8th International Olympiad on Astronomy and Astrophysics
Suceava - Gura Humorului - August 2014

Content

Indications

1. The problems were elaborated concerning two aspects:
    a. To cover merely all the subjects from the syllabus;
    b. The average time for solving the items is about 15 minutes per a short problem;
2. In your folder you will find out the following:
    c. Answer sheets
    d. Draft sheets
    e. The envelope with the subjects in English and the translated version of them in your mother tongue;
3. The solutions of the problems will be written down only on the answer sheets you receive on your desk. **PLEASE WRITE ONLY ON THE PRINTED SIDE OF THE PAPER SHEET. DON'T USE THE REVERSE SIDE**. The evaluator will not take into account what is written on the reverse of the answer sheet.
4. The draft sheets is for your own use to try calculation, write some numbers etc. BEWARE: These papers are not taken into account in evaluation, at the end of the test they will be collected separately . Everything you consider as part of the solutions have to be written on the answer sheets.
5. Each problem have to be started on a new distinct answer sheet.
6. On each answer sheet please fill in the designated boxes as follows:
    a. In PROBLEM NO. box write down only the number of the problem: i.e. 1 – 15 for short problems, 16 – 19 for long problems. Each sheet containing the solutions of a certain problem, should have in the box the number of the problem;
    b. In Student ID – fill in your ID you will find on your envelope, consisted of 3 leters and 2 digits.
    c. In page no. box you will fill in the number of page, starting from 1. We advise you to fill this boxes after you finish the test
7. We don't understand your language, but the mathematic language is universal, so use as more relationships as you think that your solution will be better understand by the evaluator. If you want to explain in words we kindly ask you to use short English propositions.
8. Use the pen you find out on the desk. It is advisable to use a pencil for the sketches.
9. At the end of the test:
    a. Don't forget to put in order your papers;
    b. Put the answer sheets in the folder 1. Please verify that all the pages contain your ID, correct numbering of the problems and all pages are in the right order and numbered. This is an advantage of ease of understanding your solutions.
    c. Verify with the assistant the correct number of answer sheets used fill in this number on the cover of the folder and sign.
    d. Put the draft papers in the designated folder, Put the test papers back in the envelope.
    e. Go to swim

GOOD LUCK !

Problem 1. Lagrange Points

The *Lagrange* points are the five positions in an orbital configuration, where a small object is stationary relative to two big bodies, only gravitationally interacting with them. For example, an artificial satellite relative to Earth and Moon, or relative to Earth and Sun. In the **Figure 1** are sketched two possible orbits of Earth relative to Sun and of a small satellite relative to the Sun. Find out which of the two points L_3^1 and L_3^2 could be the real Lagrange point relative to the system Earth – Sun, and calculate its position relative to Sun. You know the following data: the Earth - Sun distance $d_{ES} = 15 \cdot 10^7$ km and the Earth – Sun mass ratio $M_E / M_S = 1/332946$

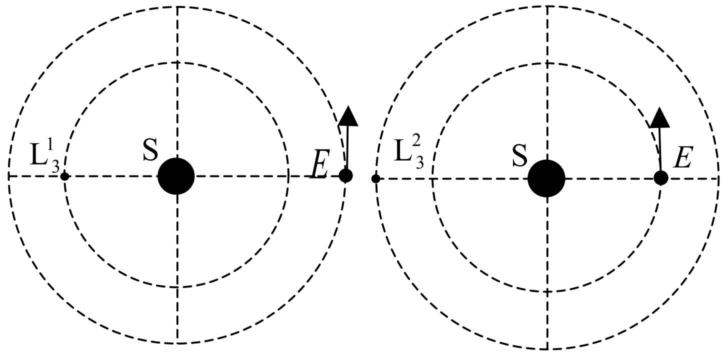


Figure 1

Problem 1. Marking scheme Lagrange Point

1.	Correct derivation of forces' equilibrium	6 points
2.	Correct identification of the Lagrange point	2 points
3.	Correct calculation of the position of Lagrange point	2 points
4.	Deduction for incorrect value	1 point

According to the notations in fig.1.1 and fig. 2.1

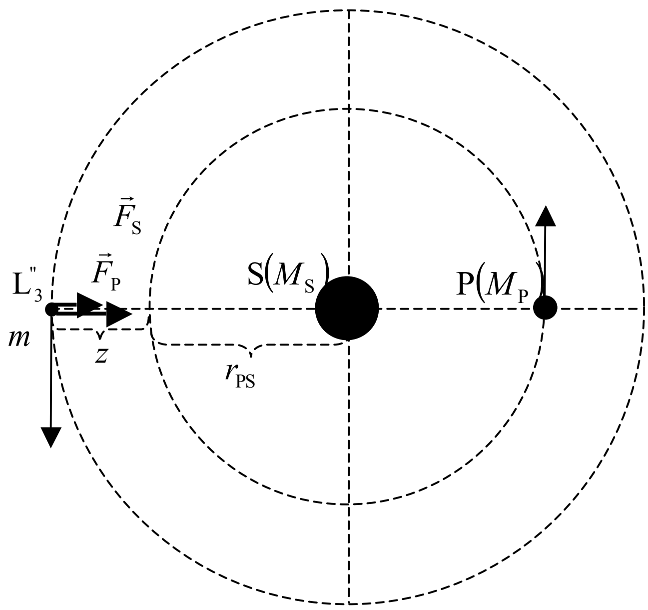


Figure 1a

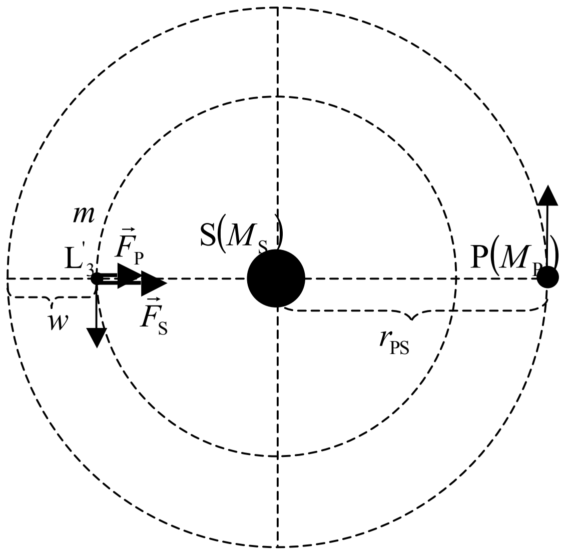


Figure 1 b

$$\vec{F}_S + \vec{F}_P = \vec{F}_{cp} = m\vec{a}_{cp}$$

$$F_S + F_P = ma_{cp} = m\omega^2(r_{PS} \pm w);$$ 2 points

The sign "+" for position L_3' and „– "for L_3''

$$K\frac{mM_S}{(r_{PS} \pm w)^2} + K\frac{mM_P}{(2r_{PS} \pm w)^2} = m\omega^2(r_{PS} \pm w);$$

Using the assumption that $w << r_{PS}$

$$\left(1 \pm \frac{w}{r_{PS}}\right)^{-2} \approx 1 \mp 2\frac{w}{r_{PS}} \quad \left(1 \pm \frac{w}{2r_{PS}}\right)^{-2} \approx 1 \mp \frac{w}{r_{PS}};$$

2points^p

The rotation speed

$$\omega^2 = \frac{KM_S}{r_{PS}^3}$$ 2p

The final relation

$$w = \mp\frac{M_P r_{PS}}{(12M_S + M_P)}$$

The value has to be positive, thus the L_3'' is the position of one Lagrange point 2p

$w \approx 37{,}54\,\mathrm{Km}$ 2p

Problem 2. Sun gravitational catastrophe!

In a gravitational catastrophe, the mass of the Sun mass decrease instantly to half of its actual value. If you consider that the actual Earth orbit is elliptical, its orbital period is $T_0 = 1\,year$ and the eccentricity of the Earth orbit is $e_0 = 0{,}0167$.

Find the period of the Earth`s orbital motion, after the gravitational catastrophe, if it occurs on: a) 3rd of July b) 3rd of January.

Problem 2. Marking scheme Sun gravitational catastrophe!

- Correct analyze of the initial conditions when the catastrophe occurs **(A)** **5 points**
- Correct calculations **(B)** **5 points**
  - Correct use of laws of conservation 2 points
  - Finding out that in the first case the orbit will be elliptic, relations (1)

	and (2)	2 points
o	Correct conduct of calculations	1 point

Detailed solution

(A) The orbit of Earth is elliptical, so the shape of the orbit after the solar catastrophe will depend on the moment when the decrees of the mass of the Sun will occur.

Initial analysis of the problem

a) In 3rd July the Earth is at the aphelion. The speed of the Earth is smaller than the speed of Earth on a circular orbit with radius $r_{0,\max} = a_0(1+e_0)$.

b) In 3rd January the Earth is at perihelion. The speed of the Earth is bigger than the speed of Earth on a circular orbit with radius $r_{0,\max} = a_0(1-e_0)$.

Conclusion (A) the period should be calculated only for situation a). The expected trajectory in this case is an elliptic one. The possibility that Earth hit the Sun is available too.

