Model Solution

We will assume the relationship of the form:

$$P = f(W)\,h(\theta)$$

$$M_p = f(M_w)\,h(\theta)$$

M_w represents the mass in the hanger (Load)
M_p represents the mass in the pan + the mass of the pan (i.e. $M'_p + M_{pan}$) (Effort)

The relation between these variables can be found in two parts:

- Relation between M_P and M_W
- Relation between M_P and θ

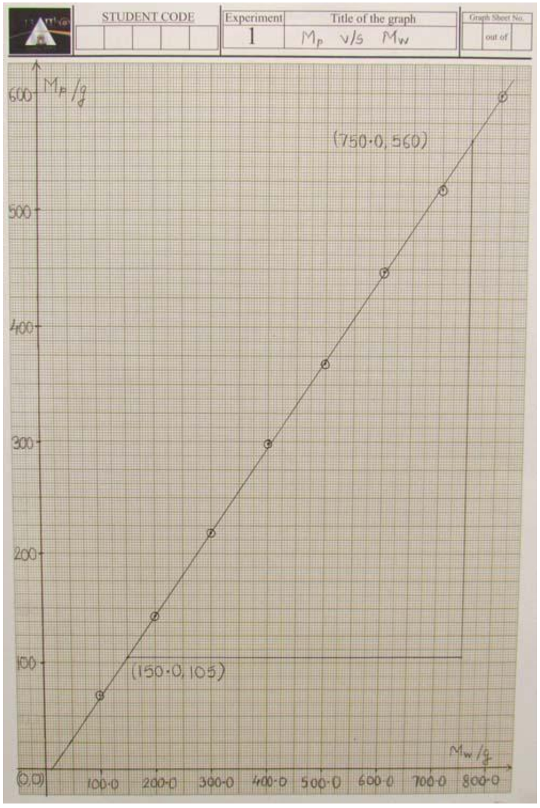
Part 1:

Mass of the pan = 28.6 g

$\theta = \pi$ radians

Obs. No.	M_w/g	M'_{p-} /g	M'_{p+} /g	$M'_{p(average)}$ /g	$M_{p(average)} = M'_{p(average)} + M_{pan}$ /g	$\Delta M_p = \frac{M'_{p+} - M'_{p-}}{2}$ /g
1	800.0	566	574	570	598.6	4
2	700.0	486	494	490	518.6	4
3	600.0	417	423	420	448.6	3
4	500.0	337	343	340	368.6	3
5	400.0	267	273	270	298.6	3
6	300.0	188	192	190	218.6	2
7	200.0	113	117	115	143.6	2
8	100.0	41	43	42	70.6	1

Graph of M_P v/s M_W:



Slope of the graph = 0.7583

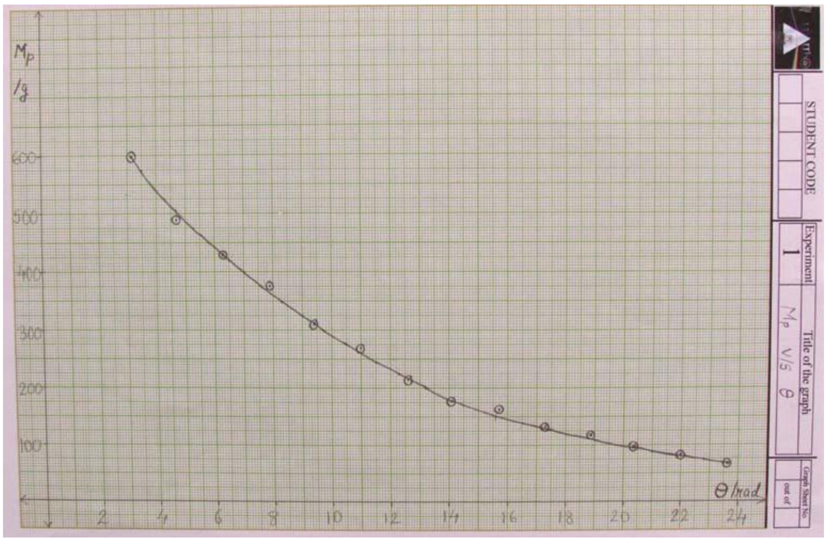
This shows that

$$P \propto W \tag{1}$$

Part 1:

