**SHORT QUESTIONS: MARKING SCHEME**

1. Several exoplanets have been observed in the Gliese 876 system as given in the following table,

|  |  |  |
| --- | --- | --- |
| **Gliese System** | **Mass** | **Semi Major Axis (AU)** |
| Gliese 876 b | 2.276 MJ | 0.2083 |
| Gliese 876 c | 0.714 MJ | 0.1296 |
| Gliese 876 d | 6.8 M⊕ | 0.0208 |
| Gliese 876 e | 15 M⊕ | 0.334 |

Where is mass of Sun, is mass of Jupiter (), and mass of Earth. Assume that all these planets revolve around Gliese 876 in the same direction. Two planets are said to be in resonant orbits if the synodic period of one planet with respect to the other planet is an integer multiple of the orbital period of the second planet.

Find if any of the exoplanets of Gliese 876 system may have resonant orbits.

**Answer and Marking Scheme:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1** | **Determine planet periods**  Synodic period of planet 1 and 2 (assume *P*1 < *P*2) is  Resonant orbit happens if  Using the data of the Gliesean system we have   |  |  |  |  | | --- | --- | --- | --- | | **Gliese System** | **Mass (kg)** | **Semi Major Axis (m)** | **Period (Earth, days)** | | Gliese 876 star | 6.64359 × 1029 |  |  | | Gliese 876 b | 4.31938 × 1027 | 3.1161 × 1010 | 60.07568 | | Gliese 876 c | 1.35564 × 1027 | 1.9388 × 1010 | 29.48304 | | Gliese 876 d | 4.079 × 1025 | 3.1116 × 109 | 1.8956 | | Gliese 876 e | 8.7194 × 1025 | 5.0011 × 1010 | 122.1432 | | **50** |
| **2** | **Calculate resonant orbit for each planet**  From the results above we see that   * Gliese 876 c and Gliese 876 b have resonant orbit      * Gliese 876 b and Gliese 876 e have resonant orbit | **25**  **25** |

1. Volcanic activity on Io, which has a synchronous orbital period, was proposed to be the result of tidal heating mainly from Jupiter. The resultant tidal force on a body is the difference in gravitational force experienced by the near and far sides of that body due to another body. Measurements of the surface distortion of Io via satellite radar altimeter mapping indicate that the surface rises and falls by up to 100m during one-half orbit. Only the surface layers will move by this amount. Interior layers within Io will move by a smaller amount, and thus we assume that on average the entire mass of Io is moved through 50m. Calculate the *average power* of the tidal heating on Io.

**Hint:** you can use the following approximation  for small *x*.

The mass of Io is .

The perijove distance is *rperi* = 420000 km

The apojove distance is *rapo* = 423400 km.

The orbital period of Io is 152853 s.

The radius of Io is *R*Io = 1821.6 km.

