Problem
Conductors in Conducting Liquid

A system consisted of two conductor bodies is immersed in a uniform dielectric and weakly conducting liquid. When a constant voltage difference is applied between both conductors, the system has both electric and magnetic fields. In this problem we will investigate this system.

1. **(0.4 pts)** First consider an infinitely long line with charge per unit length λ in vacuum. Calculate the electric field $\mathbf{E}(\mathbf{r})$ due to the line.

2. **(0.4 pts)** The potential due to the line charge could be written as
$$V(r) = f(r) + K,$$
where K is a constant. Determine $f(r)$.

3. **(0.7 pts)** Calculate the potential in all space $V(x,y,z)$due to an infinitely long line with charge per unit length λ at $x = -b, y = 0$ and another infinitely long line with charge per unit length $-\lambda$ at $x = b, y = 0$. Both lines are parallel to the z-axis. Take $V = 0$ at the origin. Sketch the equipotential surfaces.

   For the following questions, ignore any edge effects.

4. **(2.0 pts)** Now consider two identical conducting cylinders, both with radius $R = 3a$ in vacuum. The length of each cylinders are the same and much larger than its radius ($l \gg R$). The axis of both cylinders are on the xz-plane and parallel to the z-axis, one at $x = -5a, y = 0$ and the other at $x = 5a, y = 0$. An electrical potential difference of V_0 is applied between the two cylinders (the cylinder at $x = -5a$ has the higher potential) by connecting them to a battery. Calculate the potential in **all regions**. Take $V = 0$ at the origin.

5. **(0.5 pts)** Calculate the capacitance C of the system.

6. **(1.0 pts)** Now both cylinders are totally immersed in a weakly conducting liquid with conductivity σ. Calculate the total current that flows between both cylinders. Assume the permittivity of the liquid is equal to that of vacuum, $\epsilon = \epsilon_0$.

7. **(0.5 pts)** Calculate the resistance R of the system. Calculate RC of the system.

8. **(1.5 pts)** Calculate the magnetic field due to the current in question 6. Assume that the permeability of the liquid is equal to that of vacuum $\mu = \mu_0$.

   Notes $\int \frac{\alpha dx}{\alpha^2+x^2} = \arctan\frac{x}{a} + const$