Solution and Marking Scheme
Experiment
II. Cylindrical Bore

a) Derivation of moment of inertia I (0.5 points)

Configuration Fig. 1.2(a)

$$I_1 = \frac{1}{6}Ma^2 - \frac{1}{2}mb^2 = \frac{1}{6}(\rho a^3)a^2 - \frac{1}{2}(\rho\pi b^2 a)b^2$$

$$= \frac{1}{6}\rho a^5 - \frac{1}{2}\rho\pi ab^4$$

Configuration Fig. 1.2(b)

$$I_2 = \frac{1}{6}Ma^2 - \frac{1}{12}ma^2 - \frac{1}{4}mb^2 = \frac{1}{6}(\rho a^3)a^2 - \frac{1}{12}(\rho\pi b^2 a)a^2 - \frac{1}{4}(\rho\pi b^2 a)b^2$$

$$= \frac{1}{6}\rho a^5 - \frac{1}{12}\rho\pi a^3 b^2 - \frac{1}{4}\rho\pi ab^4$$

Derivation of period of oscillation T

For both configurations:

the restoring torque $\tau = Fd$

where $F = \frac{1}{2}m_0 g\frac{\delta s}{\ell}$ and $\frac{\delta s}{d/2} \approx \theta$

$$F \approx \frac{1}{2}m_0 g\frac{d}{2\ell}\theta$$ (0.5 points)

net mass $m_0 = \rho a^3\left(1 - \pi\frac{b^2}{a^2}\right) = \rho a^3(1-\pi x^2)$ where $x \equiv \frac{b}{a}$

since $\tau = I\alpha$, $\alpha = \dfrac{\frac{1}{4}m_0 g\frac{d^2}{\ell}\theta}{I}$ (0.5 points)

$$\omega^2 = \frac{4\pi^2}{T^2} = \frac{\frac{1}{4}m_0 g\frac{d^2}{\ell}}{I}$$

$$T^2 = \frac{4\pi^2 I\ell}{\frac{1}{4}m_0 gd^2} = \left(\frac{16\pi^2 I}{m_0 gd^2}\right)\ell$$

For configuration in Fig. 2.2(a),

$$T_1^2 = \left(\frac{16\pi^2}{gd^2}\frac{\frac{1}{6}\rho a^5 - \frac{1}{2}\rho\pi ab^4}{\rho a^3(1-\pi x^2)}\right)\ell$$

$$= \frac{8\pi^2}{3g}\left(\frac{a}{d}\right)^2\left(\frac{1-3\pi x^4}{1-\pi x^2}\right)\ell \qquad \text{(0.5 points)}$$

For configuration in Fig. 2.2(b), $$T_2^2 = \left(\frac{16\pi^2 \frac{1}{6}\rho a^5\left(1-\frac{\pi}{2}\frac{b^2}{a^2}-\frac{3\pi}{2}\frac{b^4}{a^4}\right)}{\rho a^3(1-\pi x^2)gd^2}\right)\ell$$

$$= \frac{8\pi^2}{3g}\left(\frac{a}{d}\right)^2\left(\frac{1-\frac{\pi x^2}{2}-\frac{3\pi}{2}x^4}{1-\pi x^2}\right)\ell$$