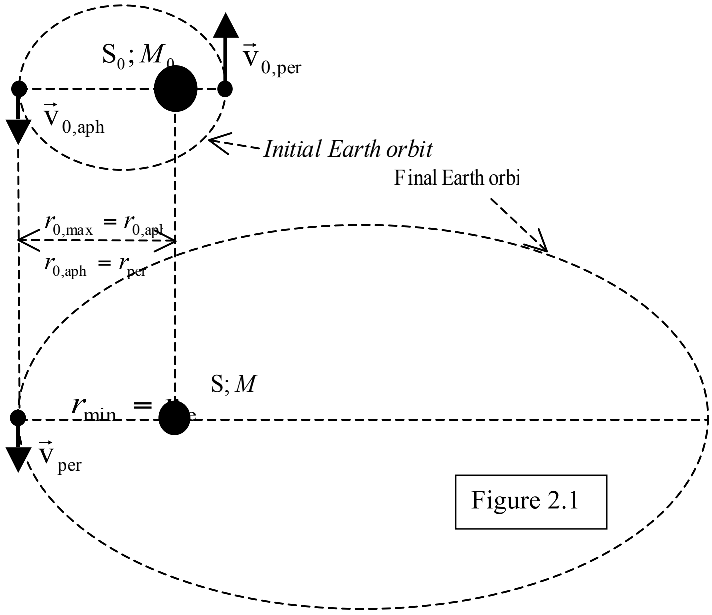


Figure 2.1

(B) Calculations:

In 3rd July the distance from Sun is maximum: fig. 2.1,

$$r_{0,\max} = a_0(1+e_0).$$

Before the catastrophe:

$v_{0,aph}$ - the speed of Earth on aphelion,

a_0 - big Earth's elliptical orbit semi axis

v_0 - the speed of Earth if its orbit is circular with radius $r_0 = a_0$

According to Keppler's second law and the law of energy conservation (see figure 2.1) the following relations can be written :

$$v_{0,per}\, r_{0,per} = v_{0,aph}\, r_{0,aph};$$

$$\frac{v_{0,per}^2}{2} - K\frac{M_0}{r_{0,per}} = \frac{v_{0,aph}^2}{2} - K\frac{M_0}{r_{0,aph}}$$

$$r_{0,\min} = r_{0,per} = a_0(1-e_0)$$

$$r_{0,\max} = r_{0,aph} = a_0(1+e_0)$$

$$KM_0 = v_0^2 r_0 = v_0^2 a_0$$

$$v_0 = \sqrt{K\frac{M_0}{r_0}} = \sqrt{K\frac{M_0}{a_0}}$$

$$v_{0,per} = v_0\sqrt{\frac{1+e_0}{1-e_0}} > v_0$$

$$v_{0,per} > v_0 \qquad \textbf{(1)}$$

$$v_{0,aph} = v_0\sqrt{\frac{1-e_0}{1+e_0}}$$

$$v_{0,aph} < v_0 \qquad \textbf{(2)}$$

Conclusion – According to the relations (1) and (2) the new orbit of the Earth could be an elliptic one.

For the new elliptical Earth orbit:

$$r_{\text{per}} = r_{0,\text{aph}};$$
$$r_{\min} = r_{\text{per}} = a(1-e);$$
$$a_0(1+e_0) = a(1-e);\ a = a_0\frac{1+e_0}{1-e};$$
$$\text{v}_{\text{per}} = \text{v}_{0,\text{aph}};$$
$$\text{v}_{\text{per}} = \text{v}\sqrt{\frac{1+\text{e}}{1-\text{e}}},$$

Where v este is the Earth's speed on a circular orbit with the radius $r = a$, when the mass of the Sun becomes $M = M_0/2$;

$$\text{v}\sqrt{\frac{1+\text{e}}{1-\text{e}}} = \text{v}_0\sqrt{\frac{1-e_0}{1+e_0}};$$
$$\text{v} = \sqrt{K\frac{M}{r}} = \sqrt{K\frac{M_0}{2a}} = \sqrt{K\frac{M_0}{a_0}}\sqrt{\frac{a_0}{2a}} = \text{v}_0\sqrt{\frac{a_0}{2a}};$$
$$e = 1-2e_0;\ a = a_0\frac{1+e_0}{2e_0}.$$

Conclusion

$$\text{T}_0 = \frac{2\pi \text{r}_0}{\text{v}_0} = \frac{2\pi \text{a}_0}{\text{v}_0}$$
$$T = \frac{2\pi r}{\text{v}} = \frac{2\pi a}{\text{v}};$$
$$\frac{T}{T_0} = \frac{a}{a_0}\frac{\text{v}_0}{\text{v}} = \frac{1+e_0}{2e_0}\sqrt{\frac{2a}{a_0}} = \frac{1+e_0}{2e_0}\sqrt{2}\sqrt{\frac{1+e_0}{2e_0}};$$
$$\text{T} = \text{T}_0\sqrt{2}\left(\frac{1+\text{e}_0}{2\text{e}_0}\right)^{3/2} \approx 230\,\text{years}$$

b) In 3rd of January the Earth is at perihelion. In that moment the Erath speed is larger than the speed necessary for an Earth's circular orbit. Thus the trajectory of the Earth after the catastrophe will be an open trajectory, i.e. an hyperbolic or parabolic orbit.

Conclusion it is not necessary to calculate the period of revolution or could be issued as infinite

Problem 3. Cosmic radiation

During studies concerning cosmic radiation, a neutral unstable particle – the π^0 meson was identified. The rest-mass of meson π^0 is much larger than the rest-mass of the electron. The studies reveal that during its flight, the meson π^0 disintegrates into 2 photons.

Find an expression for the initial velocity of the meson π^0, if after its disintegration, one of the photons has the maximum possible energy $E_{\max}$ and, consequently, the other photon has the minimum possible energy $E_{\min}$. You may use as known **c -** the speed of light.

Problem 3. Marking scheme Cosmic radiation

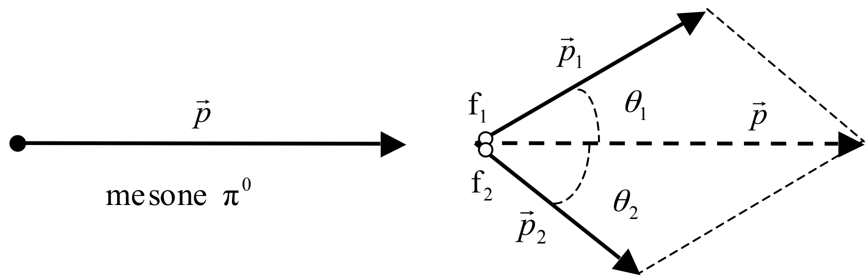
- Correct use of general laws of conservation **(A)** — **5 points**
- Correct applying of the laws of conservation for the conditions stated in the problem **(B)** — **4 points**
- Correct conduct of calculations and final solution (C) — **1 point**

Detailed solution

(A)

In the disintegration process the laws of energy conservation and the law of the conservation of momentum are both obeyed.

In the general case the law of conservation of the momentum is represented in the down below figure.



the total initial energy of the π^0 meson is

$$E^2 = p^2c^2 + m_0^2c^4$$

And its kinetic energy is

$$E_c = E - m_0c^2$$

The expressions of the 2 conservation laws written after the disintegration are:

$$\vec{p} = \vec{p}_1 + \vec{p}_2;$$

$$E + m_0c^2 = E_1 + E_2,$$

The energy of the photon 1 can be calculated using the notations in the figure

$$E = E_c + m_0c^2;$$

$$p_1 \sin\theta_1 = p_2 \sin\theta_2;$$

$$\frac{E_1}{c}\sin\theta_1 = \frac{E_2}{c}\sin\theta_2;$$

$$E^2 = p^2c^2 + m_0^2c^4;$$

$$p^2c^2 = E^2 - m_0^2c^4;$$

$$E = E_1 + E_2;$$

$$E_2 = E - E_1 = \left(E_c + m_0c^2\right) - E_1;$$

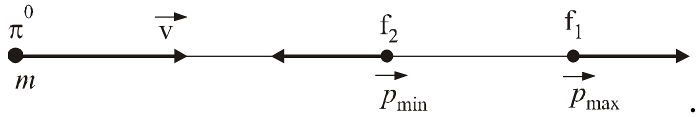
$$E_1 = \frac{m_0{}^2c^4}{2}\frac{1}{E_c + m_0c^2 - \cos\theta_1\sqrt{E_c\left(E_c + m_0c^2\right)}}.$$

Similar the second photon energy is:

$$E_2 = \frac{m_0{}^2c^4}{2}\frac{1}{E_c + m_0c^2 - \cos\theta_2\sqrt{E_c\left(E_c + m_0c^2\right)}}$$

(B)

If one of the photon has the maximum possible energy E_{max} and consequently the other photon has the minimum possible energy E_{min} the law of momentum conservation is sketched:



Thus the relations become very simple:

$$mv = \frac{m_0\mathbf{v}}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}$$

$$E_{max} = \frac{m_0c^2}{2\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}\left(1 + \frac{\mathbf{v}}{c}\right);$$

$$E_{min} = \frac{m_0c^2}{2\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}\left(1 - \frac{\mathbf{v}}{c}\right);$$

$$v = c\frac{E_{max} - E_{min}}{E_{max} + E_{min}}. \qquad \textbf{(C)}$$

Problem 4. Mass function of a visual binary stellar system

For a visual binary stellar system consisted of the stars σ_1 and σ_2, the following relation represents the mass function of the system:

$$f(M_1;M_2) = \frac{M_2^3\sin^3 i}{(M_1 + M_2)^2},$$

where M_1 is the mass of star σ_1, M_2 is the mass of star σ_2 and i is the angle between the plane of the stars' orbits and a plane perpendicular on the direction of observation.

The recorded spectrum of radiations emitted by the star σ_1, during several months, reveals a sinusoidal variation of radiation wavelength, with the period T = 7 days and a shift factor $z = (\Delta\lambda)/\lambda = 0{,}001$.

a. Prove that the mass function of the system is:

$$f(M_1;M_2) = \frac{M_2^3\sin^3 i}{(M_1 + M_2)^2} = \frac{T}{2\pi K}(v_1 \cdot \sin i)^3,$$

Where: $v_1 \cdot \sin i$ is the maximum speed of star σ_1 relatively to the observer; $K-$ the gravitational constant , i is the angle between the plane of the orbits and the plane normal to the observation direction.
Assumptions: The orbits of the stars are circular,

b. Derive an expression for the mass function of the system. The following values are known: $c = 3 \text{x} 10^8$ m/s; $K = 6{,}67 \text{x} 10^{-11}$ Nm^{2}kg^{-2}.

Problem 5. The Astronaut saved by … ice from a can!

An astronaut, with mass $M = 100$ kg, get out of the space ship for a repairing mission. He has to repair a satellite standing still relatively to shuttle, at about $d = 90$ m distance away from the shuttle. After he finished his job he realizes that the systems designated to assure his come-back to shuttle were broken. He also observed that he has air only for 3 minutes. He also noticed that he possessed a hermetically closed cylindrical can (base section $S = 30$ cm^2) firmly attached to its glove, with $m = 200$ g of ice inside. The ice did not completely fill the can.

Determine if the astronaut is able to arrive safely to the shuttle, before his air reserve is empty. Briefly explain your calculations. Note that he cannot throw away anything of its equipment, or touch the satellite.

