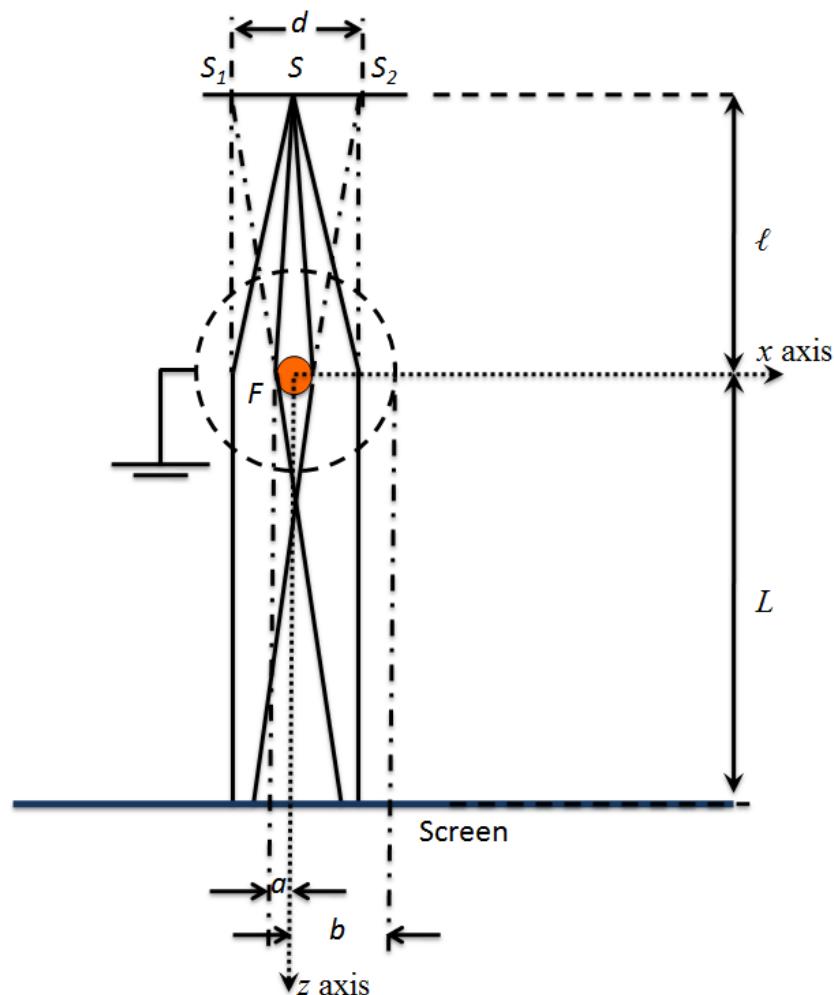


Question 2

The two-slit electron interference experiment was first performed by Möllenstedt *et al*, Merli-Missiroli and Pozzi in 1974 and Tonomura *et al* in 1989. In the two-slit electron interference experiment, a monochromatic electron point source emits particles at S that first passes through an electron “biprism” before impinging on an observational plane; S_1 and S_2 are virtual sources at distance d . In the diagram, the filament is pointing into the page. Note that it is a very thin filament (not drawn to scale in the diagram).



The electron “biprism” consists of a grounded cylindrical wire mesh with a fine filament F at the center. The distance between the source and the “biprism” is ℓ , and the distance between the “biprism” and the screen is L .

- (a) **(2 points)** Taking the center of the circular cross section of the filament as the origin O , find the electric potential at any point (x,z) very near the filament in terms of V_a , a and b where V_a is the electric potential of the surface of the filament, a is the radius of the filament and b is the distance between the center of the filament and the cylindrical wire mesh. (Ignore mirror charges.)

$$\begin{aligned}\text{Writing out } |E| &= \frac{\lambda}{2\pi\epsilon_0 r} = -\frac{\partial}{\partial r} V(r) \\ &= -\frac{\partial}{\partial r} \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{r} \quad \text{(1 point)}\end{aligned}$$

Note that

$$V(r) = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{r} \quad (= 0 \text{ at the mesh})$$

Also at the edge of the filament, $V_a = V(r = a)$, so

$$V_a = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}$$

Giving together

$$V(r) = V_a \frac{\ln(b/r)}{\ln(b/a)} \quad \text{where } r = \sqrt{x^2 + z^2}$$

(1 point) for final expression)

- (b) **(4 points)** An incoming electron plane wave with wave vector k_z is deflected by the “biprism” due to the x -component of the force exerted on the electron. Determine k_x the x -component of the wave vector due to the “biprism” in terms of the electron charge, e , v_z , V_a , k_z , a and b , where e and v_z are the charge and the z -component of the velocity of the electrons ($k_x \ll k_z$). Note that $\vec{k} = \frac{2\pi\vec{p}}{h}$ where h is the Planck constant.

