Theoretical Question 3: Electron and Gas Bubbles in Liquids

MARKING SCHEME

Total	Mark(s)	Marking Scheme for Answers
Part A 4.0	(a) 0.4	Relation between P_{He}, P_{e}, and σ. ➢ 0.1 for force on interface from surface tension. →(a-1)† ➢ 0.1 for force on interface from pressure. →(a-1) ➢ 0.1 for condition of static equilibrium. →(a-1) ➢ 0.1 for $P_{\text{e}} = P_{\text{He}} + 2\sigma/R$ →(a-2)
	1.0	Relation between E_k and P_{e}. ➢ 0.1 for $P \propto 1/R$ ➢ 0.2 for $E_{\text{k}} \propto 1/R^2$ →(a-3) ➢ 0.2 for $dE_{\text{k}}/dR \propto -2E_{\text{k}}/R$ →(a-4) ➢ 0.2 for work-energy relation $dE_{\text{k}} = dW$ →(a-4) ➢ 0.1 for work supplied to bubble $dW = -P_{\text{e}}dV$ →(a-5) ➢ 0.2 for $P_{\text{e}} = E_{\text{k}}/2\pi R^3$ →(a-6)
	(b) 0.8	The smallest possible kinetic energy E_0 as a function of R. ➢ 0.2 for uncertainties = mean-squared values →(b-2)(b-3) ➢ 0.1 for $(\Delta x)^2 = (\Delta y)^2 = (\Delta z)^2$ →(b-4)(b-5) ➢ 0.1 for $3(\Delta x)^2 = \overline{r^2}$ →(b-4)(b-5) ➢ 0.2 for $\overline{r^2}\,\overline{p^2} \geq 9\hbar^2/4$ →(b-6) ➢ 0.2 for $E_0 = 9\hbar^2/8mR^2$ →(b-8)
	(c) 0.6	The bubble's equilibrium radius R_{e} when $E_{\text{k}} = E_0$ and $P_{\text{He}} = 0$. ➢ 0.2 for establishing a valid equation for R_{e}. →(c-1) ➢ 0.2 for $R_{\text{e}} = (9\hbar^2/32\pi m\sigma)^{1/4}$ →(c-2) ➢ 0.1 for significant figure (or mantissa) with first 2 digits correct. ➢ 0.1 for unit and exponent
	(d) 0.6	Condition satisfied by R and P_{He} for stable equilibrium at R. ➢ 0.2 for indicating stable equilibrium requires restoring force. →(d-1) ➢ 0.4 for $P_{\text{He}} > -8\sigma/(5R)$ →(d-4)
	(e) 0.6	The threshold pressure P_{th} for no possibility of equilibrium. ➢ 0.2 for an inequality relating P_{He} and R. →(e-1) ➢ 0.4 for expression of P_{th}. →(e-5)

Part B 6.0	(f) 0.4	Amount of work dW done on the liquid from R to $R+dR$. ➢ 0.1 for work on a sphere is $dW = FdR = PdV$ →(f-1) ➢ 0.1 for change of liquid volume is zero. →(f-1) ➢ 0.2 for expression of dW →(f-1)
	0.4	Values of m and n. ➢ 0.2 for m = 3 →(f-4) ➢ 0.2 for n = 2 →(f-4)
	(g) 0.4	Pressure $P \equiv P(R)$ as a function of R. ➢ 0.2 for obtaining initial pressure P_i →(g-1) ➢ 0.1 for $PV^\gamma = \text{constant}$ →(g-2) ➢ 0.1 for expression of $P(R)$ →(g-2)
	0.2	Temperature $T \equiv T(R)$ as a function of R. ➢ 0.1 for $TV^{\gamma-1} = \text{constant}$ →(g-3) ➢ 0.1 for expression of $T(R)$ →(g-3)
	(h) 0.6	The coefficient μ in terms of R_i and P_0. ➢ 0.2 for an integrable equation for $\dot{\beta}^2$ →(h-2) ➢ 0.2 for carrying out integration leading to $U(\beta)$ →(h-3) ➢ 0.2 for $\mu = P_0/(3R_i^2)$ →(h-5)
	(i) 0.4	Values of the constant C_m. ➢ 0.4 for $C_m = 1$ →(i-2)
	0.3	The minimum radius R_m for $R_i = 7R_0$. ➢ 0.2 for significant figure (or mantissa) with first 2 digits correct. (i-5) ➢ 0.1 for unit and exponent
	0.3	The temperature T_m of the gas at the minimum radius $\beta = \beta_m$. ➢ 0.2 for significant figure (or mantissa) with first 2 digits correct. (i-6) ➢ 0.1 for unit and exponent
	(j) 0.6	The radius β_u at which $u \equiv \lvert\dot{\beta}\rvert$ reaches its maximum value. ➢ 0.1 for an equation satisfied by β_u →(j-2) ➢ 0.2 for obtaining β_u as a function of Q →(j-4) ➢ 0.2 for significant figure (or mantissa) with first 2 digits correct. (j-4) ➢ 0.1 for unit and exponent
	0.4	The value $\bar{u}$ of radial speed u at $\beta = \bar{\beta} \equiv (\beta_m + \beta_u)/2$. ➢ 0.1 for value of $\beta = \bar{\beta} \equiv (\beta_m + \beta_u)/2$ ➢ 0.2 for significant figure (or mantissa) with first 2 digits correct. (j-6) ➢ 0.1 for unit and exponent

	0.6	The time duration Δt_m for β to diminish from β_u to β_m. ➢ 0.1 for a formula to compute Δt_m →(j-7) ➢ 0.1 for choosing a reasonable radial speed such as $\bar{u}$ in $\Delta t_m = (\beta_u - \beta_m)/\bar{u}$ →(j-7) ➢ 0.1 for the value of a reasonable radial speed. →(j-6) ➢ 0.2 for significant figure (or mantissa) with first 2 digits correct. (j-7) ➢ 0.1 for unit and exponent
	(k) 0.6	The power $\dot{E}$ supplied to the bubble at β. ➢ 0.2 for a formula of $\dot{E}$ in terms of derivatives $\dot{V}$ or $\dot{T}$. →(k-2) ➢ 0.2 for expressing $\dot{V}$ or $\dot{T}$ in terms of $\dot{\beta}$ and β. →(k-2) ➢ 0.2 for $\dot{E} = -4\pi R_i^3 P_i \dot{\beta}/\beta^3$ →(k-2)
	0.8	The upper bound of the emissivity a. ➢ 0.2 for radiant power in terms of temperature →(k-1) ➢ 0.2 for radiant power in terms of β →(k-1) ➢ 0.2 for an upper bound of a →(k-3) ➢ 0.2 for value of $a < 0.0107$ →(k-4)

†The equation number(s) at the end of a line refers to equation(s) in the SOLUTION sheets.