# Problem2: Oscillation of Water-Filled Vessel

## Section I

i) $m_1 = \rho\pi\left[R^2-(R-t)^2\right]L = \rho\pi\left(2Rt-t^2\right)L$ ………………….. (i)

ii) $m_2 = \rho\pi(0.6\,\text{cm})R^2$ ………………….. (ii)

iii) $m_3 = \pi(R-t)^2 L$ ………………….. (iii)

iv) $M = m_1+2m_2+m_3$ ………………….. (iv)

v) Water, as an **ideal** fluid, does not take part in the oscillatory motion of the water-filled vessel. We therefore shall not include contribution of water in the expressions for the moments of inertia.

$$I_y = \frac{1}{2}m_1\left[R^2+(R-t)^2\right]+2\left[\frac{1}{2}m_2R^2\right]$$ ………………….. (v)

$\ell=35.6$ cm, $g = 978$ cm s$^{-2}$, $a=R=2.5$ cm,

$\rho=2.70$ g cm$^{-3}$, $h = 9.2$ cm, $L=9.2-1.2=8.0$ cm

$$m_1 = 339.3t-67.86t^2 \text{ g},$$

$$m_2 = 31.8 \text{ g},$$

$$m_3 = 157.1-125.7t+25.13t^2 \text{ g},$$

$$M = 220.7+213.6t-42.73t^2 \text{ g}$$

$$I_y = 198.8+2121t-1273t^2+339.3t^3-33.93t^4$$

## Section II

**a)**

$$T_y = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_y}{Ma^2}}$$ ………………….. (vi)

Time for 50 oscillations is 43.3 s, hence

$$T_y = 0.866 \text{ s} \pm 0.004 \text{ s}$$

Substituting these values into equation (vi), we get

$$t^4-10t^3+33.42t^2-41.97t+15.36 = 0$$ ………………….. (vii)

The solution of (vii), by numerical iteration, is $t = 0.62 \text{ cm} \pm 0.02 \text{ cm}$

(The error can be estimated from a repeat of the procedure or a differential equation for $dT$ and $dt$ )

Hence, we get

$$m_1 = 184 \pm 5 \text{ g},$$

$$m_2 = 31.8 \pm 0.2 \text{ g},$$

$$m_3 = 89 \pm 2 \text{ g},$$

$$M = 337 \pm 6 \text{ g}.$$

**b)**

$$T_x = 2\pi \sqrt{\frac{\ell}{g} \cdot \frac{I_x}{Ma^2}} \qquad \text{......................... (viii)}$$

$\ell = 33.6 \text{ cm}, \quad a = \frac{h}{2} = \frac{9.2}{2} = 4.6 \text{ cm}$

Time for 50 oscillations is 38.0 s $\pm$ 0.2 s, hence

$$T_x = 0.760 \text{ s} \pm 0.004 \text{ s}$$

And from $T_x = 2\pi \sqrt{\frac{\ell}{g} \cdot \frac{I_x}{Ma^2}}$ where $\ell = 33.6 \text{ cm}, \ a = \frac{h}{2} = \frac{9.2}{2} = 4.6 \text{ cm}$,

$g = 978 \text{ cm s}^{-2}$, $M = 337 \text{ g}$, we get

$$I_x^{\text{Exp}} = 3036 \text{ g cm}^2 \pm 94 \text{ g cm}^2$$

Also $I_x^{\text{Theo}} = 3261 \text{ g cm}^2 \pm 68 \text{ g cm}^2$

**c)**

$$\Delta I_x = I_x^{\text{Theo}} - I_x^{\text{Exp}} = 225 \text{ g cm}^2$$

The experimental value $I_x^{\text{Exp}}$ is smaller than $I_x^{\text{Theo}}$ by 225 g $\text{cm}^2$.

This difference is probably significant and it is due to low viscosity of water. The mass of water in the middle section does not take part in the oscillatory motion of the vessel.

$\Delta I_x$ can be estimated to be due to a stationary cylindrical portion of water in the middle.

$$\Delta I_x = \pi (R-t)^2 L_{water} \left( \frac{L_{water}{}^2}{12} + \frac{(R-t)^2}{4} \right)$$

$L_{water} \approx 5.7\,\text{cm}$

This corresponds to the water mass of $\approx 63.8\ g$

The percentage of the water that takes part in the oscillation is $\approx 28.5\%$

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