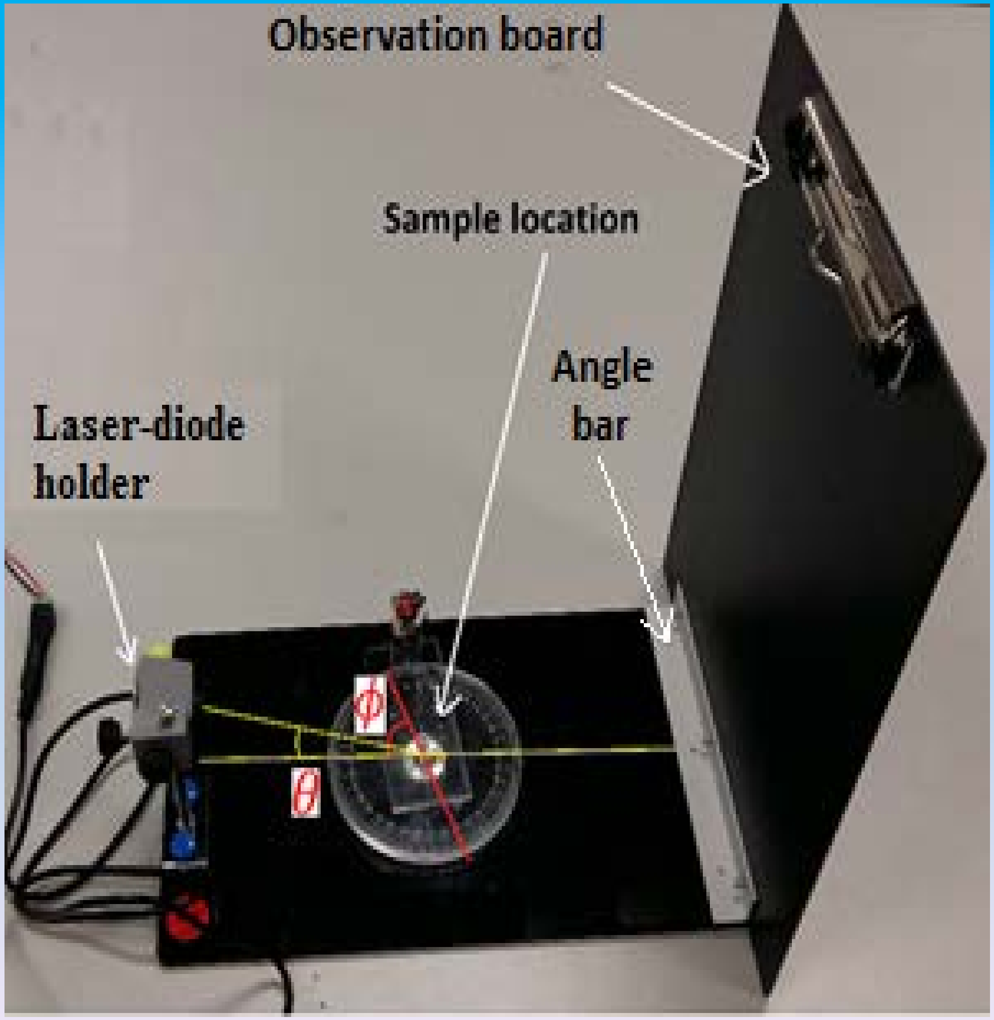
After a rotation of 90^0

Optical Analogy

Onsite Demo!