You may use the following data: $T = 272$ K / the temperature of the ice in the can, $p_s = 550$ Pa - the pressure of the saturated water vapors at the temperature $T = 272$ K; $R = 8300$ J/(kmol·K) - the constant of perfect gas; $\mu = 18$ kg/kmol - the molar mass of the water.

Problem 5. Marking scheme The Astronaut saved by … ice from a can!

- **A.** For the use with an adequate justify of one of the relationships (4)	3 points
- **B. Reasoning** The student describe correctly the processes before and after the can is opened.	4points
- **C.** Calculations according to the reasoning, and/or as support for reasoning	2 points
- **D.** Correct result	1 point

Detailed solution

Theoretical considerations:

The incident particle flux on a wall (i.e. a certain direction on a surface) is:

$$\Phi = m_0 \cdot \Omega = \frac{1}{6} \cdot m_0 \cdot n \cdot S \cdot \bar{v} \qquad (3)$$

$$m_0 \cdot n = m_0 \cdot \frac{N}{a^3} = \frac{m_0 N}{a^3} = \frac{m}{V} = \rho,$$

where: m_0 is the mass of one molecule; $m-$ mass of the gas in the cube ; $V-$ volume of the cube ; $\rho-$ the density of the gas ;

$$\rho = \frac{\mu p}{RT},$$

where p – the pressure of the gas in the cube; The relation (3) – the mass flux relation becomes:

A $$\Phi = \frac{1}{6}\cdot\rho\cdot S\cdot\overline{\text{v}} = \frac{1}{6}\cdot\frac{\mu p}{RT}\cdot S\cdot\sqrt{\frac{3RT}{\mu}} = \frac{1}{6}\cdot p\cdot S\cdot\sqrt{\frac{3\mu}{RT}}. \qquad (4)$$

$$\Phi = \frac{1}{6}\cdot\rho\cdot S\cdot\overline{\text{v}} = \frac{1}{6}\cdot\frac{\mu p}{RT}\cdot S\cdot\sqrt{\frac{3RT}{\mu}} = \frac{1}{6}\cdot p\cdot S\cdot\sqrt{\frac{3\mu}{RT}}. \qquad (5)$$ 3points

B. **Reasoning**

Because the cylindrical can is not full of ice, in the empty part of it there are saturated vapors, i.e the mass flux of the molecule which sublimate is equal with mass flux of gass which transform into ice. Thus the pressure in the can is the saturated vapor pressure p_s and it has the corresponding maximum density ρ_s See figure 6.2

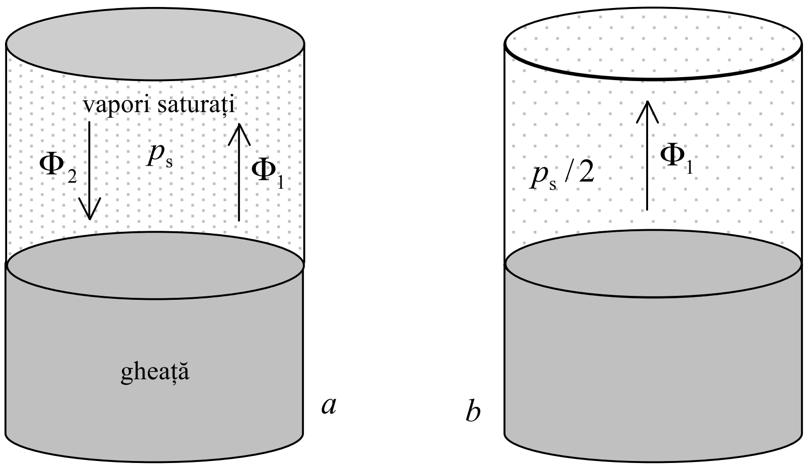


Fig. 6.2

$$\Phi_1 = \Phi_{sublimation} = \frac{1}{6}\cdot\rho_s\cdot S\cdot\overline{\text{v}} = \frac{1}{6}\cdot p_s\cdot S\cdot\sqrt{\frac{3\mu}{RT}};$$

$$\Phi_2 = \Phi_{solidification} = \frac{1}{6}\cdot\rho_s\cdot S\cdot\overline{\text{v}} = \frac{1}{6}\cdot p_s\cdot S\cdot\sqrt{\frac{3\mu}{RT}};$$

$$\Phi_1 = \Phi_2.$$

After the can was opened, there be no molecules which sublimate thus the mass flux of the molecules which gather the ice become null. So the pressure becomes $(p_s/2)$.

Thus the force acting on the astronaut will be

C. **Calculations according to the reasoning**, and/or as support for reasoning 2 points

$$F = \frac{p_s}{2}\cdot S,$$

Opening the can the astronaut will be accelearated with:

$$a = \frac{F}{M} = \frac{p_s\cdot S}{2M} = \frac{550\,\text{Nm}^{-2}\cdot 30\cdot 10^{-4}\ \text{m}^2}{2\cdot 10^2\ \text{kg}} = 0{,}00825\,\text{ms}^{-2}.$$

The total time of the acceleration movement will be the total time of ice sublimation:

$$\tau = \frac{m}{\Phi_1} = \frac{m}{\frac{1}{6}\cdot p_s \cdot S \cdot \sqrt{\frac{3\mu}{RT}}} = \frac{6m}{p_s \cdot S}\cdot\sqrt{\frac{RT}{3\mu}} \approx 150\,\mathrm{s}.$$

D. Correct result

The travel distance in this time will be :

$$L = \frac{a\tau^2}{2} = \frac{0{,}00825\,\mathrm{ms^{-2}}\cdot 225\cdot 10^2\,\mathrm{s}^2}{2} \approx 93\,\mathrm{m},$$

The astronaut could arrive safely in at the shuttle if he didn't lose to much time by solving the problem.

Problem 6. The life –time of a star from the main sequence

The plot of the function $\log(L/L_S) = f(\log(M/M_S))$ for data collected from a large number of stars is represented in figure 3. The symbols represents: L and M the luminosity respectively the mass and of a star and L_S and respectively M_S the luminosity and the respectively the mass of the Sun.

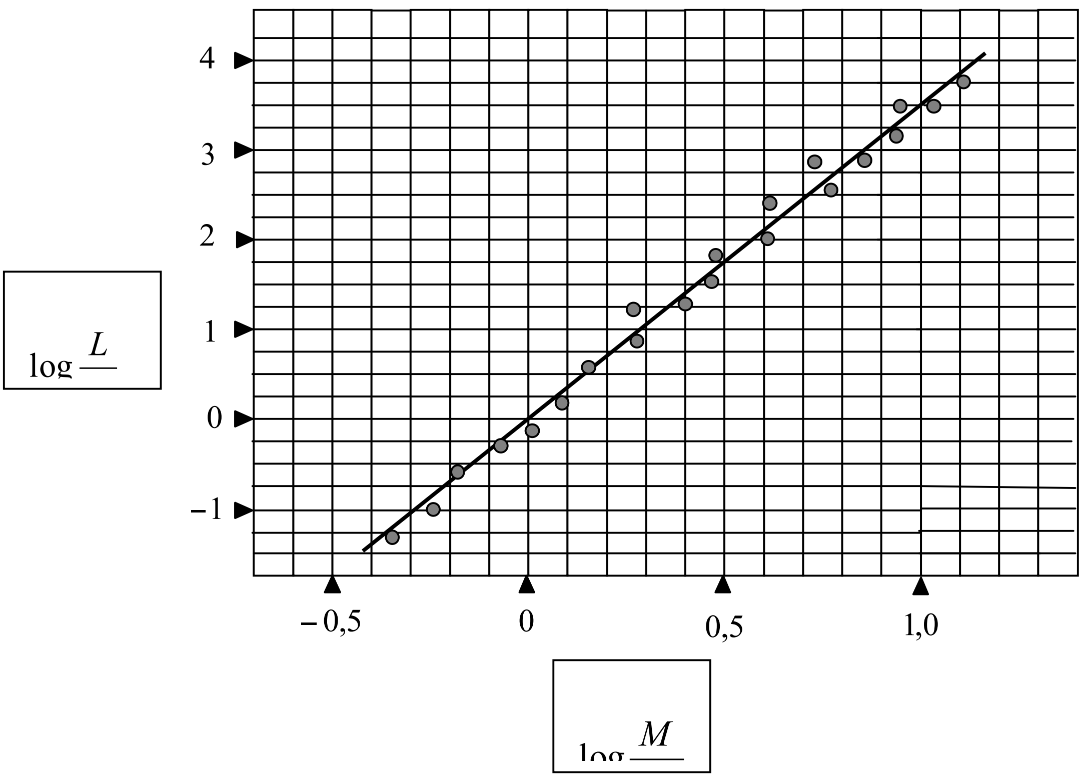


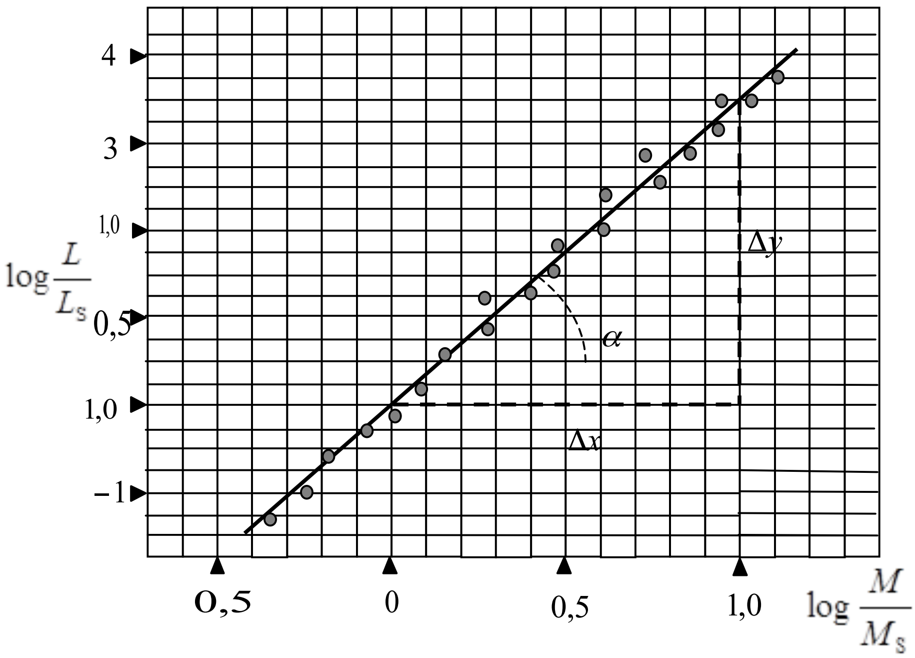
Figure 6

Find an expression for the life- time for each star in the Main Sequence from Hertzprung – Rossell diagram if the time spent by Sun in the same Main Sequence is τ_S. Consider the following assumptions: for each star the percentage of its mass which changed into energy is η, the percent of the mass of Sun which changes into energy is η_S, the mass of each star is $M = nM_S$ and the luminosity of each star remains constant, during its entire life time.

