



III.1. **[1.0 mark]** Let the phase difference between two rays making an angle  $\theta$  with  $z$  direction be  $\delta$ . Clearly

$$\delta = \omega \Delta t = \omega d \sin \theta / c \quad (1)$$

Then the intensity is given by

$$I(\theta) = \beta |\mathbf{E}_1 + \mathbf{E}_2|^2$$

Here

$$\begin{aligned} |\mathbf{E}_1 + \mathbf{E}_2|^2 &= |E_0 \cos(\omega t) + E_0 \cos(\omega t + \delta)|^2 \\ &= |E_0 \cos(\omega t) (1 + \cos \delta) - E_0 \sin(\omega t) \sin \delta|^2 \\ &= E_0^2 [\cos^2(\omega t) (1 + 2 \cos \delta + \cos^2 \delta) + \sin^2(\omega t) \sin^2 \delta \\ &\quad - 2 \cos(\omega t) (1 + \cos \delta) \sin(\omega t) \sin \delta] \end{aligned}$$

Since  $\overline{\cos^2 \omega t} = \overline{\sin^2 \omega t} = 1/2$  and  $\overline{\sin(\omega t) \cos(\omega t)} = 0$ , we get the intensity to be

$$I(\theta) = \beta E_0^2 (1 + \cos \delta)$$

where  $\delta$  is given by Eq. (1).

(a) Alternate:

$$\begin{aligned} |\mathbf{E}_1 + \mathbf{E}_2|^2 &= |E_0 \cos(\omega t) + E_0 \cos(\omega t + \delta)|^2 \\ &= |E_0 \cos(\omega t) (1 + \cos \delta) + E_0 \sin(\omega t) \sin \delta|^2 \\ &= |E_0 A \cos(\omega t - \phi)|^2 \end{aligned}$$

where

$$\begin{aligned} A^2 &= (1 + \cos \delta)^2 + \sin^2 \delta \\ &= 2(1 + \cos \delta) \end{aligned}$$

Since  $\overline{\cos^2(\omega t - \phi)} = 1/2$ , we have

$$I(\theta) = \frac{\beta}{2} E_0^2 A^2 = \beta E_0^2 (1 + \cos \delta)$$

where  $\delta$  is given by Eq. (1).

III.2. **[1.0 marks]** The beam 1 has travelled extra optical path  $= (\mu - 1)w$ . Thus the net phase difference between two beams when they emerge at the angle  $\theta$  is

$$\delta = \frac{\omega}{c} (d \sin \theta - (\mu - 1)w)$$

and  $I(\theta) = \beta E_0^2 (1 + \cos \delta)$ .



III.3. **[2.0 marks]** The two beams have travelled exactly same paths upto the slits. Thus when they emerge at an angle  $\theta$ , they have a net phase difference of  $\delta = \omega d \sin \theta / c$ . Then, notice

$$\begin{aligned}
 |\mathbf{E}_1 + \mathbf{E}_2|^2 &= \left| \mathbf{i} \left[ \frac{E_0}{\sqrt{2}} \cos(\omega t) + E_0 \cos(\omega t + \delta) \right] + \mathbf{j} \left[ \frac{E_0}{\sqrt{2}} \sin(\omega t) \right] \right|^2 \\
 &= \left| E_0 \cos(\omega t) \left( \frac{1}{\sqrt{2}} + \cos \delta \right) + E_0 \sin(\omega t) \sin \delta \right|^2 + \frac{E_0^2}{2} \sin^2(\omega t) \\
 &= E_0^2 \left[ \cos^2(\omega t) \left( \frac{1}{2} + \sqrt{2} \cos \delta + \cos^2 \delta \right) + \sin^2(\omega t) \sin^2 \delta \right. \\
 &\quad \left. + 2 \cos(\omega t) \left( \frac{1}{\sqrt{2}} + \cos \delta \right) \sin(\omega t) \sin \delta \right] + \frac{E_0^2}{2} \sin^2(\omega t)
 \end{aligned}$$

Thus, after taking time averages, the intensity will be

$$\begin{aligned}
 I(\theta) &= \beta E_0^2 \left[ \frac{1}{2} \left( \frac{1}{2} + \sqrt{2} \cos \delta + \cos^2 \delta \right) + \frac{1}{2} \sin^2 \delta \right] + \beta \frac{E_0^2}{4} \\
 &= \beta E_0^2 \left[ 1 + \frac{1}{\sqrt{2}} \cos \delta \right] \\
 &= \beta E_0^2 \left[ 1 + \frac{1}{\sqrt{2}} \cos \left( \frac{\omega d \sin \theta}{c} \right) \right]
 \end{aligned}$$

The maximum value of the intensity is  $\beta E_0^2 \left[ 1 + \frac{1}{\sqrt{2}} \right]$ .