APhO Experimental Problem E1

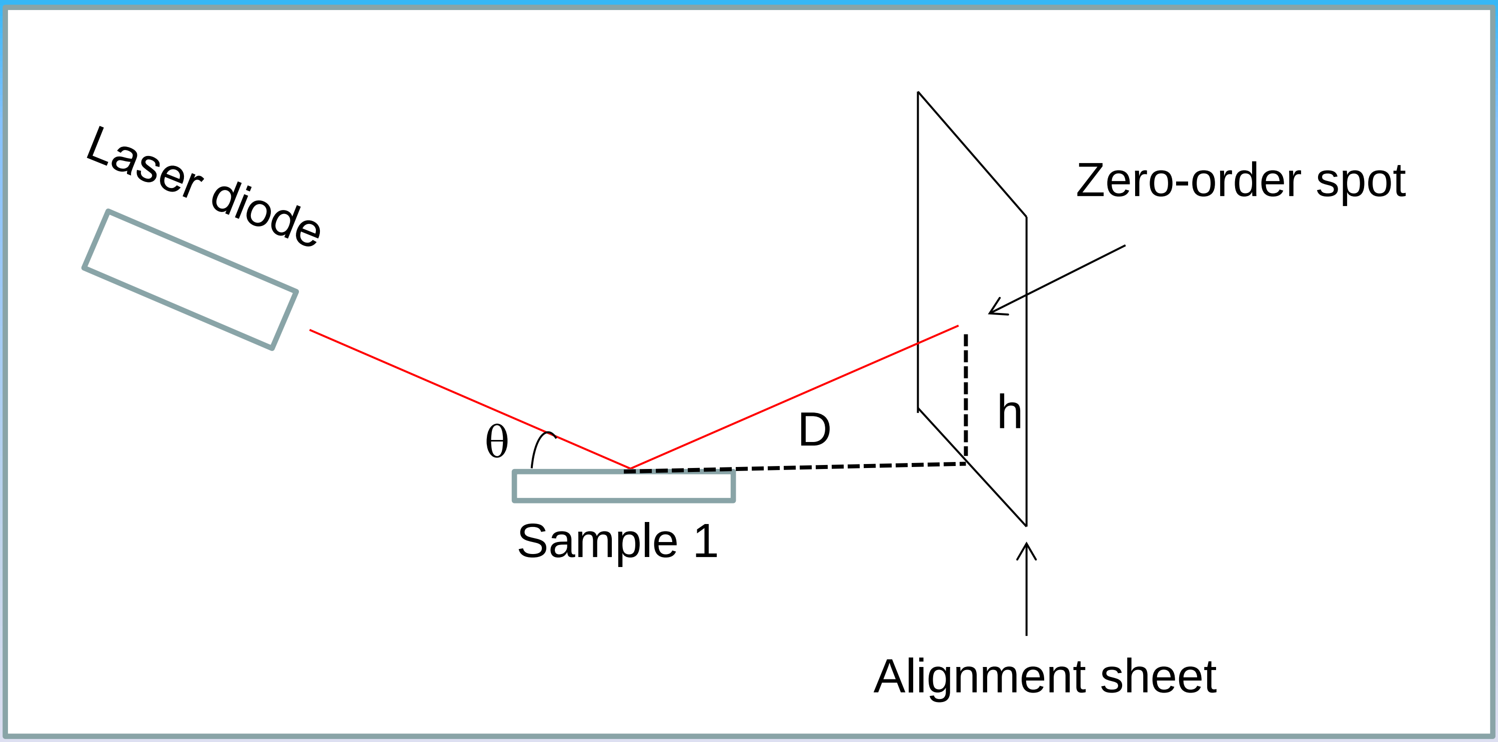
Reflected optical diffraction patterns from one-dimensional structures



The setup used in Problem E1

Part A: Alignment of the setup

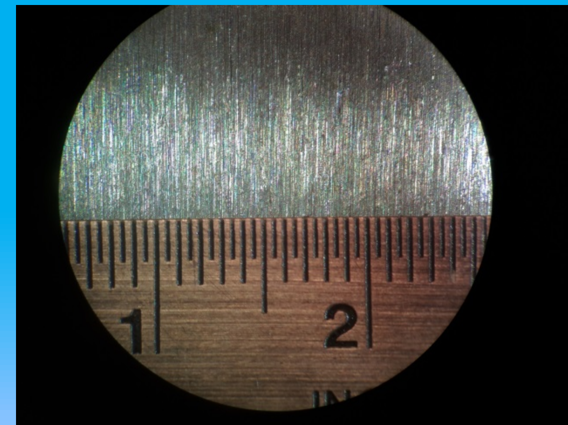
Sample 1: a plane mirror (Si wafer)



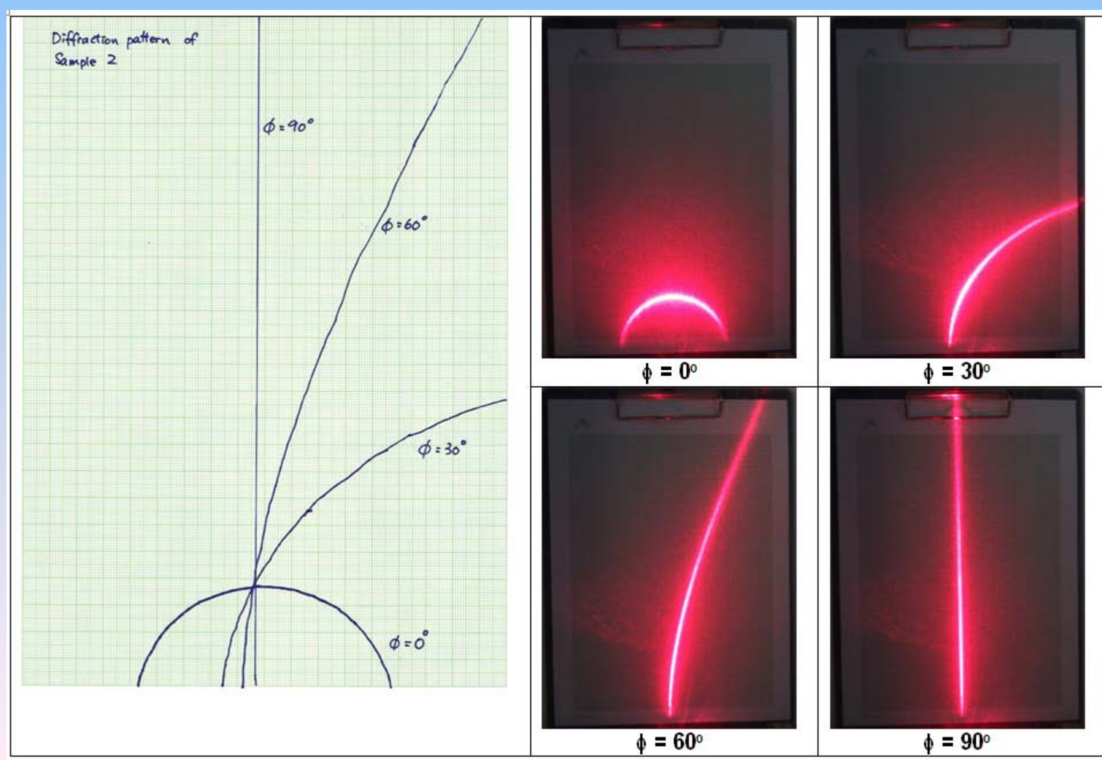
Students are asked to measure the height **h (5.5 cm)** and then find out the angle θ**(~ 20.1^0)**