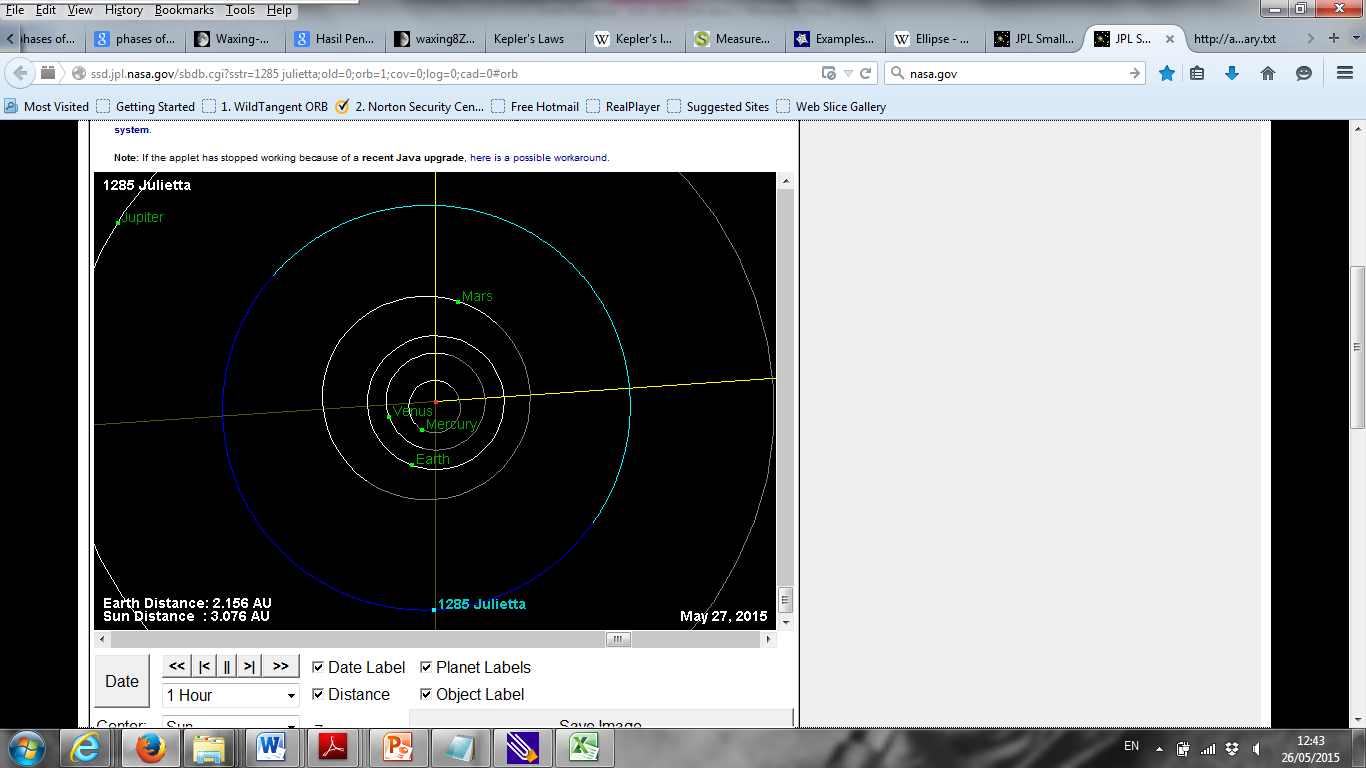
**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Calculate the tidal force**  The tidal force,  **Correction: The solution should use the elementar mass of Io. It will be corrected accordingly.**              , so that | **40**  **10** |
| **2** | **Calculate the difference of the tidal force**  The difference in the tidal force experienced at perijove (closest to Jupiter) and apojove (furthest from Jupiter) | **15**  **15** |
| **3** | **Calculate the average power of work done**  Hence, the average power of the work done on the rock during one-half orbit is | **10**  **10** |

1. On 27 May 2015 at 02:18:49, the occultation of the star HIP 89931 (Sgr) by the asteroid 1285 Julietta was observed from Borobudur temple, which was located at the center of the asteroid shadow path. It lasted for only 6.201 s. Assume that Earth’s orbit is circular and the orbit of Julietta is on the ecliptic plane and revolves in the same direction as Earth. At the occultation, Julietta is near its aphelion. At the time of the occultation the distances of Julietta from the Sun and the Earth are AU and 2.156 AU respectively. Find the approximate diameter of asteroid Julietta, if the semi major axis of Julietta is AU.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Calculate the Earth revolution speed**  Earth revolution speed is | **20** |
| **2** | **Calculate Julietta revolution speed**  Julietta revolution is (since the given data is semimajor axis, the orbit of Julietta is ellipse) | **25** |
| **3** | **Calculate the approximate dimension of Julietta**  *vE*  *vJ*  *rE*  *rJ*  *rJ-E*  **  *S*  *E*  *J*  From the geometry    Thus, Julietta’s relative projected speed to Earth is    Hence, the approximate dimension of Julietta is | **40**  **15** |

****

http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=1285%20julietta;old=0;orb=1;cov=0;log=0;cad=0#orb

1. Let us assume that an observer using a hypothetical, far-infrared, Earth-sized telescope (wavelength range 20 to 640 m) found a static and neutral supermassive black hole with a mass of 2.1 x 1010 . Determine the maximum distance at which this black hole can be resolved by the observer.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Identify the relationship between the telescope’s resolution and the angular size of black hole**  This problem can be solved by using relationship between the telescope’s resolution and the angular size of black holes which can be seen in this equation: | **25** |
| **2** | **Identify the relation for determining the distance**  Then, it is assumed that the radius of black hole is the Schwarszchild radius; and the telescope is also Earth-sized telescope. Therefore: | **50** |
| **3** | **Calculating the maximum-observer distance**  To find the maximum distance, then we should use the minimum .  Therefore:    6.6 | **25** |