The minimum value of the intensity is  $\beta E_0^2 \left[ 1 - \frac{1}{\sqrt{2}} \right]$ .

III.4. **[2.0 marks]** The electric field at  $z = b$  is given by

$$\begin{aligned}
 \mathbf{E}_1(z = b) &= \left( \frac{E_0}{\sqrt{2}} (\cos(\omega t - kb) \mathbf{i} \cdot \mathbf{i}' + \sin(\omega t - kb) \mathbf{j} \cdot \mathbf{i}') \right) \mathbf{i}' \\
 &= \frac{1}{\sqrt{2}} [E_0 \cos \gamma \cos(\omega t - kb) + E_0 \sin \gamma \sin(\omega t - kb)] \mathbf{i}' \\
 &= \frac{1}{\sqrt{2}} [E_0 \cos(\omega t - kb - \gamma)] \mathbf{i}'
 \end{aligned}$$

and at  $z = c$

$$\begin{aligned}
 \mathbf{E}_1(z = c) &= \frac{E_0}{\sqrt{2}} (\cos(\omega t - kc - \gamma) \mathbf{i}' \cdot \mathbf{i}) \mathbf{i} \\
 &= \frac{1}{\sqrt{2}} \cos \gamma E_0 \cos(\omega t - kc - \gamma) \mathbf{i}
 \end{aligned}$$

Where as

$$\mathbf{E}_2(z = c) = E_0 \cos(\omega t - kc) \mathbf{i}$$

So, the net phase difference between beam 1 and beam 2 is now

Phase difference  $\alpha = \gamma$ .

III.5. **[0.5 marks]**

(a) At equator  $e = 0$ . Then

$$\mathbf{E} = \mathbf{E}_{\text{Eq}} = \mathbf{i}' E_0 \cos(\omega t).$$

(b) Clearly the beam is linearly polarised along  $\mathbf{i}'$ .

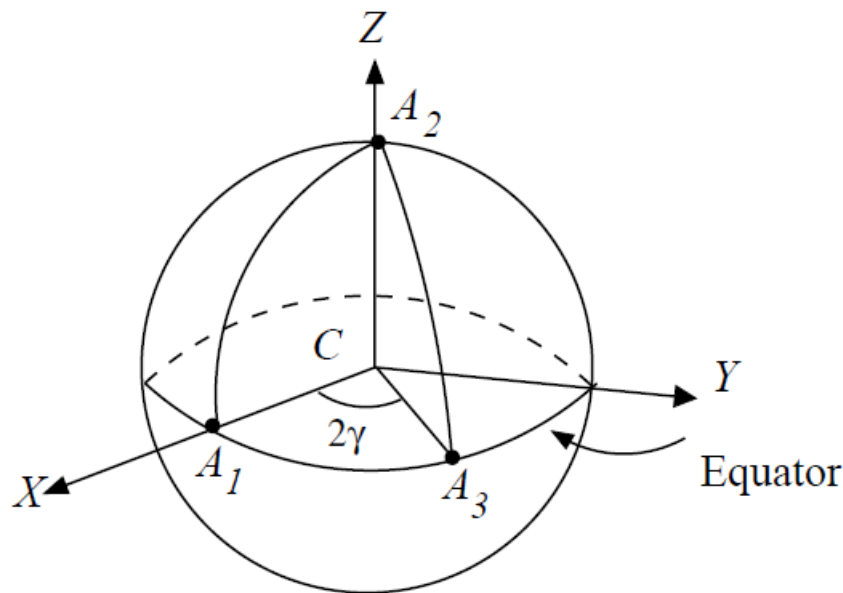
III.6. [0.5 marks]

(a) At north pole  $e = \pi/4$  and  $\gamma$  can be taken to be 0. Then  $\mathbf{i}' = \mathbf{i}$  and  $\mathbf{j}' = \mathbf{j}$ . The electric field

$$\mathbf{E}_{\text{NP}} = \mathbf{i} \frac{E_0}{\sqrt{2}} \cos(\omega t) + \mathbf{j} \frac{E_0}{\sqrt{2}} \sin(\omega t),$$

(b) Which represents circular polarisation.

III.7. [1.5 marks]



III.8. [1.5 mark] From figure it is clear that the area of the spherical triangle  $A_1 A_2 A_3$  is  $2\pi \times \left(\frac{2\gamma}{2\pi}\right) = 2\gamma$  and the phase difference was  $\gamma$ . Thus the phase difference is half the area of the spherical triangle  $A_1 A_2 A_3$  on Poincare sphere.

Thus  $S = 2\alpha$ .

Here, the beam 1 passes through various states of polarization and returns to its original state. Though there has been no additional path difference, the beam has picked up a phase  $\gamma$  with respect to the beam 2. This is called as Pancharatnam phase.