Задачи – 5 Май 2019г.

Problem 3
Bohr's model of a hydrogen atom (10 points)

Consider a hydrogen atom, which consists of a proton, whose mass can be assumed virtually infinite, and an electron of mass $m_e = 9.11 \times 10^{-31}$ *kg*. The proton has a positive charge of $e = 1.61 \times 10^{-19}$ Cl and the electron has a negative charge $-e$, so that the atom is totally neutral.

1. **[2.0 points]** It is known that most of the atoms are stable and can exist for a long time. This means that the electron being in the hydrogen atom cannot be at rest since the attraction force would inevitably make it fall onto the proton. Let the distance between the electron and the proton be equal to $r_0 = 5.00 \times 10^{-11}$ *m*/ Assuming that the laws of classical physics are applicable, find the time t_1 that the initially motionless electron needs to fall onto the proton.

Hence, to guarantee the stability of the hydrogen atom, the electron must move around the proton like the planets revolve around the Sun. Let the electron orbit be circular and assume again that the laws of classical physics can still be applied.

2. **[1.0 points]** Find the electron velocity v as a function of the orbit radius r.
3. **[0.5 points]** Find the angular momentum L of the electron as a function of the orbit radius r.

Danish physicist Niels Bohr postulated that the angular momentum of the electron is an integer of the Planck constant $\hbar = 1.05 \times 10^{-34}$ Дж·с, that is $L = n\hbar$, where n is the main quantum number.

4. **[0.5 points]** Find the possible radii r_n of the electron orbits in the hydrogen atom.
5. **[0.5 points]** Calculate the numerical value of the minimum radius r_n of the electron orbit in the hydrogen atom.
6. **[1.0 points]** Find the possible values of the total energy E_n of the electron in the hydrogen atom.
7. **[0.5 points]** Calculate the numerical value of the minimum total energy E_1 of the electron in the hydrogen atom.

According to classical electrodynamics, if the electron is accelerated, it loses its energy due to electromagnetic radiation. The power of the radiation is given by

$$P = \frac{1}{6\pi\varepsilon_0} \frac{e^2 a^2}{c^3},$$

where a is the electron acceleration, $\varepsilon_0 = 8.85 \times 10^{-12}$ *F/m* is the dielectric constant, $c = 3.00 \times 10^8$ *m / s* is the speed of light in vacuum.

Electromagnetic radiation would lead to the fact that in the Bohr model the electron would fall onto the proton. In the following assume that the electron orbit is almost circular.

8. **[1.5 points]** Assuming that at the time moment $t = 0$ the electron moves in the orbit with the radius r_1, find the dependence of the orbit radius on time t.
9. **[0.5 points]** Find and calculate the time τ_1 that is needed for the electron to fall from the orbit with the radius r_1 onto the proton.
10. **[2.0 points]** How many revolutions around the proton can the electron make during its fall for the time interval τ_1?

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Example 2

An interesting variant of Example 1 is one in which three bodies (each of the type described in Example 1, with $U = CT$) have initial temperatures of 300 K, 350 K, and 400 K, respectively. It is desired to raise *one* body to as high a temperature as possible, independent of the final temperatures of the other two (and without changing the state of any external system). What is the maximum achievable temperature of the single body?

2.59. For a Van der Waals gas find:
(a) the equation of the adiabatic curve in the variables T, V;
(b) the difference of the molar heat capacities $C_p - C_V$ as a function of T and V.

2.57. Find the work performed by one mole of a Van der Waals gas during its isothermal expansion from the volume V_1 to V_2 at a temperature T.

Example 1

A particular system is constrained to constant mole number and volume, so that no work can be done on or by the system. Furthermore, the heat capacity of the system is C, a constant. The fundamental equation of the system, for constant volume, is $S = S_0 + C \ln(U/U_0)$, so $U = CT$.

Two such systems, with equal heat capacities, have initial temperatures T_{10} and T_{20}, with $T_{10} < T_{20}$. An engine is to be designed to lift an elevator (i.e., to deliver work to a purely mechanical system), drawing energy from the two thermodynamic systems. What is the maximum work that can be so delivered?

Задача 3: На фигура 2 е показана схемата на свързване на три съпротивления и идеален диод към източник на променливо напрехение с амплитуда $U_0 = 10V$. Стойностите на съпротивленията са $R_1 = 10\Omega$, $R_2 = 20\Omega$ и $R_3 = 15\Omega$. Да се определи средната мощност, която се отделя върху съпротивленията R_1 и R_2.