Part B: Reflected Diffraction from Sample 2

Sample 2: a steel plate with non-uniformly spaced scratch grooves



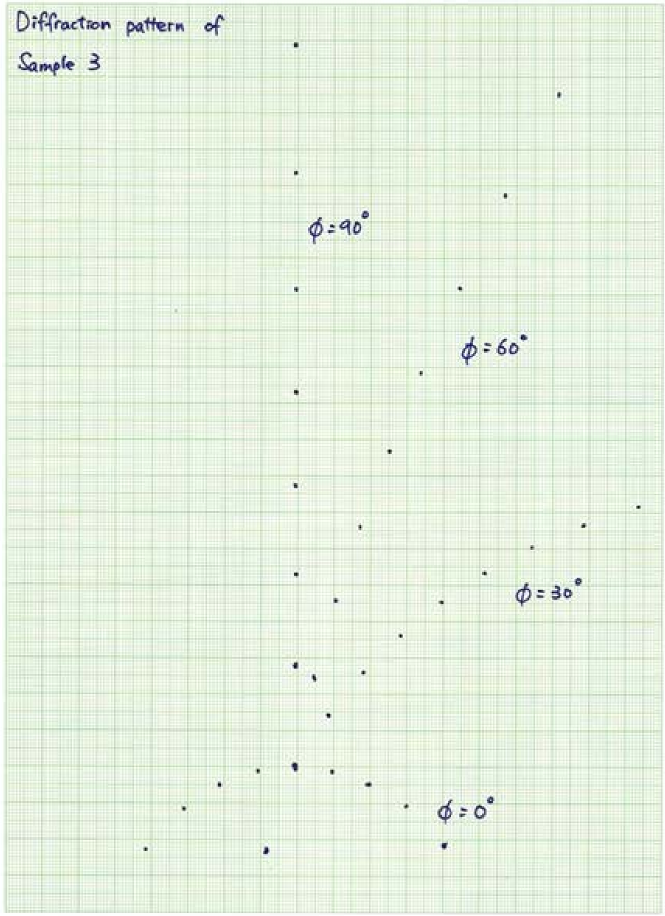
Task B1: Students are asked to record diffraction patterns for four ϕ values



Part C: Reflected Diffraction from a regular grating

Sample 3: a regular grating with equal-spacing grooves
(a Si wafer with periodic 1D photoresist pattern)

Task C 1: Students are asked to record diffraction patterns for four ϕ values



$\phi = 0^o$

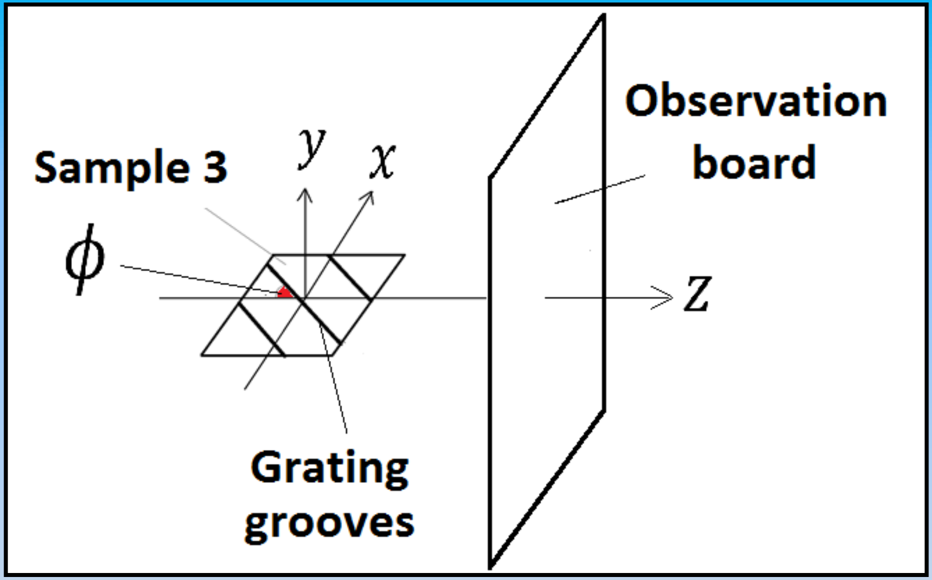
$\phi = 30^o$

$\phi = 60^o$

$\phi = 90^o$

Part D: Theory behind the reflected diffraction patterns from Sample 3 (regular grating)