Problem 6. Marking scheme The life –time of a star from the main sequence

- **A.** The analysis of the graph — 6 points
  - Obtaining the formula (1) from the linearity of the graph
  - Correct use of the luminosity formula (2) for finding out the final formula — 4 points

Detailed solution



A. The analysis of the graph :

The graph is linear:

$y = ax + b = ax;$

From the graph it can be obtain the following data:

$$\log \frac{L}{L_S} = a \cdot \log \frac{M}{M_S};$$

$$a = \tan \alpha = \frac{\Delta y}{\Delta x} = \frac{3{,}5}{1} = 3{,}5;$$

$L \sim M^{3,5}$. **(1)**

The total energy of the star is:

$E = Mc^2$,

So the emitted energy due to the mass variation of the star is:

$\Delta E = c^2 \Delta M$,

According to the text

$\Delta M = \eta M;$

$\Delta E = c^2 \eta M.$

By using the definition of the luminosity :

$$\frac{\Delta E}{\Delta t} = L; \ \textbf{(2)}$$

$\Delta t = \tau;$

$$\frac{c^2 \eta M}{\tau} = L;$$

$$\tau = \frac{c^2 \eta M}{L}, \textbf{ (2)}$$

Which represents the life-time of the star.

By using the results from the graph analysis

$$L = \frac{L_S}{M_S^{3,5}} \cdot M^{3,5},$$

Thus :

$$\tau = \frac{c^2 \eta M_S^{3,5}}{L_S} \cdot M^{-2,5}.$$

If use the same calculations for the Sun it can be obtain

$$E_S = M_S c^2;$$

$$\tau_S = \frac{c^2 \eta_S M_S}{L_S},$$

Which is the life-time of the Sun

$$\tau = \frac{\eta}{\eta_S} \cdot \tau_S \cdot M_S^{2,5} \cdot M^{-2,5};$$

$$\frac{\tau}{\tau_S} = \frac{\eta}{\eta_S} \cdot \left(\frac{M}{M_S}\right)^{-2,5}; M = nM_S;$$

$$\tau = \frac{\eta}{\eta_S} (n)^{-2,5} \tau_S.$$

Problem 7. The effective temperature on the surface of a star

A star emits radiation with wavelength values in a narrow range $\Delta\lambda << \lambda$, i.e. the wavelength have values between λ and λ and $\lambda + \Delta\lambda$. According to Planck's relationship (for an absolute black body), the following relation define, the energy emitted by star in the unit of time, through the unit of area of its surface, per length-unit of the wavelength range:

$$r = \frac{2\pi hc^2}{\lambda^5 \left(e^{hc/k\lambda c} - 1\right)}.$$

The spectral intensities of two radiations with wavelengths λ_1 and respectively λ_2, both in the range $\Delta\lambda$ measured on Earth are $I_1(\lambda_1)$ and, respectively $I_2(\lambda_2)$.

a. Establish the equation which, in a general case, allows determining the effective temperature on the surface of the star using only spectral measurements.
b. Find out the approximate value of the effective temperature on the star surface if $hc >> \lambda kT$.
c. Find out the relation between wavelength λ_1 and λ_2, if $I_1(\lambda_1) = 2I_2(\lambda_2)$, when $hc << \lambda kT$.

You know: h – Planck's constant; k – Boltzmann's constant; c – speed of light in vacuum.

Problem 7. Marking scheme The effective temperature on the surface of a star

a. Identifying the expression of spectral intensity and correct use of the given relation for obtaining the relation **(1)** 4 points

b. correct use of the assumption $hc >> \lambda kT$ and find out relation (2) 3 points

c. correct use of the assumption $hc << \lambda kT$. and find out relation (3) 3 points

Detailed solution

a. We start from the definition of r:

$$r = \frac{\Delta E}{\Delta t \cdot S_{stea} \cdot \Delta\lambda} = \frac{\Delta E}{\Delta t \cdot 4\pi R^2 \cdot \Delta\lambda}$$ where R is the radius of the star

$$r = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/k\lambda T} - 1\right)}$$

Considering **d** as the distance from the star to the Earth, the definition- relation of the spectral intensity can be written as follows:

$$I(\lambda) = \frac{\frac{\Delta E}{\Delta t \cdot \Delta\lambda}}{4\pi d^2}$$

$$I(\lambda) = \frac{2\pi hc^2 R^2}{d^2\lambda^5\left(e^{hc/\lambda kT} - 1\right)};$$

Particularly for each wavelength:

$$I_1(\lambda_1) = \frac{2\pi hc^2 R^2}{d^2\lambda_1^5\left(e^{hc/\lambda_1 kT} - 1\right)};\ I_2(\lambda_2) = \frac{2\pi hc^2 R^2}{d^2\lambda_2^5\left(e^{hc/\lambda_2 kT} - 1\right)};$$

The ratio of the 2 above relations

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot \frac{e^{hc/\lambda_2 kT} - 1}{e^{hc/\lambda_1 kT} - 1} \qquad (1)$$

Represents an equation which allow to find out the temperature of star's surface **T by using spectral measurements**

b. If we consider that

$hc >> \lambda kT$, then:

$e^{hc/\lambda_1 kT} - 1 \approx e^{hc/\lambda_1 kT}$ and $e^{hc/\lambda_2 kT} - 1 \approx e^{hc/\lambda_2 kT}$

The relation (1) becomes

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot \frac{e^{hc/\lambda_2 kT}}{e^{hc/\lambda_1 kT}} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot e^{hc/\lambda_2 kT - hc/\lambda_1 kT}$$

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)}\cdot\left(\frac{\lambda_1}{\lambda_2}\right)^5 = e^{hc/\lambda_2 kT - hc/\lambda_1 kT}$$

$$T = \frac{hc(\lambda_1 - \lambda_2)}{k\lambda_1\lambda_2 \cdot \ln\left[\frac{I_1(\lambda_1)}{I_2(\lambda_2)}\cdot\left(\frac{\lambda_1}{\lambda_2}\right)^5\right]}$$

c. If $hc << \lambda kT$, then:

$$\frac{hc}{k\lambda_1 T} << 1;\ e^{hc/\lambda_1 kT} - 1 \approx 1 + \frac{hc}{k\lambda_1 T} - 1 = \frac{hc}{k\lambda_1 T}$$

$$\frac{hc}{k\lambda_2 T} << 1;\ e^{hc/\lambda_2 kT} - 1 \approx 1 + \frac{hc}{k\lambda_2 T} - 1 = \frac{hc}{k\lambda_2 T}$$

The relation (1) becomes:

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot \frac{\frac{hc}{k\lambda_2 T}}{\frac{hc}{k\lambda_2 T}} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$$

$$I_1 = 2I_2;\ \left(\frac{\lambda_2}{\lambda_1}\right)^4 = 2;\ \lambda_2 = \lambda_1 \cdot \sqrt[4]{2} \approx 1{,}2 \cdot \lambda_1.$$

Problem 8. Gradient temperatures

The spectra of two stars with different temperatures T_1 and respectively T_2 were compared. In the spectrum of each star, two very close spectral lines corresponding to the wavelength with values λ_1 and respectively λ_2 were found. For each line of this spectral lines, the difference between the corresponding visual apparent magnitudes of the stars are $\Delta m_{\lambda_1} = m_{1,\lambda_1} - m_{2,\lambda_1}$ and $\Delta m_{\lambda_2} = m_{1,\lambda_2} - m_{2,\lambda_2}$. m_{1,λ_1} is the apparent magnitude of the star 1 for the wavelength λ_1, m_{1,λ_2} is the apparent magnitude of the star 1 for the wavelength λ_2, m_{2,λ_1} is the apparent magnitude of the star 2 for the wavelength λ_1, m_{2,λ_2} is the apparent magnitude of the star 2 for the wavelength λ_2.

Determine the temperature T_1 of one of the two stars, if the temperature T_2 of the other star is already known, by using the Plank expression of black body radiation:

$$r(\lambda)=\frac{2\pi hc^2}{\lambda^5}\left(e^{hc/k\lambda c}-1\right)^{-1},$$

where: h – Planck's constant; k – Boltzmann's constant; c – speed of light in vacuum. You will consider that $hc >> k\lambda T$.

Problem 8. Marking scheme . Gradient temperatures

- Finding out the relations (1) and (2) by the correct using of the approximation $hc >> k\lambda T$.	4 points
- Correct using of the Pogson's formula	3 points
- For correct conduct of calculations and obtaining the final formula (5)	3points

Detailed solution

By using the Plank expression the spectral intensities for the two wavelengths in the doublet emitted by the star number 1 are:

$$r_1(\lambda_1)=\frac{2\pi hc^2}{\lambda_1^5\left(e^{hc/\lambda_1 kT_1}-1\right)};\ r_1(\lambda_2)=\frac{2\pi hc^2}{\lambda_2^5\left(e^{hc/\lambda_2 kT_1}-1\right)};$$

And by considering $hc >> k\lambda T$;

$$r_1(\lambda_1)=\frac{2\pi hc^2}{\lambda_1^5}e^{-hc/\lambda_1 kT_1};\ r_1(\lambda_2)=\frac{2\pi hc^2}{\lambda_2^5}e^{-hc/\lambda_2 kT_1}. \quad (1)$$

Respectively, the spectral intensities for the two wavelengths in the doublet emitted by the star number 2 are:

$$r_2(\lambda_1)=\frac{2\pi hc^2}{\lambda_1^5\left(e^{hc/\lambda_1 kT_2}-1\right)};\ r_2(\lambda_2)=\frac{2\pi hc^2}{\lambda_2^5\left(e^{hc/\lambda_2 kT_2}-1\right)};$$

