Spherical coordinates $(r, \theta, \phi) \in \mathbb{R}_0^+ \times [0, \pi] \times [0, 2\pi)$

$$x = r\sin(\theta)\cos(\phi), y = r\sin(\theta)\sin(\phi), z = r\cos(\theta)$$
$$\hat{\mathbf{r}} = \sin(\theta)\cos(\phi)\hat{\mathbf{x}} + \sin(\theta)\sin(\phi)\hat{\mathbf{y}} + \cos(\theta)\hat{\mathbf{z}}$$
$$\hat{\boldsymbol{\theta}} = \cos(\theta)\cos(\phi)\hat{\mathbf{x}} + \cos(\theta)\sin(\phi)\hat{\mathbf{y}} - \sin(\theta)\hat{\mathbf{z}}$$
$$\hat{\boldsymbol{\phi}} = -\sin(\phi)\hat{\mathbf{x}} + \cos(\phi)\hat{\mathbf{y}}$$

Gradient:

$$\boldsymbol{\nabla} T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin(\theta)}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence:

$$\boldsymbol{\nabla} \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(\sin(\theta)v_\theta) + \frac{1}{r\sin(\theta)}\frac{\partial v_\phi}{\partial \phi}$$

Curl:

$$\boldsymbol{\nabla} \times \mathbf{v} = \frac{1}{r\sin(\theta)}\left[\frac{\partial}{\partial \theta}(\sin(\theta)v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin(\theta)}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\theta}}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 T}{\partial \phi^2}$$

Cylindrical coordinates $(s, \phi, z) \in \mathbb{R}_0^+ \times [0, 2\pi) \times \mathbb{R}$

$$x = s\cos(\phi), y = s\sin(\phi), z = z$$
$$\hat{\mathbf{s}} = \cos(\phi)\hat{\mathbf{x}} + \sin(\phi)\hat{\mathbf{y}}$$
$$\hat{\boldsymbol{\phi}} = -\sin(\phi)\hat{\mathbf{x}} + \cos(\phi)\hat{\mathbf{y}}$$
$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

Gradient:

$$\boldsymbol{\nabla} T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

Divergence:

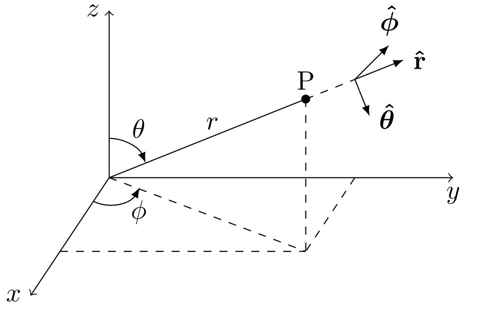
$$\boldsymbol{\nabla} \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl:

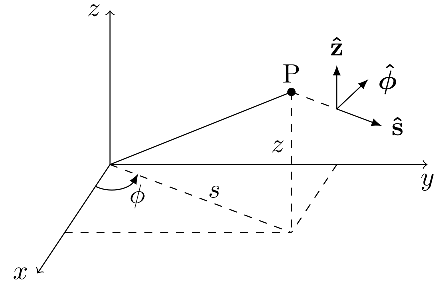
$$\boldsymbol{\nabla} \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$



Spherical coordinates



Cylindrical coordinates

General

Identities:

$$
\begin{aligned}
\boldsymbol{\nabla}(fg) &= f\boldsymbol{\nabla}g + g\boldsymbol{\nabla}f \\
\boldsymbol{\nabla}(\mathbf{A}\cdot\mathbf{B}) &= \mathbf{A}\times(\boldsymbol{\nabla}\times\mathbf{B}) + \mathbf{B}\times(\boldsymbol{\nabla}\times\mathbf{A}) + (\mathbf{A}\cdot\boldsymbol{\nabla})\mathbf{B} + (\mathbf{A}\cdot\boldsymbol{\nabla})\mathbf{A} \\
\boldsymbol{\nabla}\cdot(f\mathbf{A}) &= f(\boldsymbol{\nabla}\cdot\mathbf{A}) + \mathbf{A}\cdot(\boldsymbol{\nabla}f) \\
\boldsymbol{\nabla}\cdot(\mathbf{A}\times\mathbf{B}) &= \mathbf{B}\cdot(\boldsymbol{\nabla}\times\mathbf{A}) - \mathbf{A}\cdot(\boldsymbol{\nabla}\times\mathbf{B}) \\
\boldsymbol{\nabla}\times(f\mathbf{A}) &= f(\boldsymbol{\nabla}\times\mathbf{A}) - \mathbf{A}\times(\boldsymbol{\nabla}f) \\
\boldsymbol{\nabla}\times(\mathbf{A}\times\mathbf{B}) &= (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{A} - (\mathbf{A}\cdot\boldsymbol{\nabla})\mathbf{B} + \mathbf{A}(\boldsymbol{\nabla}\cdot\mathbf{B}) - \mathbf{B}(\boldsymbol{\nabla}\cdot\mathbf{A})
\end{aligned}
$$

Fundamental theorems:

$$
\begin{aligned}
\int_{\mathbf{a}\atop\mathcal{P}}^{\mathbf{b}} (\boldsymbol{\nabla}T)\cdot \mathrm{d}\mathbf{l} &= T(\mathbf{b}) - T(\mathbf{a}) \\
\int_{\mathcal{V}} (\boldsymbol{\nabla}\cdot\mathbf{v})\mathrm{d}\tau &= \oint_{\mathcal{S}} \mathbf{v}\cdot \mathrm{d}\mathbf{a} \\
\int_{\mathcal{S}} (\boldsymbol{\nabla}\times\mathbf{v})\cdot \mathrm{d}\mathbf{a} &= \oint_{\mathcal{P}} \mathbf{v}\cdot \mathrm{d}\mathbf{l}
\end{aligned}
$$