Using wave optics, (**credits go to Prof. Michael KY Wong**), one can derive the following two equations for the x and y coordinates of the diffraction spots:



$$y^2 = \frac{(D\cos\phi + x\sin\phi)^2}{\cos^2\theta\cos^2\phi} - x^2 - D^2 \qquad (1)$$

$$x = \frac{Dm\lambda\cos\phi}{a\cos\theta - m\lambda\sin\phi} \qquad (2)$$

where “a” is the grating constant of Sample 3

Tasks D1 (cancelled during the board meeting)

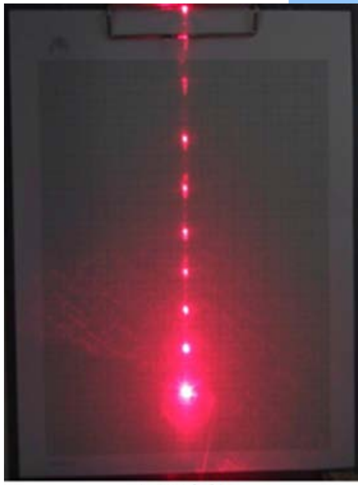
Tasks		Marks
D1	Based on Eqs. (1) and (2), the diffraction spots for $\phi = 90^0$ should lie along the y-axis at $x = 0$. Derive an equation for the y values of the diffraction spots for $\phi = 90^0$ as a function of a, θ, λ, m and D. Write down your result in the answer sheet.	1.4

Solution of D1 :

For $\phi = 90^{\circ}$, Eq. (2) gives the value of x equals to 0. To obtain the required equation, substitute Eq. (2) into the first term of Eq. (1). We have

$$y^2 = \frac{\left(D\cos\phi + \dfrac{Dm\lambda\cos\phi}{(a\cos\theta - m\lambda)}\right)^2}{\cos^2\theta\cos^2\phi} - D^2$$

$$y^2 = \frac{(Da\cos\phi\cos\theta - Dm\lambda\cos\phi + Dm\lambda\cos\phi)^2}{\cos^2\theta\cos^2\phi(a\cos\theta - m\lambda)^2} - D^2$$



$\phi = 90^{o}$

Cancellation of some terms leads to

$$y^2 = \frac{D^2a^2}{(a\cos\theta - m\lambda)^2} - D^2$$

$$\therefore y = D\sqrt{\frac{a^2}{(a\cos\theta - m\lambda)^2} - 1}$$

Tasks D2 - D4 (changed to D1 – D3 during the board meeting)

D2	The solution for Task D1 can be rearranged to obtain a quadratic equation for the grating constant a of Sample 3, as $$Aa^2 + Ba + C = 0. \quad (3)$$ Derive the expressions for A, B and C. Enter your results in the corresponding table in the answer sheet.	0.9
D3	By solving this quadratic equation and using the measured y values of the diffraction spots for Sample 3 at $\phi = 90^0$ (See Task C1), together with the known values of D, θ and λ, determine the grating constant a of Sample 3 in meters to three significant figures for each diffraction order from the 1st order ($m = 1$) up to the 6th order ($m = 6$) [*Hints: These orders correspond to the six spots above the zero-order spot*]. Enter your results in the corresponding table in the answer sheet.	1.8
D4	Calculate the mean and the standard error of the mean for the grating constant a in meters to three significant figures. Enter your results in the corresponding table in the answer sheet.	0.8

Solutions of D2-D4 involve solving a quadratic equation of “a” and getting the its mean value and standard error of the mean → straightforward!