1. An observer is trying to determine an approximate value of the orbital eccentricity of a man-made satellite. When the satellite was at apogee, it was observed to have moved by in a short time. When the radius vector connecting Earth and the satellite is perpendicular to the major axis (true anomaly is equal to 90o), within the same duration of time, it was observed to have moved by Assume that the observer is located at the center of the Earth. Find an approximate value of the eccentricity of the satellite’s orbit.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Identify expression for** .  We know that  . Hence | **30** |
| **2** | **Estimate area of sectors using area of triangles.**  Estimate the area of swept out sectors as the area of sectors of circles | **20** |
| **3** | **Use Kepler’s second law to find a relation between the ratio** **and the eccentricity** .  Use Kepler’s second law to obtain that  Thus, | **30** |
| **4** | **Obtain an estimate value of the eccentricity.**  Thus, the eccentricity is | **20** |

1. At the start of every observation, a radio telescope is pointed at a point-source calibrator that has a known flux density of 21.86 Jy outside the Earth’s atmosphere. However, on a certain date, the measured flux density of the calibrator source was 14.27 Jy. If the calibrator source was at an altitude of 35 degrees, estimate the zenith atmospheric optical depth, *τz*.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | Atmospheric optical depth τA can be determined from the measured flux, that is:  Smeas = exp(-τA) Sreal  14.27 = exp(-τA) × 21.86 🡪 exp(-τA) = 14.27/21.86 = 0.65  Then  τA = −ln(0.65) = 0.43 | **50** |
| **2** | Now we have:  τA = τZ sec z  τz = τA cos z  = 0.43 × cos (90 − 35)° = 0.25  where z is zenith angle. | **50** |

1. A galaxy at the boundary of a galaxy cluster of radius 10 Mpc is expected to escape from the cluster if it has an initial velocity of at least 700 km/s relative to the center of the cluster. Calculate the density of the cluster.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | We have the total energy of the escaping galaxy: | **15** |
| **2** | Where *V* is velocity of galaxy and *R* radius of galaxy cluster  Galaxy will escape from galaxy cluster if the total energy, | **10** |
| **3** |  | **15** |
| **4** | Escape velocity | **20** |
| **5** | Density of cluster | **15** |
| **6** | V=700 km/s  R=10 Mpc  G=6.6378×10-11Nm2kg  ρ=9.20121×10-27kg/m3 | **25** |

1. A strong continuum radio signal from a celestial body has been observed as a burst with a very short duration of 700 μs. The observed flux density at a frequency of MHz is measured to be kJy. If the distance from the source is known to be 2.3 kpc, estimate the brightness temperature of this source.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | During 700 μs, the radio wave travels about r = c t = 3 × 1010 × 700 × 10-6 = 2.1 × 107 cm. The region from where the burst originates must be no larger than the distance that light can travel during the duration of the burst. So we estimate that r is the size of the source. (r = 2.1 × 107 cm, R = 2.3 kpc)  Flux density observed at 1660 MHz is  S1660MHz = 0.35 kJy  = 0.35 × 103 × 10-23 ergs s-1 cm-2 Hz-1  = 3.5 × 10-21 ergs s-1 cm-2 Hz-1 | **10**  **10** |
| **2** | The solid angle subtended by this source of radiation is  Ω = π (r/R)2 = 3.14 (2.1 × 107/2.3 × 103 × 3.086 × 1018)2  = 2.75 × 10-29 sr | **30** |
| **3** | The flux density is related to the total brightness by the relation:  S1660MHz = B1660MHz Ω  while at this frequency, the total brightness can be approximated from the Rayleigh-Jeans formula:  B1660MHz = 2kTbν2/c2  where Tb is the brightness temperature. Then we have:  Tb = S1660MHzc2/2kν2Ω  Tb = [3.5 × 10−21 × (3 × 1010)2]/[2 × (1.38 × 10−16) × (1660 × 106)2 × (2.75 × 10−29)]  = 1.5 × 1026 K | **20**  **20**  **10** |

1. Assume that the Sun is a perfect blackbody. Venus is also assumed to be a blackbody, with temperature *T*V, and it is in thermal equillibrium (i.e. it is radiating about as much energy as it receives from the Sun) at its orbital distance of 0.72 AU. Suppose that at closest approach to Earth, Venus has an angular diameter of about 66 arcsec. What is the flux density of Venus at the closest approach to Earth as observed by a radio telescope at an observing frequency of 5 GHz?

