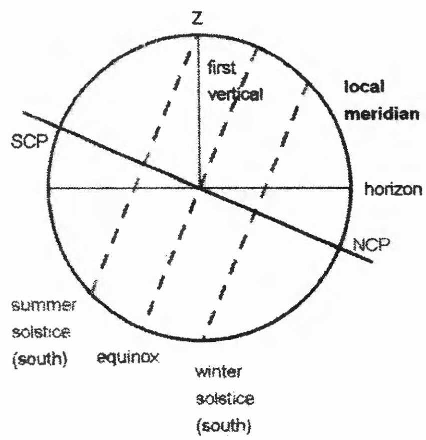
Theory - General Solutions

1 As the dial is vertical, the rod lies on the east-zenith-west plane. The clock works only when the Sun is in the northern hemisphere, $\delta_{Sun} > \delta_{zenith}$, because only in this case there will be a shadow projected over the dial.



3 pt declination

2 pt months& seasons

i) When $\delta_{Sol} \leq 0°$, the Sun does not project a shadow when it is near the horizon (after rising and before setting). It corresponds to the time between the equinoxes: between September and March, during spring and summer.

3 pt declination

2 pt months& seasons

ii) When $\delta_{Sun} =< -22°54'$, the solar clock doesn't work at all. It corresponds to the small time intervalclose to the Summer solstice, during December and January.

2 Let be T_{sol} the duration of the solar day, T_{sid} the duration of the sidereal day and T_{year} the length of the year. In the present situation, we have

$$\frac{1}{T_{sol}} = \frac{1}{T_{sid}} - \frac{1}{T_{year}}$$

3 pt calc. T_sid

Using the given data, $T_{sid} = 0.997270\ days = 23\ h\ 56\ m\ 4\ s$

3 pt sidereal doesn't change

The sidereal day depends only of the magnitude of Earth's rotation speed, and the angular velocity of the sun's annual movement is opposite. So we have

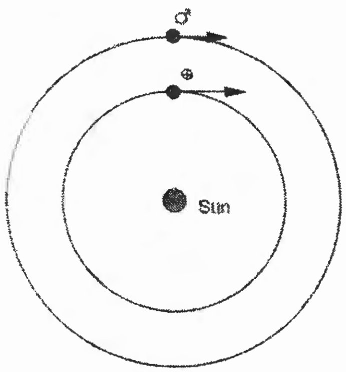
$$\frac{1}{T'_{sol}} = \frac{1}{T_{sid}} + \frac{1}{T_{year}}$$

2 pt - to +

$$\therefore\ T'_{sol} = 0.994555\ d = \mathbf{23h\ 52m\ 9.5s}$$

2 pt right number

3.



From Kepler's Third Law, we have:

$$\frac{a_{♂}^3}{T_{♂}^2} = \frac{a_{\oplus}^3}{{T_{\oplus}}^2} \Rightarrow T_{♂} = 1{,}874\,year$$

5 pt calculating the period of Mars

Thus

$$\frac{1}{T_{op}} = \frac{1}{T_{\oplus}} - \frac{1}{T_{♂}} \Rightarrow \boldsymbol{T_{op} = 2{,}14 year}$$

5 pt synodic period

4. The Moon's flux is proportional to its albedo, so $\frac{F_1}{a} = \frac{F_2}{1}$, being F_1 its actual flux, a the albedo and F_2 the hypothetical flux in the case the Moon had an albedo 1.

5 pt flux relation

Being m_1 the Moon apparent magnitude and m_2 the desired value, we have:

$$m_2 - m_1 = -2{,}5 \log \frac{F_2}{F_1} \Rightarrow m_2 = m_1 + 2{,}5 \log a$$

5 pt all the rest

Replacing m_1 = -12,74 and a = 0,14 we have: $\boldsymbol{m_2 = -14.9}$

5. *Fact 1.* $\Delta\phi = 1° = \frac{\pi}{180}$ $\quad R_{\oplus} = \frac{L}{\Delta\phi}$
$L = 111\,km$

2 pt relations of fact 1

Fact 2: $g = 9.8\,m/s^2$ $\quad g = \frac{GM_{\oplus}}{R_{\oplus}^2} = \frac{4\pi G\rho_{\oplus}R_{\oplus}}{3} \quad \therefore \quad \rho_{\oplus} = \frac{3g\Delta\phi}{4\pi GL}$

2 pt relations of fact 2

Fact 3: $T_{\oplus} = 1\,year$ $\quad \frac{T_{\oplus}^2}{a_{\oplus}^3} = \frac{4\pi^2}{GM_{\odot}} \quad \therefore \quad T_{\oplus}^2 = \frac{3\pi a_{\oplus}^3}{G\rho_{\odot}R_{\odot}^3}$

$$M_{\odot} = \frac{4}{3}\pi\rho_{\odot}R_{\odot}^3$$

2 pt relations of fact 3

Fact 4: $\alpha = 30'$ $\quad \frac{\alpha}{2} \approx \frac{R_{\odot}}{a_{\oplus}} \quad \therefore \quad T_{\oplus}^2 = \frac{3\pi a_{\oplus}^3}{G\rho_{\odot}R_{\odot}^3} = \frac{24\pi}{G\rho_{\odot}\alpha^3}$

