I.1. Equation of motion for the magnet is

$$m\ddot{z} = mg - k\dot{z} \tag{1}$$

For terminal velocity $\ddot{z} = 0$ **0.2 mark**

which gives $v_T = \dot{z} = \dfrac{mg}{k}$ **0.3 mark**

I.2. Rewriting Eq. (1)

$$\frac{dv}{dt} = g - \frac{k}{m}v(t)$$

Given that $v(t=0) = 0; z(t=0) = 0$ which yields

$$v(t) = \frac{mg}{k}(1 - e^{-kt/m}) = \frac{dz}{dt}$$ **0.5 mark**

$$\int_0^z dz = \int_0^t \frac{mg}{k}(1 - e^{-kt/m})dt$$

$$z(t) = \frac{mg}{k}\left[t + \frac{m}{k}(e^{-kt/m} - 1)\right]$$ **0.5 mark**

I.3. Method - I : Because of the relative speed v between the magnet and the ring, in the field $\vec{B} = B_z\hat{k} + B_\rho\hat{\rho}$ of the magnet, the induced emf is given by

$$e_i = \int (\vec{v} \times \vec{B}).d\vec{l}$$ **0.8 mark**

$$e_i = vB_a 2\pi a$$ **0.4 mark**

where $$B_a = \frac{\mu_0}{4\pi}\frac{3pa(z_0 - z)}{[a^2 + (z_0 - z)^2]^{5/2}} \tag{2}$$

0.3 mark

Method - II :Magnetic flux (ϕ) through the ring is

$$\phi = \int_0^a B_z 2\pi\rho d\rho$$ **0.2 mark**

$$= 2\pi\int_0^a \frac{\mu_0}{4\pi}\frac{\rho p}{(\rho^2 + (z_0 - z)^2)^{3/2}}\left[\frac{3(z_0 - z)^2}{\rho^2 + (z_0 - z)^2} - 1\right]d\rho$$ **0.2 mark**

$$\phi = \frac{\mu_0 pa^2}{2(a^2 + (z_0 - z)^2)^{3/2}}$$ **0.3 mark**

Induced emf $e_i = \dfrac{-d\phi}{dt} = -v\dfrac{d\phi}{dz}$ **0.4 mark**

which gives $e_i = \dfrac{\mu_0 3pa^2 v(z_0 - z)}{2[a^2 + (z_0 - z)^2]^{5/2}}$ **0.4 mark**

I.4. B_z component will cause a radially outward force on the ring and by symmetry this yields a null force. 0.4 mark

Only B_ρ will contribute to

$$\vec{df}_{em} = i(\vec{dl} \times \vec{B})$$
$$\left|\vec{f}_{em}\right| = i2\pi a B_a$$ 0.6 mark

where B_a is given by Eq. (2).

I.5. By Newton's third law, equal and opposite force will be exerted by the ring on the magnet. Hence the magnitude of the force on the magnet by the ring is f_{em}. 0.5 mark

I.6. $e_i = L\dfrac{di}{dt} + iR$ 0.5 mark

I.7. Potential energy is converted to three parts:

(a) $mv^2/2$ (kinetic energy) 0.3 mark

(b) $Li^2/2$ (magnetic energy) 0.3 mark

(c) $i^2R\Delta t$ (Joule loss due to the current in time Δt). 0.4 mark

I.8. The magnetic field does no work in the process.

Yes	
No	✓

0.5 mark

I.9. Resistance of the ring

$$\Delta R = \frac{2\pi a}{\sigma w \Delta z'}$$ 0.5 mark

I.10. Now, the net force on the magnet, due to one ring at z' is given by

$$f_{em} = (2\pi a) i B'_a$$

where

$$B'_a = \frac{\mu_0}{4\pi} \frac{3pa(z'-z)}{(a^2 + (z'-z)^2)^{5/2}}$$ 0.3 mark

and i is the induced current in the ring which is given by

$$i = \frac{e_i}{\Delta R} = \frac{\sigma w e_i}{2\pi a} \Delta z'$$ 0.5 mark

Then the net force on the magnet due to the entire pipe is given by

$$F = \int_{-\infty}^{\infty} f_{em} = \int_{-\infty}^{\infty} B'^2_a (2\pi a) w\sigma dz'.\dot{z}$$ 0.2 mark

Since the pipe is very long the limits of integration can be taken as $-\infty$ and ∞. Substituting B'_a, we get

$$F = \left(\frac{\mu_0}{4\pi}\right)^2 18p^2a^3\pi w\sigma\dot{z}\int_{-\infty}^{\infty}\frac{(z'-z)^2}{((z'-z)^2+a^2)^5}dz'$$ 0.5 mark

Let $u=(z'-z)/a$. Finally,

$$F = \left(\frac{\mu_0}{4\pi}\right)^2 \frac{18p^2\pi\sigma w\dot{z}}{a^4}\int_{-\infty}^{\infty}\frac{u^2}{(1+u^2)^5}du$$

Thus damping parameter

$$k = \left(\frac{\mu_0}{4\pi}\right)^2 \frac{18p^2\pi\sigma w}{a^4}\int_{-\infty}^{\infty}\frac{u^2}{(1+u^2)^5}du$$ 0.5 mark

I.11. Given that,

$$k = f(\mu_0, p, R_0, a)$$

Dimensions of various parameters involved are

$$[\mu_0] = I^{-2}MLT^{-2}$$ 0.2 mark

$$[p] = IL^2$$ 0.1 mark

$$[R_0] = I^{-2}ML^2T^{-3}$$ 0.2 mark

$$[a] = L$$

$$[k] == MT^{-1}$$

which gives

$$k = \frac{p^2\mu_o^2}{a^4R_0}$$ 0.5 mark