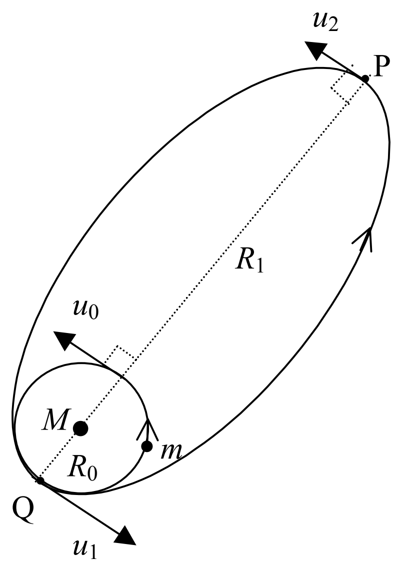
Theoretical Competition

I. Satellite's orbit transfer

In the near future we ourselves may take part in launching of a satellite which, in point of view of physics, requires only the use of simple mechanics.



a) A satellite of mass m is presently circling the Earth of mass M in a circular orbit of radius R_0. What is the speed (u_0) of mass m in terms of M, R_0 and the universal gravitation constant G? (1 point)

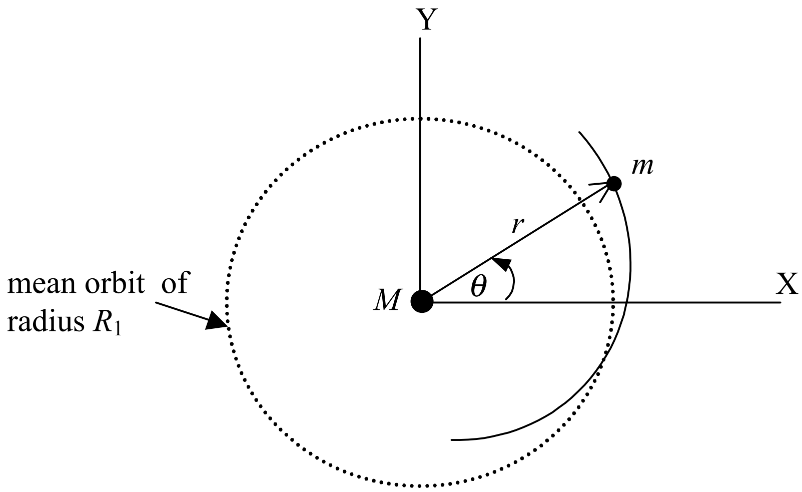
b) We are to put this satellite into a trajectory that will take it to point P at distance R_1 from the centre of the Earth by increasing (almost instantaneously) its velocity at point Q from u_0 to u_1. What is the value of u_1 in terms of u_0, R_0, R_1? (2 points)

c) Deduce the minimum value of u_1 in term of u_0 that will allow the satellite to leave the Earth's influence completely. (1 point)

d) (Referring to part b.) What is the velocity (u_2) of the satellite at point P in terms of u_0, R_0, R_1? (1 point)

e) Now, we want to change the orbit of the satellite at point P into a circular orbit of radius R_1 by raising the value of u_2 (almost instantaneously) to u_3.
What is the magnitude of u_3 in terms of u_2, R_0, R_1? (1 point)

f)



If the satellite is slightly and instantaneously perturbed in the radial direction so that it deviates from its previously perfectly circular orbit of radius R_1, derive the period of its oscillation T of r about the mean distance R_1.

Hint: Students may make use (if necessary) of the equation of motion of a satellite in orbit:

$$m\left[\frac{d^2}{dt^2}r - \left(\frac{d}{dt}\theta\right)^2 r\right] = -G\frac{Mm}{r^2} \qquad \text{..............................} \quad (1)$$

and the conservation of angular momentum:

$$mr^2\frac{d}{dt}\theta = \text{constant} \qquad \text{..............................} \quad (2)$$

(3 points)

g) Give a rough sketch of the whole perturbed orbit together with the unperturbed one.

(1 point)