And by considering $hc >> k\lambda T$;

$$r_2(\lambda_1)=\frac{2\pi hc^2}{\lambda_1^5}e^{-hc/\lambda_1 kT_2};\ r_2(\lambda_2)=\frac{2\pi hc^2}{\lambda_2^5}e^{-hc/\lambda_2 kT2}. \quad (2)$$

Using the Pogson formula for star 1 and 2 and for λ_1, result:

$$\log\frac{r_1(\lambda_1)}{r_2(\lambda_1)}=-0{,}4\left(m_{1,\lambda_1}-m_{2,\lambda_1}\right)=-0{,}4\cdot\Delta m_{\lambda_1};$$

$$\log e^{hc/\lambda_1 k(1/T_2-1/T_1)}=-0{,}4\cdot\Delta m_{\lambda_1} \quad (3)$$

Similar the Pogson formula for λ_2,:

$$\log\frac{r_1(\lambda_2)}{r_2(\lambda_2)}=-0{,}4\left(m_{1,\lambda_2}-m_{2,\lambda_2}\right)=-0{,}4\cdot\Delta m_{\lambda_2}$$

$$\log e^{hc/\lambda_2 k(1/T_2-1/T_1)}=-0{,}4\cdot\Delta m_{\lambda_2} \quad (4)$$

Using the relations (3) and (4)

$$\log e^{hc/\lambda_2 k(1/T_2-1/T_1)}-\log e^{hc/\lambda_1 k(1/T_2-1/T_1)}=-0{,}4\cdot\Delta m_{\lambda_2}+0{,}4\cdot\Delta m_{\lambda_1};$$

$$\log e^{hc/\lambda_2 k(1/T_2-1/T_1)} + \log e^{-hc/\lambda_1 k(1/T_2-1/T_1)} = -0{,}4\cdot\Delta m_{\lambda_2} + 0{,}4\cdot\Delta m_{\lambda_1};$$

$$\left[\frac{hc}{k}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\cdot\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)\right]\cdot\log e = 0{,}4\left(\Delta m_{\lambda_1}-\Delta m_{\lambda_2}\right);$$

Because $\log e \approx 0{,}43;\ \dfrac{0{,}4}{\log e}\approx 0{,}93;$

$$\frac{1}{T_2}-\frac{1}{T_1} = -0{,}93\cdot\frac{\left(\Delta m_{\lambda_1}-\Delta m_{\lambda_2}\right)}{\dfrac{hc}{k}\left(\dfrac{1}{\lambda_2}-\dfrac{1}{\lambda_1}\right)} = -0{,}93\cdot\frac{k\lambda_1\lambda_2\left(\Delta m_{\lambda_1}-\Delta m_{\lambda_2}\right)}{hc\left(\lambda_1-\lambda_2\right)};$$

$$T_1 = T_2\cdot\frac{hc\left(\lambda_1-\lambda_2\right)}{hc\left(\lambda_1-\lambda_2\right)+0{,}93\cdot k\lambda_1\lambda_2T_2\left(\Delta m_{\lambda_1}-\Delta m_{\lambda_2}\right)}. \quad \textbf{(5)}$$

Problem 9. Pressure of light

One particle of star dust is in static equilibrium at a certain distance from Sun. Assuming that the particle is spherical and its density is ρ, calculate the diameter of the particle.

The following assumption may be useful for solving the problem:

The pressure of electromagnetic radiation is equal with the volume density of the electromagnetic radiations

Problem 9. Marking scheme . Pressure of light

- Correct use of the formula (1) for the pressure of light — 3 points
- Correct identify of the equilibrium condition — 3 points
- Correct solution and reasoning — 4 points

The pressure of the emitted radiation is

$$p_{\text{rad}} = w = \frac{\phi_{planet,D}}{c}$$

$$p_{rad} = \frac{\sigma T^4_{planet} R^2_{planet}}{cD^2} \quad (1)$$

As seen in the below image, the pressure due to the solar radiation is effectively acting on an equivalent plane disc with the diameter *d* of the spherical star dust particle

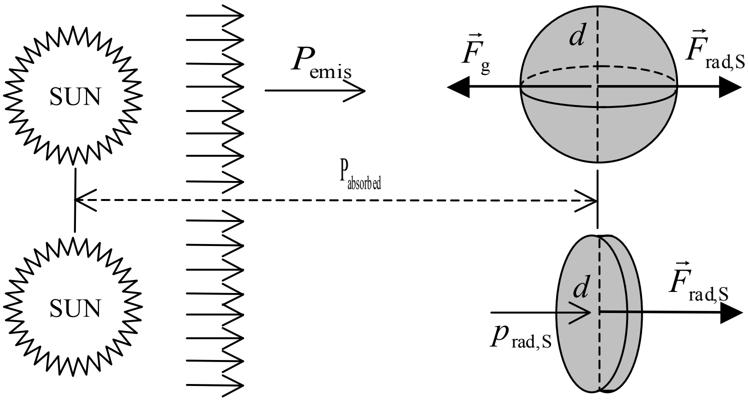


Fig.12 b

Thus the force acting by the Sun radiation on the star-dust particle is:

$$F_{\text{rad,S}} = p_{\text{rad,S}} \cdot \pi r^2 = p_{\text{rad,S}} \cdot \frac{\pi d^2}{4} = \frac{\sigma T_S^4 R_S^2}{c L_S^2} \cdot \frac{\pi d^2}{4},$$

The equilibrium condition is

$$F_{\text{rad,S}} = F_g,$$

Where F_g is the gravitational attraction force between Sun and the star-dust particle.

$$\frac{\sigma T_S^4 R_S^2}{c D_S^2} \cdot \frac{\pi d^2}{4} = K \frac{m M_S}{D_S^2};$$

$$m = \rho V = \rho \frac{4\pi r^3}{3} = \rho \frac{4\pi}{3} \frac{d^3}{8} = \rho \frac{\pi d^3}{6};$$

$$\frac{\sigma T_S^4 R_S^2}{c D_S^2} \cdot \frac{\pi d^2}{4} = K \rho \frac{\pi d^3}{6} \frac{M_S}{D_S^2};$$

$$d = \frac{3}{2} \cdot \frac{\sigma}{\rho K} \cdot \frac{T_S^4 R_S^2}{M_S}.$$

Problem 10. The density of the star

In a very simple model, a star is assumed to be a sphere of gas in a state of equilibrium in its on gravitational field. The stellar gas is consisted of plasma, i.e. hydrogen and helium atoms, completely ionized. Find an expression for the value of the mass of the star if you know: r – radius of the star; T – the temperature of the star; n – the relative proportion of hydrogen in the mass of the star; μ_H – molar mass of the hydrogen; μ_{He} – molar mass of the helium; R – universal gas constant; K – gravitation constant. You may use the formula of the pressure of radiation inside the star $p_{\text{rad}} = \frac{1}{3} a T^4$, where a is a known constant. The rotation of the star is negligible.

Problem 10. Marking scheme The density of the star

- Correct reasoning for finding the expression of the inner stellar gas (2) — 2 points
- Correct expression for the equilibrium condition — 2 puncte
- The correct conduct of calculations, and obtain the final correct result — 2 puncte
- Correct solution and reasoning — 4 points

Detailed solution

The hydrostatic equilibrium inside the star means that in each point of the inside of the star the gravitational forces are compensated by the hydrostatic pressure forces. That means that the mater of the star remains localized in a region of space.

The total pressure of the stellar gas has two components: the pressure due to the movement of the stellar –gas particles (p_{gaz}) and the pressure due to the emitted radiation by the stellar-gas particles (p_{rad}), thus:

$$p_{\text{total}} = p_{\text{gaz}} + p_{\text{rad}};$$

$$p_{\text{gaz}} = p_{\text{H}} + p_{\text{He}};$$

$$p_{\text{rad}} = \frac{1}{3}aT;$$

$$p_{\text{total}} = \rho\frac{n\mu_{\text{He}} + (1-n)\mu_{\text{H}}}{\mu_{\text{H}}\mu_{\text{He}}}RT + \frac{1}{3}aT.\mathbf{(1)}$$

In order to calculate the gravitational pressure of the stellar /gas let's consider a narrow cylinder with section area ΔS ,along the radius of the star . See the figure bellow. If the total gravitational force acting on this cylinder is $\vec{F}_g$ than the gravitational pressure exert by the gas –column is $p_{\text{grav}} = F_g / \Delta S$

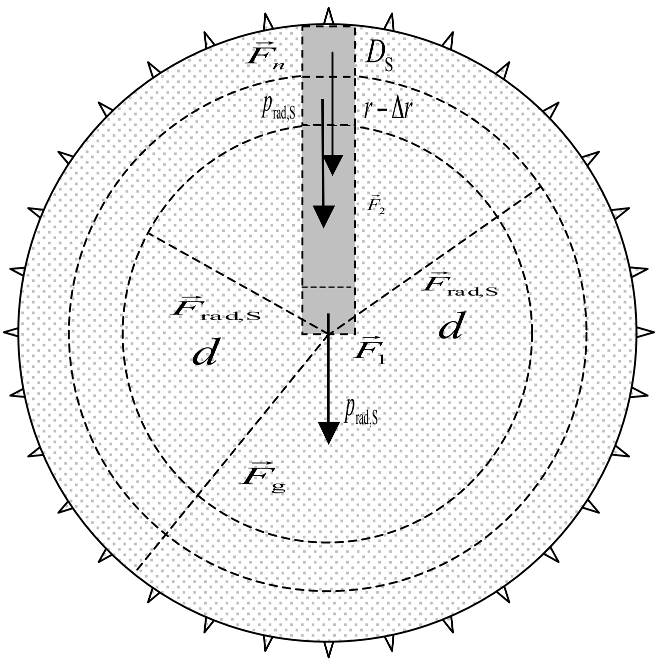


Fig. 15 a

In order to calculate $\vec{F}_{\mathrm{g}}$ let divide the cylinder in n identically small cylinders each of it with height Δr and mass Δm Considering an homogenous star :

$$F_1 = K\frac{\Delta m\cdot M_1}{\left(r-\frac{\Delta r}{2}\right)^2} = K\frac{\Delta m\cdot M_1}{r^2\left(1-\frac{\Delta r}{2r}\right)^2} = K\frac{\Delta m\cdot M_1}{r^2}\left(1-\frac{\Delta r}{2r}\right)^{-2};$$

$$\frac{M}{r^3} = \frac{M_1}{(r-\Delta r)^3};\ M_1 = M\cdot\frac{(r-\Delta r)^3}{r^3} = M\cdot\left(1-\frac{\Delta r}{r}\right)^3;$$

$$F_1 \approx K\frac{\Delta m\cdot M}{r^2}\left(1-2\frac{\Delta r}{r}\right);$$

$$F_2 \approx K\frac{\Delta m\cdot M}{r^2}\left(1-3\frac{\Delta r}{r}\right);$$

……………………………………………………..