2 pt relations of fact 4

$$\therefore \frac{\rho_{\oplus}}{\rho_{\odot}} = \frac{\frac{3g\Delta\phi}{4\pi GL}}{\frac{24\pi}{GT_{\oplus}^2\alpha^3}} = \frac{g\Delta\phi\alpha^3T_{\oplus}^2}{32\pi^2 L} = \mathbf{3.23}$$

2 pt answer

6. The reaction chain is

$$2He^3 \rightarrow He^4 + 2H^1 + E$$

So the energy is

$$E = (2m_{He^3} - m_{He^4} - 2m_{H^1}) \cdot c^2 = \mathbf{12.66\ MeV}$$

7 pt energy

$$\frac{\Delta m}{m} = \frac{12.66\ MeV/c^2}{2m_{He^3}} = \mathbf{0{,}002254}$$

3 pt percentage

7. $T = 30000\,K$

4 pt

$T_2 = 5000\ K$

$m_{bol} = constant \quad \Leftrightarrow \quad L = constant$ — m_{bol}const = L const

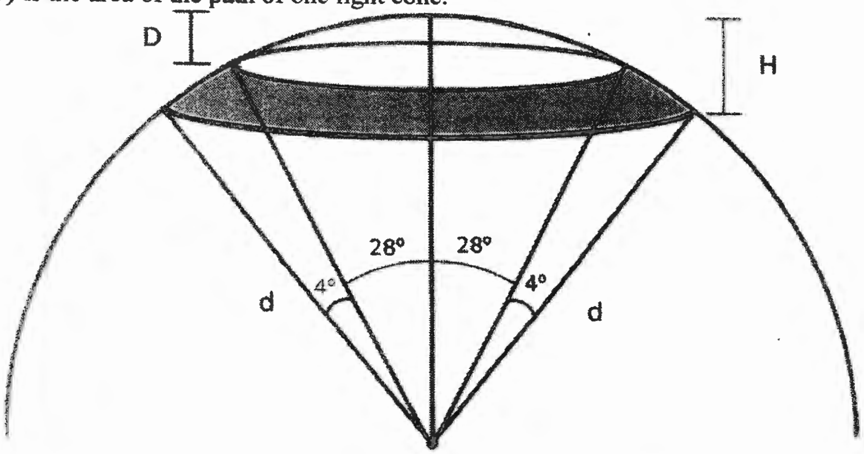
As $L = 4\pi\sigma R^2 T^4$ and $L_1 = L_2$, we have: — 4 pt Equating

$$4\pi\sigma R_0^2 T_1^4 = 4\pi\sigma R_{shell}^2 T_2^4 \quad \Rightarrow \quad \frac{R_{shell}}{R_0} = 36$$ — 2 pt Answer

8. As for the probability, supposing that the neutron star space orientation is random, is

$$p = \frac{2A_p(d)}{4\pi d^2}$$ — 2 pt stating p

where $A_p(d)$ is the area of the path of one light cone.



3 pt evaluating p

As the figure above shows, the area is the difference of the spherical caps of height H and D:

$$A = 2\pi d(H - D) = 2\pi d^2 \left[\cos\left(\theta - \frac{\alpha}{2}\right) - \cos\left(\theta + \frac{\alpha}{2}\right)\right]$$

$$p = \cos\left(\theta - \frac{\alpha}{2}\right) - \cos\left(\theta + \frac{\alpha}{2}\right) = \mathbf{3.5 \cdot 10^{-2}} \quad or \quad \mathbf{3.5\%}$$ — 2 pt stating the flux

The light waves are spread along the double cone and not along a sphere, thus:

$$F = \frac{L}{2A(d)} = \frac{10\,000\ L_\odot}{4\pi d^2 \left(1 - \cos\frac{\alpha}{2}\right)}$$