Part E: Determination of the unknown angle ϕ^* for Sample 4

Sample 4: a steel piece with unknown angle ϕ^*

Observed reflected diffraction on the observation board

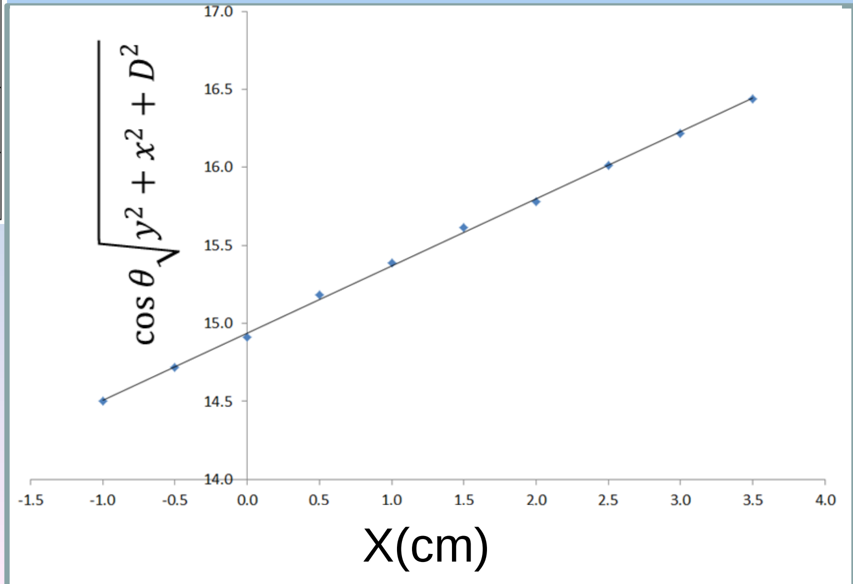
Tasks E1-E2

Tasks		Marks
E1	Along the continuous diffracted curve of Sample 4 projected on the graph paper, measure the *y*-coordinates in cm for ten points starting from x = - 1.0 cm to 3.5 cm with a step of 0.5 cm. Enter your results in the corresponding table in the answer sheet.	0.3
E2	Based on Eq. (1) given in Task D, construct a linear equation in the form of $$M(y,x,D,\theta) = I(D) + S(\phi^*)x \,. \quad (4)$$ Determine the functional forms for $M(y,x,D,\theta)$, $I(D)$ and $S(\phi^*)$. Plot M against x, using the data recorded in E1. Determine the unknown angle ϕ^* in degrees from this graph. Write down all the functional forms and the value of ϕ^* in the corresponding table in the answer sheet.	1.3

x co-ordinate (cm)	y co-ordinate (cm)
-1.0	3.5
-0.5	4.5
0.0	5.2
0.5	6.0
1.0	6.5
1.5	7.0
2.0	7.3
2.5	7.7
3.0	8.0
3.5	8.3

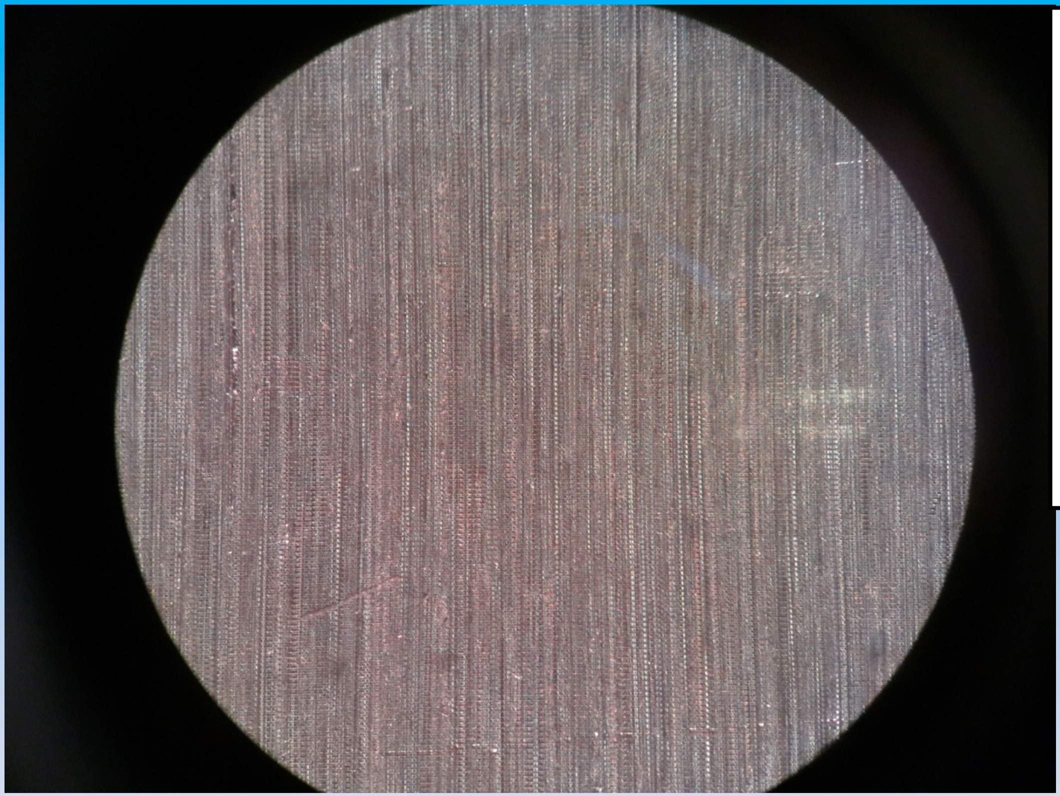
$M(y,x,D,\theta)$	$\cos\theta\sqrt{y^2+x^2+D^2}$
I	D
S	$\tan\phi^*$