M_w = 800.0 g
M_{pan} = 28.6 g

Obs. No.	θ /rad	M'_{p-} /g	M'_{p+} /g	$M'_{p(average)}$ /g	$M_{p(average)} = M'_{p(average)} + M_{pan}$ /g	$\Delta M_p = \frac{M'_{p+} - M'_{p-}}{2}$ /g
1	π	565	575	570	598.6	5
2	$3\pi/2$	455	465	460	488.6	5
3	2π	396	404	400	428.6	4
4	$5\pi/2$	316	324	320	348.6	4
5	3π	276	284	280	308.6	4
6	$7\pi/2$	236	244	240	268.6	4
7	4π	182	188	185	213.6	3
8	$9\pi/2$	147	153	150	178.6	3
9	5π	133	137	135	163.6	2
10	$11\pi/2$	104	110	107	135.6	3
11	6π	88	92	90	118.6	2
12	$13\pi/2$	68	72	70	98.6	2
13	7π	54	56	55	83.6	1
14	$15\pi/2$	39	41	40	68.6	1
15	8π	29	31	30	58.6	1
16	$17\pi/2$	18	20	19	47.6	1



The graph between M_p and θ shows a curve.

There can be possibilities of different functional relationship.

1)

The possible functions can be $\frac{1}{\theta}$, $\frac{1}{\theta^2}$, $e^{-k\theta}$

For the first two functions mentioned above, at θ = 0, M_p will reach infinite value which is not possible. For the third function we know that M_p will have some finite value.

2)

If the function is anticipated as exponential one then it can be verified using half value technique whether at every half value of Mp, and then plotting ln M_p or better still $\ln\left(M_p / M_w\right)$ against θ.

If it is a straight line with slope $-k$,

$$M_p \propto e^{-k\theta}$$

or

$$P \propto e^{-k\theta} \quad (2)$$

From (1) and (2),

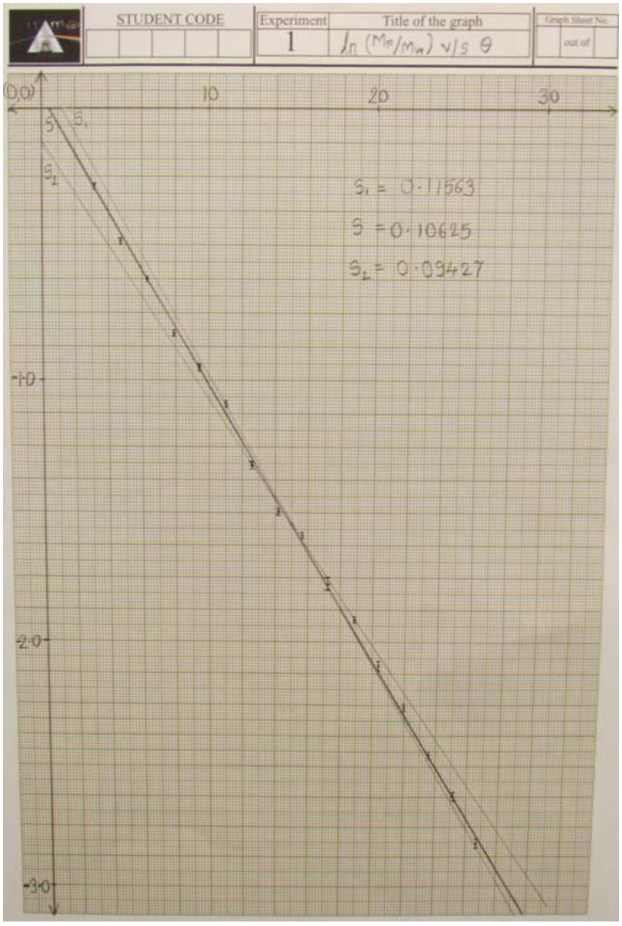
$$P \propto We^{-k\theta} \quad (3)$$

The constant k in the above expression is equated to the coefficient of friction, μ.

$$P = CWe^{-\mu\theta} \quad (4)$$

The constant of proportionality in equation (4) is 1. This is because at θ = 0 rad, $P = W$.

$$P = We^{-\mu\theta} \quad (5)$$



From the graph,

$$\ln\left(\frac{M_p}{M_w}\right) = -\mu\theta$$

where μ is the slope of the graph.