**[Add the formula on the Table of Constants (also mentioning that the Rayleigh-Jeans formula is a derivation from Planck’s formula)]**

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | Solar luminosity is  L🞊 = 4πR🞊2σT🞊4  = 4 × 3.14 × (6.96 × 1010)2 (5.67 × 10-5)(5780)4 ergs/sec  = 3.9 × 1033 ergs/sec | **10** |
| **2** | Flux of the Sun intercepted by Venus is  LV = (AV(projected)/A(sphere,0.72au)) × L🞊  Also,  LV = 4πRV2σTV4  So,  4πRV2σTV4 = (πRV2/4πd2) × L🞊 = (πRV2/4πd2) × 4πR🞊2σT🞊4 | **30** |
| **3** | Therefore,  TV4 = (R🞊2/4d2) × T🞊4 = [(6.957 × 105 km)2/4 × (0.72 × 149.6 × 106)2] × (5780)4  = (483998490000/46407499776000000) × (5780)4 = (4.84 × 1011/4.64 × 1016) × (5780)4  = 1.0429316216908190111241385226915e-5 × 1114577187760656  = 1.04 × 10-5 × 1.11 × 1015  = 11624277939.308134327737156481455 = 1.1624 × 1010  TV = 328.5 K | **20** |
| **4** | The flux density of Venus at closest approach to Earth at an observing frequency of 5 GHz:  (1” = 4.84 × 10−6 rad)  S5GHz = B5GHz Ω = (2kTbν2/c2) Ω (🡪Tb =TV)  = (2 × 1.38 × 10−16 × 328.5 × (5 ×109)2 × π (66 × 4.84 × 10−6)2)/ (3 × 1010)2  = 8.07 × 10−22 ergs s-1 cm-2 Hz-1  = 80.7 Jy | **40** |

1. A molecular hydrogen cloud is known to have a temperature *T* = 115 K. If the atoms (assumed spherical) in this cloud have masses of about 5 × 10-23 *g* and radii of 3.5 Å, and the separation distance between the hydrogen atoms is [Placeholder]. Assume that the molecules are homogenous and in thermal equilibrium. Estimate the frequency at which they will radiate due to molecular rotational excitation.

**[Add the separation distance between hydrogen atom.]**

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | The molecules rotate with angular velocity ω and for spherical molecules, their moment of inertia is I = ⅖ mr2, i.e.,  **[The formula of the moment of inertia will be corrected accordingly]**  **[Change formula into: kT]**  I = ⅖ mr2 = ⅖ (5 × 10-23 g) (3.5 × 10-8 cm)2 = 2.45 × 10-38 g cm-2 | **30** |
| **2** | In thermal equilibrium, we have:  ½ I ω2 = 3/2 kT | **30** |
| **3** | So we can estimate:  ω = (3kT/I)1/2 = (3 × 1.38 × 10−16 115/2.45 × 10-38)1/2 = 1.39 × 1012 s-1  ν = ω/2π = 2.2 × 1011 Hz | **40** |

|  |  |  |
| --- | --- | --- |
| 1. The mass density of an object is inversely proportional to the radial distance from the center of the object with a factor of proportionality . If the escape velocity at the surface of the object is , calculate the total mass of the object. | | |
|  | Answer |  |
| **1** |  | **20** |
| **2** |  | **30** |
| **3** |  | **30** |
| **4** |  | **20** |

|  |  |  |
| --- | --- | --- |
| 1. A proton with a kinetic energy of propagates out from the surface of the Sun towards the Earth. Neglecting the magnetic field of the Sun, calculate the travel time of the proton as seen from the Earth. | | |
|  | Answer |  |
| **1** |  | **20** |
| **2** |  | **20** |
| **3** | 🡪 | **30** |
| **4** |  | **30** |

|  |  |  |
| --- | --- | --- |
| 1. Two protons collide to produce deuteron, positron, and neutrino in the center of a star. Write down the reaction equation and calculate the minimum temperature needed for this reaction. Assume that the neutrino mass is negligible. | | |
|  | Answer |  |
| **1** |  | **20** |
| **2** |  | **20** |
| **3** |  | **20** |
| **4** |  | **20** |
| **5** |  | **20** |