$$F_n \approx K\frac{\Delta m\cdot M}{r^2}\left(1-(n+1)\frac{\Delta r}{r}\right);$$

$$F_{\mathrm{g}} = F_1 + F_2 + \dots\dots + F_n;$$

$$F_{\mathrm{g}} = K\frac{m\cdot M}{nr^2}\left[n-\frac{1}{n}\frac{(1+n)n}{2}\right];\ F_{\mathrm{g}} = K\frac{m\cdot M}{r^2}\left[1-\frac{(1+n)}{2n}\right];$$

$$F_{\mathrm{g}} = K\frac{m\cdot M}{r^2}\frac{n-1}{2n};\ \frac{n-1}{2n}\approx\frac{1}{2};$$

$$F_{\mathrm{g}} = K\frac{m\cdot M}{2r^2}.(2)$$

Thus the gravitational pressure is

$$p_{\mathrm{g}} = \frac{F_{\mathrm{g}}}{\Delta S} = \frac{1}{\Delta S}\frac{\rho\cdot r\cdot\Delta S\cdot\rho\cdot\frac{4\pi r^3}{3}}{2r^2}$$

$$p_{\mathrm{g}} = \frac{2}{3}\pi K r^2\rho^2$$

Using the relations (1) and (2) in the pressures equilibrium relationship

$$p_{\text{total}} = p_{\text{gravitational}}$$

Results :

$$\frac{2}{3}\pi K r^2\rho^2 = \rho\frac{n\mu_{\mathrm{He}} + (1-n)\mu_{\mathrm{H}}}{\mu_{\mathrm{H}}\mu_{\mathrm{He}}}RT + \frac{1}{3}aT;$$

$$\frac{2}{3}\pi K r^2\cdot\rho^2 - \frac{n\mu_{\mathrm{He}} + (1-n)\mu_{\mathrm{H}}}{\mu_{\mathrm{H}}\mu_{\mathrm{He}}}RT\cdot\rho - \frac{1}{3}aT = 0;$$

This is an second degree equation in ρ

$$A\rho^2 - B\rho - C = 0$$

Where the coefficients are

$$A = \frac{2}{3}\pi K r^2$$

$$B = \frac{n\mu_{\mathrm{He}} + (1-n)\mu_{\mathrm{H}}}{\mu_{\mathrm{H}}\mu_{\mathrm{He}}} RT$$

$$C = \frac{1}{3} aT$$

The positive solution is the valid one i.e.

$$\rho = \frac{B + \sqrt{B^2 + 4AC}}{2A} \quad \rho = \frac{\frac{n\mu_{\mathrm{He}} + (1-n)\mu_{\mathrm{H}}}{\mu_{\mathrm{H}}\mu_{\mathrm{He}}} RT + \sqrt{\left(\frac{n\mu_{\mathrm{He}} + (1-n)\mu_{\mathrm{H}}}{\mu_{\mathrm{H}}\mu_{\mathrm{He}}} RT\right)^2 + \frac{8}{9}\pi K r^2 aT}}{\frac{4}{3}\pi K r^2}$$

Problem 11. Space – ship orbiting the Sun

A spherical space –ship orbits the Sun on a circular orbit, and spin around one of its axes. The temperature on the exterior surface of the ship is T_N. Find out the apparent magnitude of the Sun and the angular diameter of the Sun as seen by the astronaut on board of the space – ship. The following values are known:, T_S - the effective temperature of the Sun; R_S - the radius of the Sun; d_0 - the Earth –Sun distance; m_0 - apparent magnitude of Sun measured from Earth; R_N - the radius of the space –ship.

Problem 11. Marking scheme Space – ship orbiting the Sun

- Correct use of the formulas (1) for the apparent brightness	3 points
- Correct use of the formula (2) by using Pogson formula	3 points
- Correct solution and reasoning	4 points

Detailed solution

According to the Stefan – Boltzmann law, the luminosity of the Sun is:

$$L_{sun} = Q_{sun} \cdot 4\pi R_{\mathrm{Sun}}^2 = \sigma T_{\mathrm{Sun}}^4 \cdot 4\pi R_{\mathrm{Sun}}^2, \quad (1)$$

At distance d from the Sun , where the space ship is the energy which passes the unit of surface in an unit of time is:

$$\phi_{\mathrm{Sun,d}} = \frac{L_{\mathrm{S}}}{4\pi d^2} = \frac{\sigma T_{\mathrm{S}}^4 \cdot 4\pi R_{\mathrm{S}}^2}{4\pi d^2}. \quad (2)$$

The space ship receive through its entire surface, in the unit of time, the energy:

$$P_{received} = \frac{\sigma T_{\mathrm{Sun}}^4 \cdot 4\pi R_{\mathrm{Sun}}^2}{4\pi d^2} \cdot \pi R_{ship}^2.$$

Corresponding to its temperature, T_{N}, according the Stefan - Boltzmann law, the emitted energy by starship through its hole surface in the unit of time :

$$P_{\mathrm{emis,N}} = \sigma T_{\mathrm{N}}^4 \cdot 4\pi R_{\mathrm{N}}^2.$$

When the temperature stabilized at thermic equilibrium :

$$P_{received,\mathrm{N}} = P_{\mathrm{emis,N}}$$

$$\frac{\sigma T_{\mathrm{S}}^4 \cdot 4\pi R_{\mathrm{S}}^2}{4\pi d^2} \cdot \pi R_{\mathrm{N}}^2 = \sigma T_{\mathrm{N}}^4 \cdot 4\pi R_{\mathrm{N}}^2$$

the distance of orbiting the Sun of the space ship is:

$$d = \frac{T_{\mathrm{S}}^2 R_{\mathrm{S}}}{2T_{\mathrm{N}}^2},$$

The angular diameter of the Sun as seen from the space ship :

$$\alpha / 2 = \frac{R_{\mathrm{S}}}{d}$$

$$\alpha = \frac{2R_{\mathrm{S}}}{d} = 4\left(\frac{T_{\mathrm{N}}}{T_{\mathrm{S}}}\right)^2$$

According the Pogson formula written for Sun seen from Earth and space ship the following relation occurs:

$$\lg \frac{E_{\mathrm{S,Nava}}}{E_{\mathrm{S,P}}} = -0{,}4(m - m_0) \qquad (2)$$

$$2 \cdot \lg\left(\frac{d_0}{d}\right) = -0{,}4(m - m_0)$$

The apparent magnitude of the Sun as seen from the space ship

$$m = m_0 - 5^{\mathrm{m}} \cdot \lg \frac{2d_0 T_{\mathrm{N}}^2}{R_{\mathrm{S}} T_{\mathrm{S}}^2}$$

Problem 12. The Vega star in the mirror

Inside a photo camera a plane mirror is placed along the optical axis of the lens of the objective (as seen in figure 13). The length of the mirror is half of the focal distance of the lens of the objective. The photo camera is oriented as on the photographic plate situated in the focal plane of the photo camera are captured two images with different illuminations of the Vega star. Find out the difference between the apparent photographical magnitudes of the two images of the Vega stars.

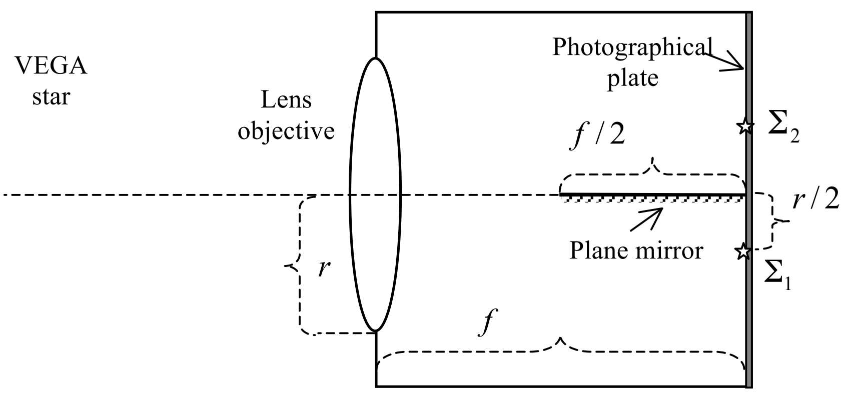


Figure 12

Problem 12. Marking scheme The Vega star in the mirror

The light beam arriving from **Vega Star** can be considered paraxial, due to the distance from it to the observer on Earth. The explanation for the existence of two distinct images of the star is that the optical axis of the objective is not parallel with the light beam from the star.

The images on the camera plate are symmetrical placed relative to the principal optical axis.

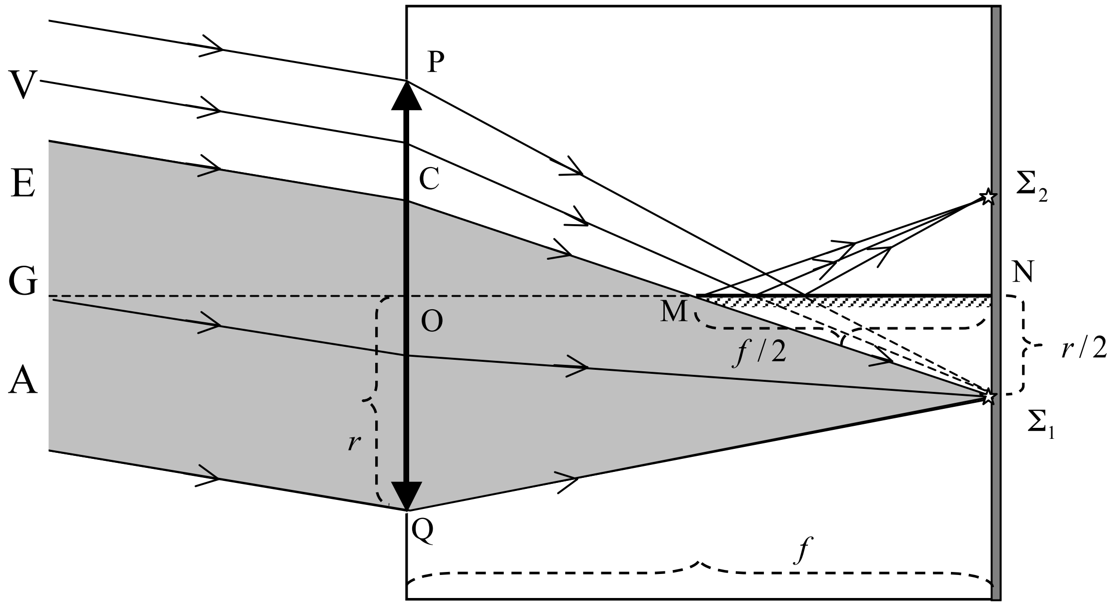


Fig. 12

Each of the point images of the Vega Star Σ_1 and Σ_2 didn't concentrate the same light fluxes. In the down below figure it can be seen the sections of the lens which correspond to each image. The sector APBC is passed by the light which concentrates in the image Σ_2 and the light passing the sector ACBQ concentrates into the point image Σ_1 See the picture in figure 13 .