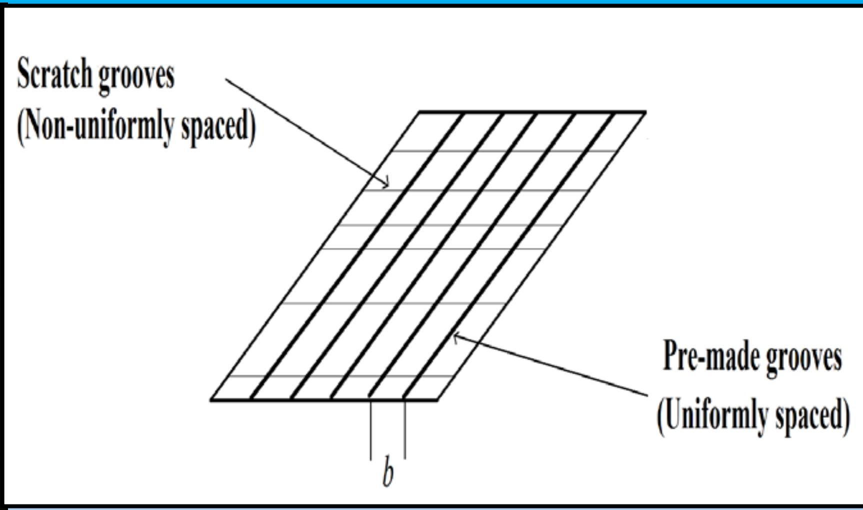
$\phi^* = 23.2^0$



Part F: Diffraction patterns from Sample 5

Sample 5: **a steel plate with non-uniformly spaced scratch grooves + uniformly spaced grooves perpendicular to the scratch grooves (photoresist pattern)**





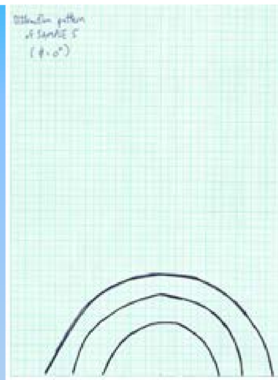
Schematic drawing of Sample 5

Sample 5 viewed under microscope

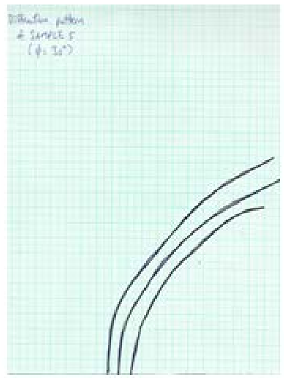
Sample 5 is the optical analogy of the nano-grating ZnSe surface !

Tasks F1

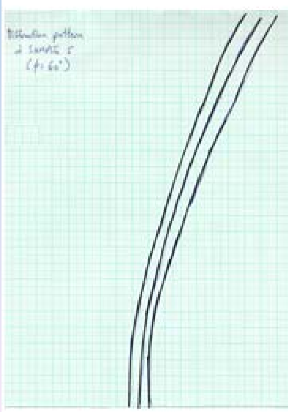
Tasks		Marks
F1	Record the diffraction patterns you observed for $\phi = 0^\circ$, 30°, 60° and 90° on separate graph papers for each value of ϕ. At the top of each graph paper, put down '#5' and the corresponding ϕ value. It is expected that you could observe more than 10 diffraction orders. However, you are required to record only three relatively brighter orders on each graph paper.	0.8