1. Suppose we live in a static and infinitely large universe where the average density of stars is and the average stellar radius is equal to the solar radius. Assume that standard Euclidean geometry holds true in this universe. How far, on average, could you see in any direction before your line of sight strikes a star? Please write your answer in Mpc.

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1.** | To decide how far one can see on average in a universe filled with spherical objects of radius *R*, it is simplest to think of a long cylinder along the line of sight. If an object is closer than *R* to the line of sight, then the line of sight intersects its surface. For a distance , the cylindrical volume which would contain such objects is . If the density of objects is *n*, then the volume that will on average contain one object is defined by and the average distance to which we see before our vision is blocked is | **50** |
| **2.** | The average radius *R* in Mpc | **25** |
| **3.** | Then | **25** |

1. An airplane was flying from Lima, capital of Peru ( and ) to Yogyakarta ( and ), near the venue of the 9th IOAA. The airplane chooses the shortest flight path from Lima to Yogyakarta. Find the latitude of the southernmost point of the flight path.

**Answer and Marking Scheme:**

**1st version**

|  |  |  |
| --- | --- | --- |
| **1** | **Determine coordinates of Lima and Yogyakarta.**  Assume that the equator is the plane and the Earth is a unit sphere.  The Cartesian coordinates of the Lima is The Cartesian coordinates of the Yogyakarta is | **15** |
| **2** | **Determine the normal of the plane defined by Lima, Yogyakarta, and center of the Earth.**  The normal of the plane containing the two places and the center of the Earth is | **20** |
| **3** | **Find the equation of the (shortest flight) path from Lima to Yogyakarta.**  Let the equation of the place be or equivalently where and . For the sake of simplicity, from now let us denote and as and , respectively.  Intersection of the Earth and the plane is the large circle or equivalently  It is also the equation for the shortest flight path from Lima to Yogyakarta. | **25** |
| **4** | **Determine the z-value of the southernmost point of the path (at the point, the discriminant of the equation**   **equals zero)**.  The plane , it intersects the large circle at at either at one or two points. The number of intersection point at the highest or at the lowest point is one, i.e. when the discriminant is zero. The discriminant is  Therefore , if . Then choose | **30** |
| **5** | **Determine the latitude of the southernmost point.**  Thus, the latitude of the point is | **10** |

2nd version (using the Law of Cosines for sides)

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Determine length of the sides of the triangle**    Using Law of Cosines for sides to obtain .rad | **20** |
| **2** | **Determine the angles of the triangle**  Then use to obtain rad. Similarly, use Law of Cosines  to get that rad. | **30** |
| **3** | **Use Law of Cosines for sides to construct an equation in** .  If P is the southernmost point, then the angle is a right angle, , the Law of  Thus,  Then substitute it to get | **20** |
| **4** | **Solve the equation**  Let . Then Multiply both sides by to get  which is a quadratic equation of . The roots are and . | **20** |
| **5** | **Determine the Latitude**  Thus, or  The latitude is or . The second one is impossible because it higher then the latitude of Lima. Thus the answer is | **10** |

3rd version (using Napier’s rule)

**Answer and Marking Scheme:**

|  |  |  |
| --- | --- | --- |
| **1** | **Determine length of the sides of the triangle**    Using Law of Cosines for sides to obtain .rad | **20** |
| **2** | **Determine the angles of the triangle**  Then use to obtain rad. Similarly, use Law of Cosines  to get that rad. | **30** |
| **3** | **Find the latitude of the point**  Let be the southernmost point on the flight path**.** Since the triangle has a right triangle at P, then use one of Napier’s rules  to get . Therefore, the latitude of is | **50** |