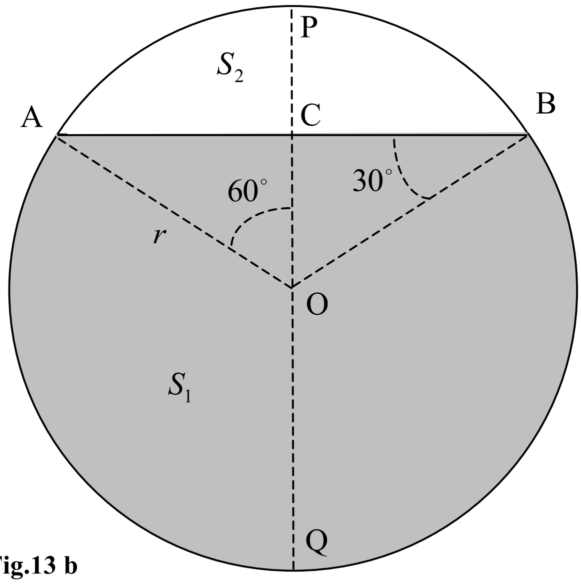


Fig.13 b

The ratio between the light fluxes concentrated into the two image points will directly depend on the ratio of the two sectors areas.

From the geometry of the figure 2 results :

$$\mathrm{MN} = \mathrm{OM};\ \mathrm{N}\Sigma_1 = \mathrm{OC} = \frac{r}{2};$$

$$\angle(\mathrm{CBO}) = 30^\circ;\ \angle(\mathrm{BOC}) = 60^\circ;\ \angle(\mathrm{AOB}) = 120^\circ;$$

$$\frac{S_1}{S_2} = \frac{8\pi + 3\sqrt{3}}{4\pi - 3\sqrt{3}} \approx 4.$$

Using the Pogson formula :

$$\log\frac{E_1}{E_2} = \log\frac{\dfrac{\sigma T_\mathrm{V}^4 \cdot 4\pi R_\mathrm{V}^2}{4\pi d_\mathrm{PV}^2}\cdot S_1}{\dfrac{\sigma T_\mathrm{V}^4 \cdot 4\pi R_\mathrm{V}^2}{4\pi d_\mathrm{PV}^2}\cdot S_2} = -0{,}4(m_1 - m_2);$$

$$\log\frac{S_1}{S_2} = -0{,}4(m_1 - m_2);$$

$$m_2 - m_1 = 1{,}5^\mathrm{m}.$$

Problem 13. Stars with Romanian names

Two Romanian astronomers Ovidiu Tercu and Alex Dumitriu from The Astronomical Observatory of the Museum Complex of Natural Sciences in Galati Romania, recently discovered – in September 2013- two variable stars. They used for that a telescope with the main mirror diameter of 40 cm and a SBIG STL-6303e CCD camera.

With the accord of the **AAVSO** (**A**merican **A**ssociation of **V**ariable **S**tars **O**bservers), the two stars have now Romanian names: Galati V1 and respectively Galati V2. The two stars are circumpolar, located in Cassiopeia and respectively in Andromeda constellation. The two stars are visible above the horizon, form the territory of Romania, all over the year. The galactic coordinates of the two stars are: $\text{Galati V1}\left(G_1 = 114.371^\circ; g_1 = -11.35^\circ\right)$ and $\text{Galati V2}\left(G_2 = 113.266^\circ; g_2 = -16.177^\circ\right)$.

Another star, discovered by the Romanian astronomer Nicolas Sanduleak, has also a Romanian name – Sanduleak -69° 202; it explodes as the supernova SN 1987. This star was localized in the Dorado constellation from the Large Magellan Cloud, by the coordinates:

$$\alpha = 5^{\mathrm{h}}35^{\mathrm{min}}28{,}03^{\mathrm{s}};\ \delta = -69^\circ 16' 11{,}79'';\ G = 279{,}7^\circ;\ g = -31{,}9^\circ.$$

Estimate the angular distance between the stars Galati V1 and Galati V2

Problem 13. Marking scheme Stars with Romanian names

1. Correct geometrical calculations 5 points
2. Correct calculations 5 points

In the figure bellow the two stars σ_1 and σ_2, are located using the galactic coordintates $\left(G_1; g_1\right)$ and respectively $\left(G_2; g_2\right)$ on the geocentric celestial sphere. The spherical triangles $\sigma_1 \mathrm{A} \sigma_2$ $\left(G_1; g_1\right)$ and respectively may be considered rectangular plane triangles because the angles $\Delta G = G_2 - G_1$ $\left(G_1; g_1\right)$ and respectively $\Delta g = g_2 - g_1$ are very small

Thus:

$$\sigma_1\sigma_2 = \sqrt{\left(\sigma_1 \mathrm{A}\right)^2 + \left(\sigma_2 \mathrm{A}\right)^2},$$

or:

$$\sigma_1\sigma_2 = \sqrt{\left(\sigma_1 \mathrm{B}\right)^2 + \left(\sigma_2 \mathrm{B}\right)^2},$$

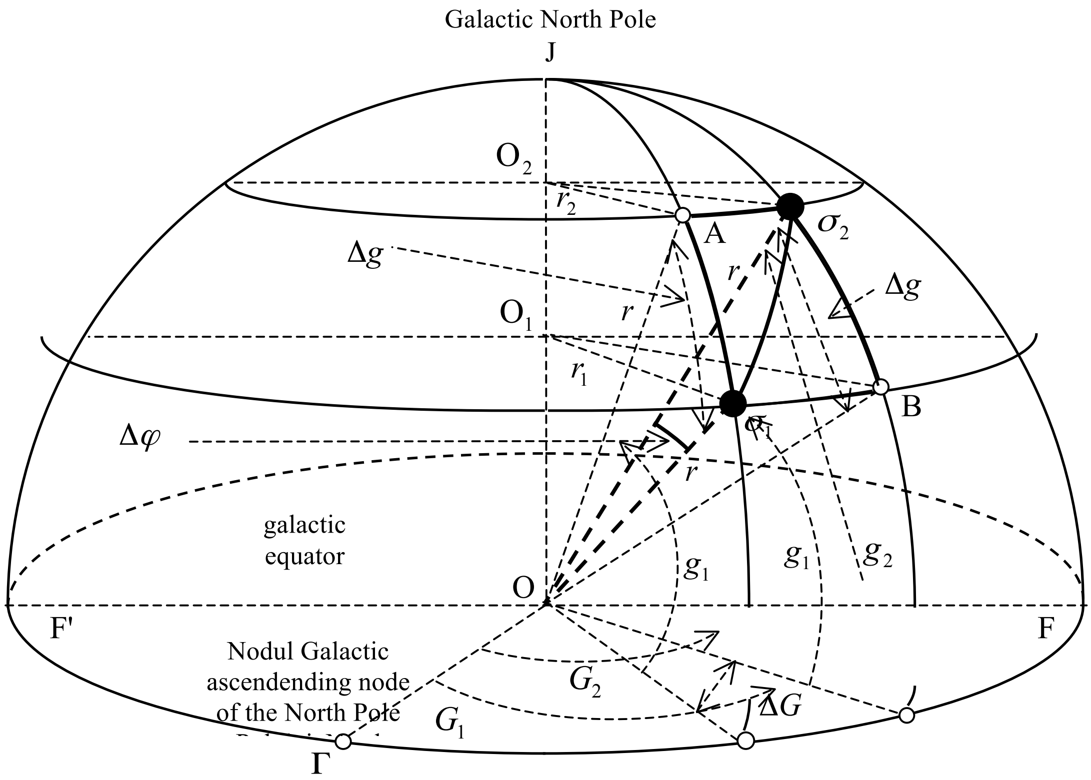


Fig.

$$\sigma_1\,\mathrm{A} = r \cdot \Delta g;$$
$$\sigma_2\,\mathrm{A} = r_2 \cdot \Delta G = r \cdot \cos g_2 \cdot \Delta G;$$
$$\sigma_1\sigma_2 = r \cdot \Delta\varphi,$$

Where $\Delta\varphi$ is the angular distance between two stars

$$r \cdot \Delta\varphi = \sqrt{(r \cdot \Delta g)^2 + (r \cdot \cos g_2 \cdot \Delta G)^2};$$
$$\Delta\varphi = \sqrt{(\Delta g)^2 + (\cos g_2 \cdot \Delta G)^2};$$
$$\sigma_1\,\mathrm{B} = r_1 \cdot \Delta G = r \cdot \cos g_1 \cdot \Delta G;$$
$$\Delta\varphi = \sqrt{(\cos g_1 \cdot \Delta G)^2 + (\Delta g)^2};$$
$$\left(G_1 = 114.371^\circ;\, g_1 = -11.35^\circ\right);\ \left(G_2 = 113.266^\circ;\, g_2 = -16.177^\circ\right);$$
$$\Delta G = G_2 - G_1 = -1{,}105^\circ;\ \Delta g = g_2 - g_1 = -4{,}827^\circ;$$
$$\cos g_1 = 0{,}98;\ \cos g_2 = 0{,}96;$$
$$\Delta\varphi = \sqrt{\left(-4{,}827^\circ\right)^2 + (0{,}96)^2 \cdot \left(-1{,}105^\circ\right)^2} \approx 4{,}942^\circ;$$
$$\Delta\varphi = \sqrt{(0{,}98)^2 \cdot \left(-1{,}105^\circ\right)^2 + \left(-4{,}827^\circ\right)^2} \approx 4{,}946^\circ,$$

The angular distance between Galați V1 and Galați V2.