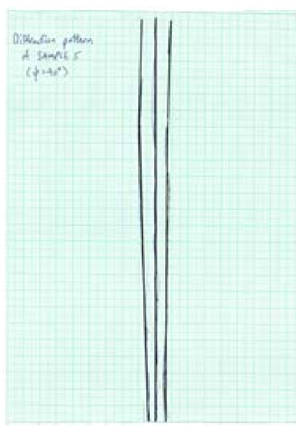
$\phi = 0^\circ$



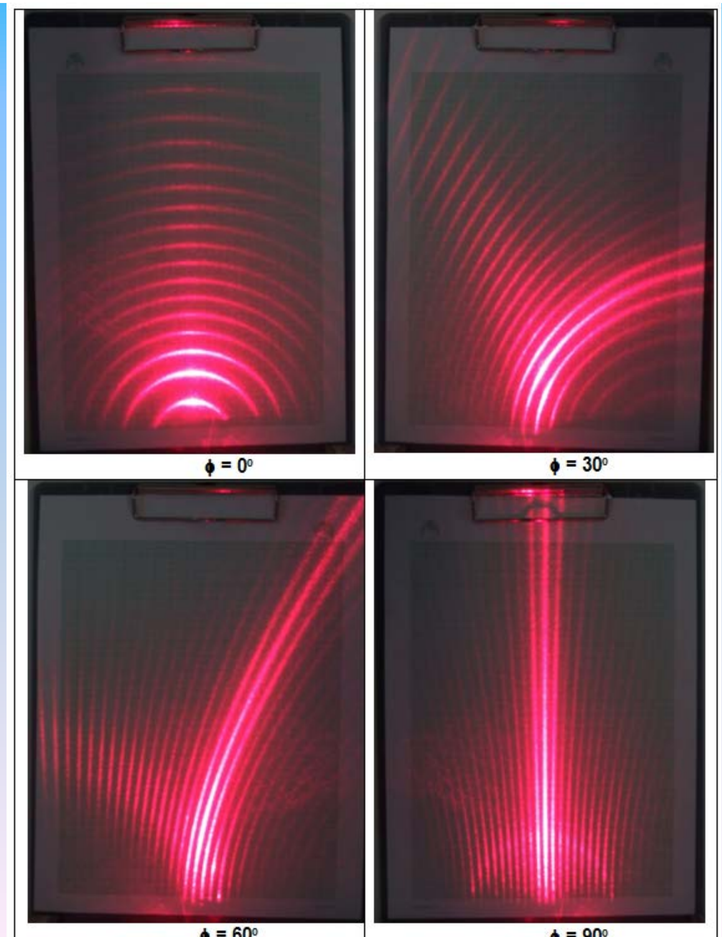
$\phi = 30^\circ$



$\phi = 60^\circ$



$\phi = 90^\circ$



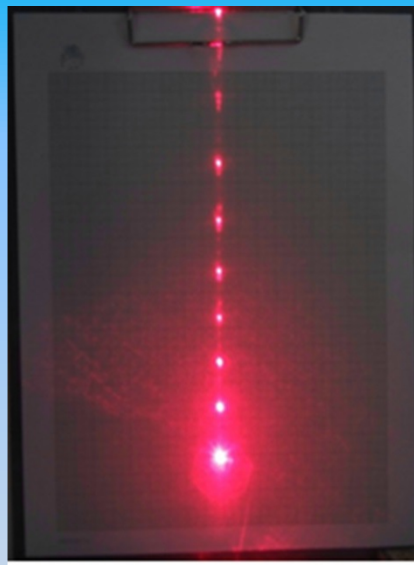
$\phi = 0^\circ$ $\phi = 30^\circ$ $\phi = 60^\circ$ $\phi = 90^\circ$

Task F2

Tasks		Marks
F2	With this understanding, estimate the spacing b in meters of the uniformly spaced pre-made grooves of Sample 5 using the recorded diffraction pattern for $\phi = 0^{\circ}$ from Task (F1). Enter the value of b in the answer sheet. *[Note that in estimating the value of b, you are only required to take the measured data of the first diffraction order and the estimated b should be rounded up to three significant figures.]*	**1.2**

$\phi = 0^0$

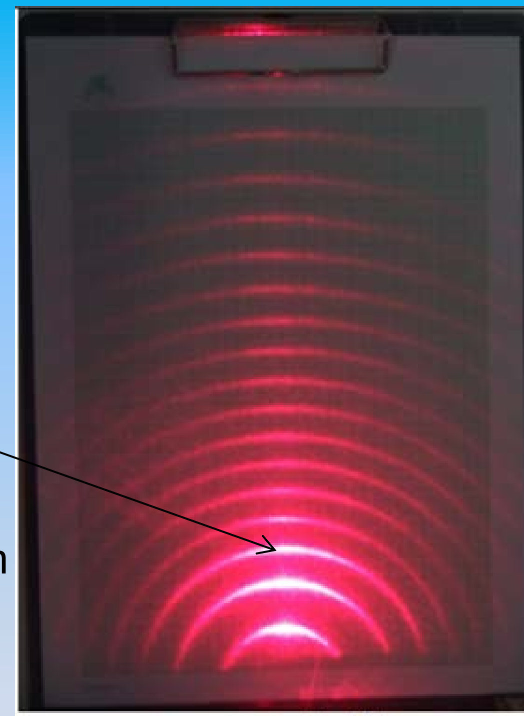
×



$\phi = 90^0$

→

1st order diffraction arc



$\phi = 0^0$

$\phi = 0^0$ for non-uniformly spaced grooves;

$\phi = 90^0$ for uniformly spaced grooves;

The y_1 value of the peak of the first order diffraction arc should satisfy the same relation as the solution of Task D:

$$y_1 = D\sqrt{\frac{b^2}{(b\cos\theta - \lambda)^2} - 1}$$

where b is the spacing of the pre-made grooves. From the recorded diffraction at $\phi = 0^\circ$ for Sample 5, the value of y_1 is measured to be 0.0695 m. Similar to what has been done in Task (D1), one can rearrange the equation for y_1 to form a quadratic equation as

$$Ab^2 + Bb + C = 0$$

Where

A	${y_1}^2\cos^2\theta + D^2\cos^2\theta - D^2 = 1.481 \times 10^{-3}$
B	$-2\lambda\cos\theta\,({y_1}^2 + D^2) = -3.320 \times 10^{-8}$
C	$\lambda^2({y_1}^2 + D^2) = 1.149 \times 10^{-14}$

Again, we take the only valid solution of b (See the solution of Task (D3) for detailed explanations) as