From the graph, $\mu = 0.106$

$$\frac{\Delta S}{S} = \frac{(S_1 \sim S_2)/2}{S} = 0.10049$$

$\Delta S = 0.01067 = \Delta\mu$

$u_C(\mu) = 0.00616$

$U(\mu) = 0.0123 \approx 0.02$

$$\mu = 0.11 \pm 0.02$$

Part 2:

When the pan is moving up:

$$M_{p1} = M_u e^{-\mu\theta} \tag{6}$$

When the pan is moving down:

$$M_{p2} = M_u e^{\mu\theta} \tag{7}$$

M'_{p1}		$M'_{p1(average)}$	$\Delta M'_{p1}$	$M_{p1} = M'_{p1} + M_{pan}$
M'_{p1+}	M'_{p1-}			
164	156	160	4	188.6

M'_{p2}		$M'_{p2(average)}$	$\Delta M'_{p2}$	$M_{p2} = M'_{p2} + M_{pan}$
M'_{p2+}	M'_{p2-}			
49	45	47	2	75.6

For M_u:

Multiplying equation (6) and (7),

$$M_u = \sqrt{M_{p1} \cdot M_{p2}}$$

$$M_u = \sqrt{188.6 \times 75.6} = 119.4075 g$$

For μ:

Dividing equation (6) by (7),

$$\frac{M_{p1}}{M_{p2}} = e^{2\mu\theta}$$

$$\mu = \frac{1}{2\theta} \ln\left(\frac{M_{p1}}{M_{p2}}\right)$$

For $\theta = \pi$

$$\mu = \frac{1}{2\pi} \ln\left(\frac{M_{p1}}{M_{p2}}\right)$$

$$\mu = \frac{1}{2\pi}\ln\left(\frac{188.6}{75.6}\right) = 0.1456$$

Uncertainty in M_u:

$$\frac{\Delta M_u}{M_u} = \sqrt{\left(\frac{1}{2}\frac{\Delta M_{p1}}{M_{p1}}\right)^2 + \left(\frac{1}{2}\frac{\Delta M_{p2}}{M_{p2}}\right)^2} = \sqrt{\left(\frac{1}{2}\frac{4}{188.6}\right)^2 + \left(\frac{1}{2}\frac{2}{75.6}\right)^2} = 0.0169$$

$$\Delta M_u = 0.0169 \times 119.4075 = 2.018$$

$$u_C(M_u) = \frac{1}{\sqrt{3}} \times 2.018 = 1.165$$

$$U(M_u) = 2 \times u_C(M_u) = 2 \times 1.165 = 2.33 \approx 3$$

$$M_u = 119 \pm 3\,\text{g}$$

Uncertainty in μ_u:

$$u_C(\mu_u) = \frac{1}{\sqrt{3}}\cdot\Delta\mu_u = \frac{1}{2\pi\sqrt{3}}\sqrt{\left(\frac{\Delta M_{p1}}{M_{p1}}\right)^2 + \left(\frac{\Delta M_{p2}}{M_{p2}}\right)^2} = \frac{1}{2\pi\sqrt{3}}\sqrt{\left(\frac{4}{188.6}\right)^2 + \left(\frac{2}{75.6}\right)^2} = 0.003117$$

$$U(\mu_u) = 2 \times u_C(\mu_u) = 2 \times 0.003117 = 0.00623 \approx 0.007$$

$$\mu_u = 0.146 \pm 0.007$$