b) For configuration in Fig. 2.2(a), $d = 7.0$ cm

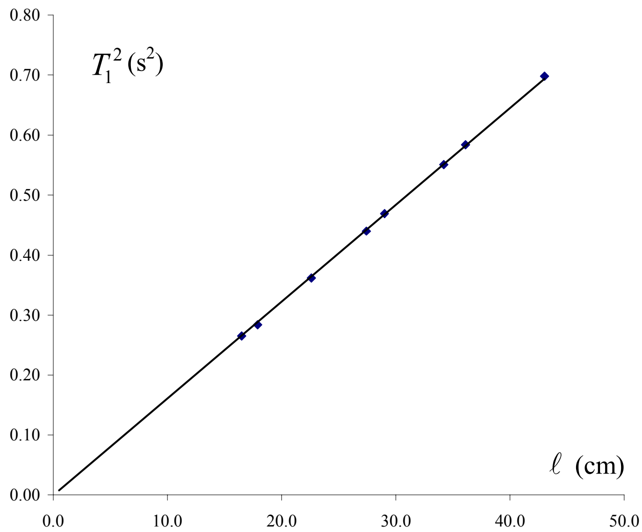
ℓ (cm)	T_1 for 40 oscillations(s)			T_1 (s)	$(T_1)^2$ (s^2)
16.5	20.60	20.50	20.70	0.515	0.265
17.9	21.35	21.35	21.30	0.533	0.284
22.6	24.05	24.00	24.00	0.601	0.362
27.4	26.55	26.45	26.55	0.663	0.440
29.0	27.40	27.40	27.40	0.685	0.469
34.2	29.75	29.70	29.65	0.743	0.551
36.1	30.60	30.60	30.50	0.764	0.584
43.0	33.40	33.35	33.50	0.835	0.698

(3 points):

3 sets of n oscillations (1 point) [2 sets –0.3, 1 set –0.7]

$n \geq 20$ (1 point) [$\geq$15, -0.3, $\geq$10, -0.7, <10, -1.0]

number of lengths, ℓ, ≥ 5 (1 point) [4, -0.3, 3, -0.5, 1 or 2, -1.0]

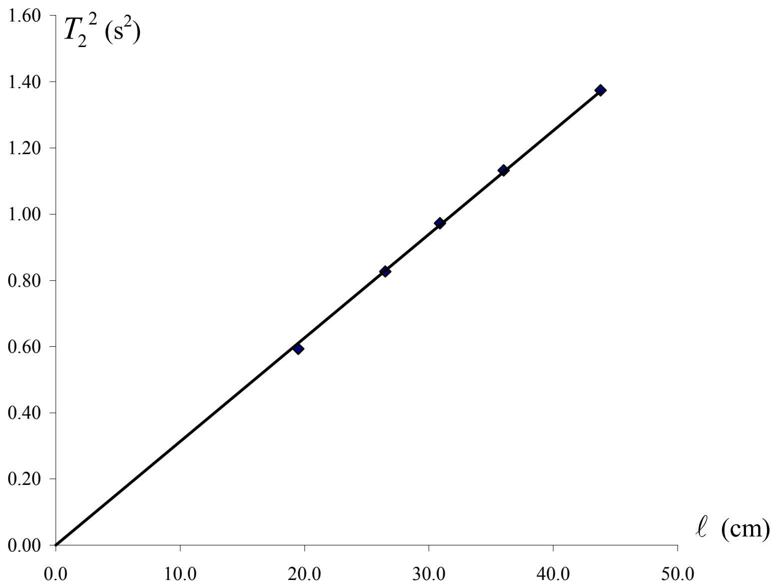


slope of graph: $$s_1 = \frac{0.698-0.265}{(43.0-16.5)\times10^{-2}} = \frac{0.433}{26.5\times10^{-2}} = 1.634 \text{ s}^2/\text{m}$$

$$x = \frac{b}{a} = 0.24 \pm 0.02$$

For configuration in Fig. 2.2(b), d = 4.9 cm

ℓ (cm)	T_2 for 40 oscillations (s)			T_2 (s)	$(T_2)^2$ (s^2)
43.8	46.95	46.90	46.80	1.172	1.374
36.0	42.70	42.45	42.50	1.064	1.132
30.9	39.60	39.40	39.35	0.986	0.973
26.5	36.40	36.30	36.45	0.909	0.827
19.5	30.80	30.85	30.75	0.776	0.593



slope of graph: $s_2 = \dfrac{1.374-0.827}{(43.8-26.5)\times10^{-2}} = \dfrac{0.547}{17.3\times10^{-2}} = 3.14$ s^2/m

$$x = \frac{b}{a} = 0.25 \pm 0.03$$

graph (3.0 points): good graph (1.5 points)
slope (1.0 point)
error of experimental points (0.5 point)

calculation for $\dfrac{b}{a}$ (1.0 point) error estimation (1.0 point)