There are several ways to work out the solution:

A charge in an electric field will experience a force and hence a change in momentum. Note that potential energy of the electron (charge = $-e_0$) is $-e_0V(r)$. Using impulse acting on the electron due to the electric field, **(2 points)**

$$\begin{aligned}\text{Impulse} &= \frac{1}{v_z} \int_{-\infty}^{\infty} (-e_0) \left(-\frac{\partial V(x, z')}{\partial x} \right) dz' \Big|_{x=a} \\ &= -\frac{1}{v_z} \int_{-\infty}^{\infty} \frac{-e_0 V_a x}{(x^2 + z'^2) \ln \frac{b}{a}} dz' \Big|_{x=a} \\ &= \frac{e_0 V_a \pi}{v_z \ln \frac{b}{a}} \\ \Rightarrow k_x &= \frac{e_0 V_a \pi}{\hbar v_z \ln \frac{b}{a}}\end{aligned}$$

(2 points for final expression)

The alternative solution is to write down the equations of motion for the electrons **(2 points)** and determine the deflection of the electron as it passes through the “biprism”:

$$\begin{aligned}\frac{\Delta x}{\Delta z} &= \frac{\lambda e}{2\epsilon_0 m v_z^2} \\ \text{Since } V_a &= \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}, \\ \frac{\Delta x}{\Delta z} &= \frac{\pi e V_a}{m v_z^2 \ln \frac{a}{b}}\end{aligned}$$

(2 points for final expression)

- (c) Before the point S , the electrons are emitted from a field emission tip and accelerated through a potential V_0 . Determine the wavelength of the electron in terms of the (rest) mass m , charge $-e_0$ and V_0 ,
- (i) **(2 points)** assuming relativistic effects can be ignored.

Equating the kinetic energy to eV_0 **(1 point)**

$$\frac{h}{\lambda} = \sqrt{2m|-e_0|V_0}$$

$$\lambda = \frac{h}{\sqrt{2me_0V_0}}$$

(1 point) for final expression)

- (ii) **(3 points)** taking relativistic effects into consideration.

Consider

$$E^2 = (pc)^2 + (mc^2)^2$$

$$= \left(\frac{h}{\lambda}c\right)^2 + (mc^2)^2$$

$$\frac{h^2c^2}{\lambda^2} = (mc^2 + |-e_0|V_0)^2 - (mc^2)^2$$

$$= 2mc^2e_0V_0\left(1 + \frac{e_0V_0}{2m_0c^2}\right)$$

$$\lambda = \frac{h}{\sqrt{2me_0V_0\left(1 + \frac{e_0V_0}{2m_0c^2}\right)}}$$

(1 point) for knowing relativistic $E - p$ relation
(1 point) for manipulating the equations
(1 point) for final expression

(d) In Tonomura et al experiment,

$$\begin{aligned}
 v_z &= c/2, \\
 V_a &= 10 \text{ V}, \\
 V_0 &= 50 \text{ kV}, \\
 a &= 0.5 \text{ }\mu\text{m}, \\
 b &= 5 \text{ mm}, \\
 \ell &= 25 \text{ cm}, \\
 L &= 1.5 \text{ m}, \\
 h &= 6.6 \times 10^{-34} \text{ Js}, \\
 \text{electron charge, } -e &= -1.6 \times 10^{-19} \text{ C}, \\
 \text{mass of electron, } m &= 9.1 \times 10^{-31} \text{ kg}, \\
 \text{and the speed of light in vacuo, } c &= 3 \times 10^8 \text{ ms}^{-1}
 \end{aligned}$$

(i) **(2 points)** calculate the value of k_x ,

Previous equation:

$$k_x = \frac{e_0 V_a \pi}{\hbar v_z \ln \frac{b}{a}}$$

Plugging the relevant numbers into the equation gives:

(1 point for plugging the correct values)

$$k_x = \frac{\pi}{907} \text{ \AA}^{-1} \text{ or } 3.46 \times 10^7 \text{ m}^{-1}$$

(1 point for final expression)

(ii) **(2 points)** determine the fringe separation of the interference pattern on the screen,

$$\text{Fringe separation is given by } \frac{1}{2} \frac{2\pi}{k_x} = 907 \text{ \AA}$$

(1 point for formula, note the factor $\frac{1}{2}$)
(1 point for final expression with units)

(iii) **(1 point)** If the electron wave is a spherical wave instead of a plane wave, is the fringe spacing larger, the same or smaller than the fringe spacing calculated in (ii)?

Larger. **(1 point for the correct answer)**

- (iv) **(2 points)** In part (c), determine the percentage error in the wavelength of the electron using non-relativistic approximation.

Non-relativistic:

$$\frac{h}{\lambda} = \sqrt{2me_0V_0}$$

$$\lambda_{nonrel} = \frac{h}{\sqrt{2me_0V_0}}$$

$$= 5.4697 \times 10^{-12}m$$

Relativistic:

$$\lambda_{rel} = \frac{h}{\sqrt{2meV_0 \left(1 + \frac{eV_0}{2m_0c^2}\right)}}$$

$$= 5.3408 \times 10^{-12}m$$

Percentage error:

$$\text{Error} = \frac{\lambda_{nonrel} - \lambda_{rel}}{\lambda_{rel}}$$

$$= 0.024$$

or 2.4 percent.

(1 point for working out non – relativistic and relativistic wavelength)
(1 point for final expression)

- (v) **(2 points)** Calculate the distance d between the apparent double slits.

The double slit formula is given by

$$y = \frac{m\lambda(\ell + L)}{d}$$

where m is the order and y is the distance for maximum intensity from the central fringe.

In this case, since the fringe spacing is 907\AA ,

$$d = 1.03 \times 10^{-4}m$$

(1 point for formula)
(1 point for final numerical answer)