.

Problem 14. Apparent magnitude of the Moon

You know that the absolute magnitude of the Moon is $M_L = 0{,}25^{\mathrm{m}}$. Calculate the values of the apparent magnitudes of the Moon corresponding to the following Moon –phases : full-moon and the first quarter. You know: the Moon – Earth distance - $d_{\mathrm{LP}} = 385000\,\mathrm{km}$, the Earth – Sun distance - $d_{\mathrm{PS}} = 1\,\mathrm{AU}$, the Moon –Sun distance, $d_{\mathrm{LS}} = 1\,\mathrm{AU}$

Problem 14. Marking scheme Apparent magnitude of the Moon

1. General analysis of the problem — 6 points
2. The analysis of the 2 particular situations — 4 points

The apparent magnitude of a planet from the Solar System depends on the phase angle $M = M(\Psi)$.
The apparent magnitude of the body is given by the relation:

$$m = M + 2{,}5 \cdot \log \frac{d_{\mathrm{C,S}}^2 \cdot d_{\mathrm{C,O}}^2}{d_0^4 \cdot p(\Psi)},$$

unde: $d_{B,\mathrm{S}}$ – the distance between the body and the Sun; $d_{B,\mathrm{O}}$ – distance between the body and observer; $d_0 = 1\,\mathrm{AU}$; Ψ – the phase angle ; $p(\Psi)$ – the phase function :

$$p(\Psi) = \frac{2}{3} \cdot \left[\left(1 - \frac{\Psi}{\pi}\right) \cos\Psi + \frac{1}{\pi} \sin\Psi \right],$$

Ψ as seen in the figure bellow is given by the cosine law.

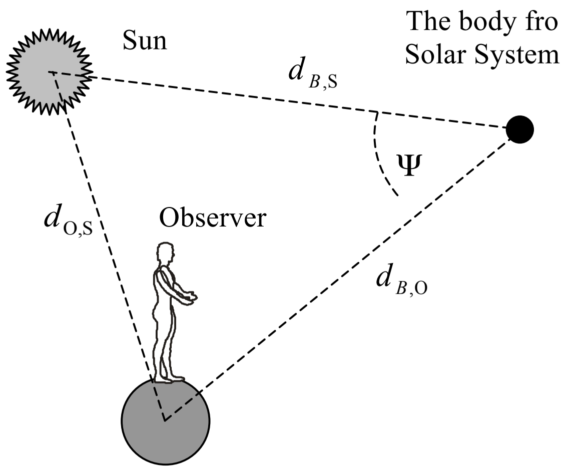
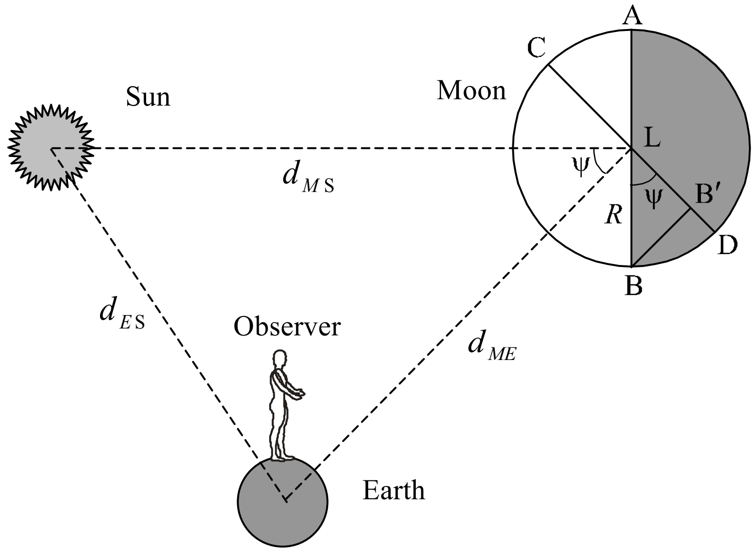


Fig.

$$\cos\Psi = \frac{d_{\mathrm{BO}}^2 + d_{BS}^2 - d_{\mathrm{OS}}^2}{2 d_{BO} \cdot d_{BS}}.$$

In particularaly for the Moon



$$\cos\Psi = \frac{d_{ME}^2 + d_{MS}^2 - d_{ES}^2}{2d_{ME}\cdot d_{MS}};$$

$$p(\Psi) = \frac{2}{3}\cdot\left[\left(1-\frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right];$$

$$m_M = M_M + 2{,}5\cdot\log\frac{d_{M,S}^2\cdot d_{ME}^2}{d_0^4\cdot p(\Psi)}.$$

Particular cases:

1) Full moon

$$\Psi = 0;$$
$$\cos\Psi = 1;\ \sin\Psi = 0;$$
$$p(\Psi) = \frac{2}{3};$$

$$d_{MS} = 1\,\text{AU};\ d_{ME} = 385000\,\text{km} \approx 0{,}00256\,\text{AU} = 256\cdot 10^{-5}\,\text{SU};\ d_0 = 1\,\text{S}U;$$

$$m_M = M_M - 12{,}5^{\text{m}} = 0{,}25^{\text{m}} - 12{,}5^{\text{m}} = -12{,}25^{\text{m}}.$$

2) First Quarter

$$\Psi = 90^\circ;$$
$$\cos\Psi = 0;\ \sin\Psi = 1;$$
$$p(\Psi) = \frac{2}{3\pi} \approx 0{,}2;$$

$$\frac{d_{M,S}^2 \cdot d_{ME}^2}{d_0^4 \cdot p(\Psi)} = \frac{65536 \cdot 10^{-10}}{0,2} = 491520 \cdot 10^{-10};$$

$$m_M = M_M - 10{,}75^{\mathrm{m}} = 0{,}25^{\mathrm{m}} - 10{,}75^{\mathrm{m}} = -10{,}5^{\mathrm{m}}.$$

Problem 15. Absolute magnitude of a cepheide

The cepheides are cariable stars, whom luminosities and luminosities varies due to volume oscillations. The period of the oscilations of a cepheide star is:

$$P = 2\pi R\sqrt{\frac{R}{KM}},$$

where: R – the radius of the cepheide; M – the mass of the cepheid (constant during oscillation);

$$R = R(t);\ P = P(t).$$

Demonstrate that the absolute magnitude of the cepheide M_{cef}, depend on the period of cepheide's pulsation P according the following relation:

$$M_{\mathrm{cef}} = -2{,}5^{\mathrm{m}} \cdot \log k - \left(\frac{10}{3}\right)^{\mathrm{m}} \cdot \log P,$$

where k *is* constant; $P = P(t);\ M_{\mathrm{cef}} = M_{\mathrm{cef}}(t).$

Problem 15. Marking scheme Apparent magnitude of the Moon

$$P = 2\pi R\sqrt{\frac{R}{KM}},$$

rezults

$$P^2 = \frac{4\pi^2 R^3}{KM};\ R = \sqrt[3]{\frac{KMP^2}{4\pi^2}} = \left(\frac{KM}{4\pi^2}\right)^{1/3} \cdot P^{2/3};$$

$$R^2 = \left(\frac{KM}{4\pi^2}\right)^{2/3} \cdot P^{4/3}.$$

The absolute brightness is:

$$L_{\mathrm{cef}} = \sigma T_{\mathrm{cef}}^4 \cdot 4\pi R^2,$$

And the apparent brightness :

$$E_{\mathrm{cef}} = \frac{L_{\mathrm{cef}}}{4\pi d_{\mathrm{P,cef}}^2} = \frac{\sigma T_{\mathrm{cef}}^4 \cdot 4\pi R^2}{4\pi d_{\mathrm{P,cef}}^2},$$

$d_{\mathrm{P,cef}}$ is the distance between the observer on Erath and the cepheide

$$E_{\mathrm{cef}} = \frac{\sigma T_{\mathrm{cef}}^4 \cdot 4\pi \cdot \left(\frac{KM}{4\pi^2}\right)^{2/3} \cdot P^{4/3}}{4\pi d_{\mathrm{P,cef}}^2}.$$

Similarly for Sun

$$E_{\mathrm{S}} = \frac{L_{\mathrm{S}}}{4\pi d_{\mathrm{PS}}^2} = \frac{\sigma T_{\mathrm{S}}^4 \cdot 4\pi R_{\mathrm{S}}^2}{4\pi d_{\mathrm{PS}}^2}.$$

By using the Pogson formula:

$$\log\frac{E_{\mathrm{cef}}}{E_{\mathrm{S}}} = -0{,}4\left(m_{\mathrm{cef}} - m_{\mathrm{S}}\right);$$

$$M_{\mathrm{cef}} = M_{\mathrm{S}} - 5^{\mathrm{m}}\log\frac{\left|d_{\mathrm{P,cef}}\right|}{\left|d_{\mathrm{PS}}\right|} - 2{,}5\cdot\log\frac{E_{\mathrm{cef}}}{E_{\mathrm{S}}};$$

$$\frac{T_{\mathrm{cef}}^4\cdot\left(\dfrac{KM}{4\pi^2}\right)^{3/2}\cdot d_{\mathrm{PS}}^2}{T_{\mathrm{S}}^4\cdot R_{\mathrm{S}}^2\cdot d_{\mathrm{P,cef}}^2} = k_1 = \text{constant};$$

$$M_{\mathrm{cef}} = M_{\mathrm{S}} - 5^{\mathrm{m}}\log\frac{\left|d_{\mathrm{P,cef}}\right|}{\left|d_{\mathrm{PS}}\right|} - 2{,}5\cdot\log k_1 - \frac{10}{3}\cdot\log P;$$

$$M_{\mathrm{S}} - 5^{\mathrm{m}}\log\frac{\left|d_{\mathrm{P,cef}}\right|}{\left|d_{\mathrm{PS}}\right|} - 2{,}5\cdot\log k_1 = -2{,}5\cdot\log k;$$

$$k = \text{constant};$$

$$M_{\mathrm{cef}} = -2{,}5\cdot\log k - \frac{10}{3}\cdot\log P.$$