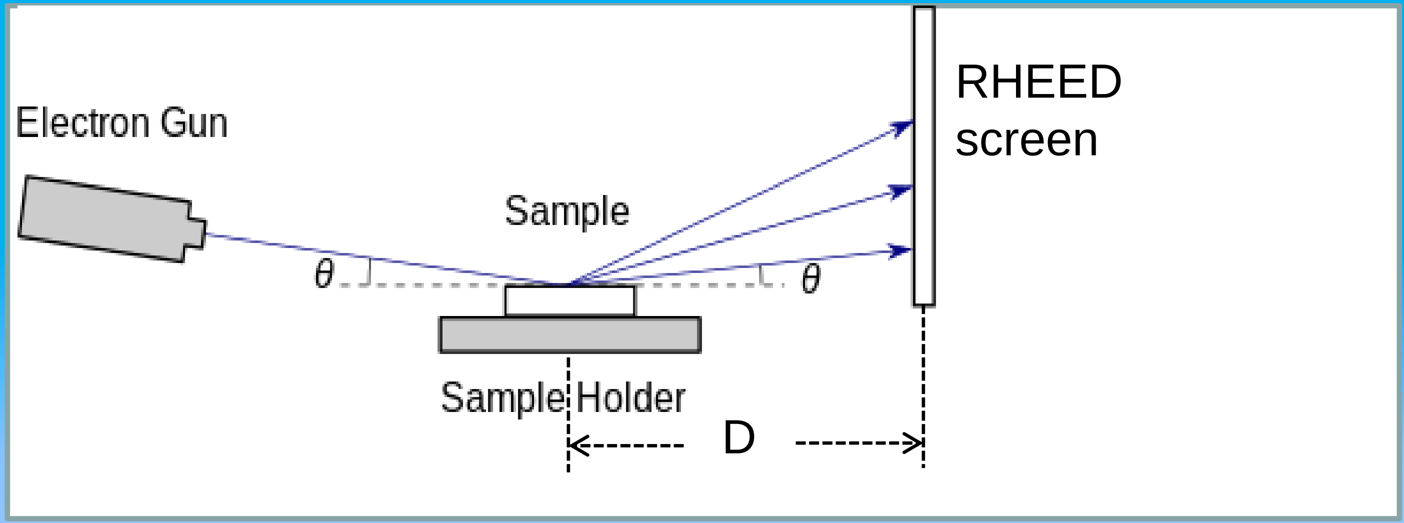
$$b = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore b \approx 2.21 \times 10^{-5}\ \text{m}$$

$\phi = 0^\circ$

Part G: Determination of the lattice-plane spacing for ZnSe

Schematic diagram of the RHEED diffraction geometry



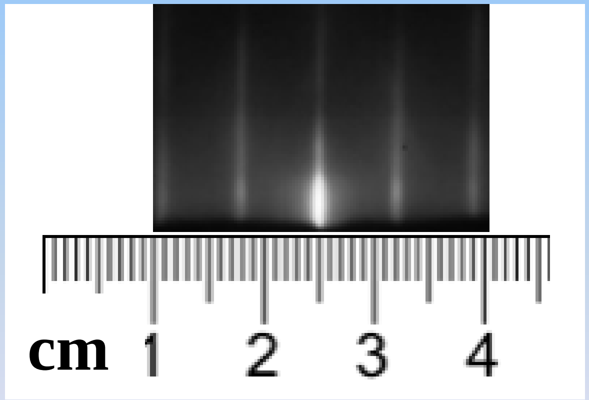


Figure 16: The reflection high energy electron diffraction (RHEED) pattern from a nano-structured surface of ZnSe, when the electron-beam is perpendicular to the nano-grooves with non-uniform spacing.

Given $$\lambda = \frac{12.247 \times 10^{-10}}{\sqrt{V(1 + 10^{-6}V)}} \text{ [m]}$$ V=13,000 volts, $\theta \approx 0^0$ and D = 26 cm

where λ is the wavelength of the incident electrons and V is the accelerating voltage.

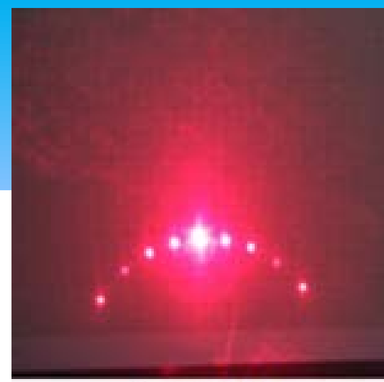
Tasks G1

Tasks		Marks
G1	For the ZnSe sample, based on Figure 16 and the experimental conditions given above, determine the lattice-plane spacing a^* of the periodic atomic lattice planes that are perpendicular to the nano-grooves with non-uniform spacing, in meters. Enter your result in the corresponding table in the answer sheet.	1.3

Solution:

Recalling Eq. (2) given in Task (D),

$$x = \frac{Dm\lambda\cos\phi}{a^*\cos\theta - m\lambda\sin\phi}$$

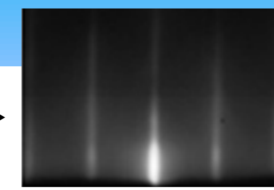


$\phi = 0^o$

×

$\phi = 90^o$

→



For the periodic atomic lattice planes, one has $\theta \approx 0^\circ$ and $\phi = 0^\circ$ and so Eq. (2) becomes

$$x = \frac{Dm\lambda}{a^*}$$

where a^* is the lattice plane spacing of the periodic atomic lattice planes that are perpendicular to nano-grooves with non-uniform spacing. Thus the average spacing of the RHEED streaks can be written as

$$\Delta x = \frac{D\lambda}{a^*}$$

Task G1 continue

Δx can be measured from the given RHEED pattern to be 0.7 cm. Given that $D = 0.26$ m and λ can be calculated using the given formula to be 0.1067×10^{-10} m. Thus, the required lattice plane spacing of ZnSe can be calculated as

$$a^* = \frac{0.26 \text{ m} \times (0.1067 \times 10^{-10} \text{ m})}{0.007 \text{ m}}$$

$$\therefore a^* = 3.96 \times 10^{-10} \text{ m}$$

Remark: For ZnSe, the actual plane spacing for the corresponding lattice plane is $a^* = 4 \times 10^{-10} m$.