3 pt evaluating magnitude

Where $A(d)$ is the area of one light cone. Using the Sun's absolute magnitude, we can find the absolute magnitude of the pulsar (m_p):

$$m_p - M_\odot = -2.5 \log\left(\frac{F}{\frac{L_\odot}{4\pi(10\ pc)^2}}\right) = -2.5 \log\left(10\,000 \cdot \frac{1}{1 - \cos\frac{\alpha}{2}} \left(\frac{10}{1000}\right)^2\right)$$

$$\mathbf{m_p = -3.24}$$

9. The intrinsic color indexes are U – V = 0.300 and U – V= 0.330. therefore
U – B = (U – V) – (B – V) = 0.030

The difference between the real and the measured color indexes are due to the absorption in the interstellar medium, as well as due the stronger absorption of the planetary nebula (in fact, during thecalculations, one can easily see that the interstellar absorption is not enough to account for the measured values). So, for the first (nearest) star:

$$(B-V)_1 = (B-V) + (E_B - E_v)\cdot D + (E'_B - E'_V)\cdot R = 0.327$$
$$(U-V)_1 = (U-V) + (E_U - E_v)\cdot D + (E'_U - E'_V)\cdot R = 0.038$$
$$(U-B)_1 = (U-B) + (E_U - E_B)\cdot D + (E'_U - E'_B)\cdot R = 0.365$$

3 pt equation

apparentintrinsic interstellar nebulaabsorption absorption

Calculating the interstellar absorption, we can find for, the nebula values:

$$(E'_B - E'_V)\cdot R = 0.0155$$
$$(E'_U - E'_V)\cdot R = 0.0100$$
$$(E'_U - E'_B)\cdot R = -0.0055$$

1 pt absorption values

So, for the second (farthest) star,

$$(B-V)_2 = (B-V) + (E_B - E_v)\cdot 3D + (E'_B - E'_V)\cdot 2R = \mathbf{0.3655}$$
$$(U-V)_2 = (U-V) + (E_U - E_v)\cdot 3D + (E'_U - E'_V)\cdot 2R = \mathbf{0.425}$$
$$(U-B)_2 = (U-B) + (E_U - E_B)\cdot 3D + (E'_U - E'_B)\cdot 2R = \mathbf{0.0595}$$

4 pt equation

2 pt final answers

10 As we have a differential equation that should be integrated, we expect that the student can solve it in two ways: using an approximation (Way 1) or calculus (Way 2).

Common

2 pt Finding final scale factor

Way 1

Using the definition of Hubble constant: $\frac{da}{dt} = Ha$, where a is the scale factor and using the approximation $\frac{da}{dt} \approx \frac{\Delta a}{\Delta t}$, and supposing that the variation of the scale factor and the Hubble constant is small compared to their values, and using that $aT =$ const:

Way 1

7 pt Using the approximation correctly

$$\frac{\Delta a}{\Delta t} = H_i a_i \Rightarrow \frac{\Delta T}{\Delta t} = -H_i T_i \Rightarrow \ \Delta t = -\frac{1}{H_i}\frac{\Delta T}{T_i} = 5.0\cdot 10^8\ years$$

1 pt final answer

Way 2

Way 2

As we have a matter dominated Universe ($\rho \propto a^{-3}$), without dark energy, and a density parameter $\Omega_0 = 1$, we have:

$$\Omega_0 = \frac{8\pi G\rho}{3H^2} = \frac{8\pi G}{3H^2}\frac{\zeta}{a^3} = 1$$

4 pt finding the diff. equation

From the Hubble law, $\frac{da}{dt} = Ha$:

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\frac{\zeta}{a} \Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G}{3}\frac{\zeta}{a}}$$

3 pt Integrating equation

Integrating this equation:

$$\frac{2}{3}\left[a_f^{3/2} - a_i^{3/2}\right] = \sqrt{\frac{8\pi G}{3}}\zeta\Delta t \Rightarrow \ \Delta t = \frac{2}{3}\left[\left(\frac{a_f}{a_i}\right)^{3/2} - 1\right]\frac{1}{\sqrt{\frac{8\pi G}{3}\frac{\zeta}{a_i^3}}} = \frac{2}{3}\frac{1}{H_i}\left[\left(\frac{a_f}{a_i}\right)^{3/2} - 1\right]$$

Obs: (7pt for using correctly that $a \propto t^{2/3}$)

Using the fact that $aT =$ const:

$$a_i T_i = a_f T_f \Rightarrow \frac{a_f}{a_i} = \frac{T_i}{T_f} = \frac{2.73}{2.63} = 1.038$$

So the time is:

$$\Delta t = 5.2 \cdot 10^8\ years$$

1 pt final answer

11 The oscillation occurs due to the fact that the center of mass (CM) of the Solar System does not lieexactly at the center of the Sun, but in another position around which the center of the Sun revolves.

Given $a_{♃} = 5.204\ AU$, the distance from CM to the center of the Sun is:

$$x_{CM} = \frac{\frac{1}{1047}}{1 + \frac{1}{1047}} 5.204 = 0.004966\ AU,$$

3 pt position of CM

Whichis the amplitude of the movement.

If the distance to Barnard's Star is $d_{bar} = 1.834\ pc$, we can write, for the angular amplitude:

$$\alpha \approx \frac{x_{cm}}{d_{bar}} = \mathbf{0{,}002708''}$$

3 pt angular amplitude

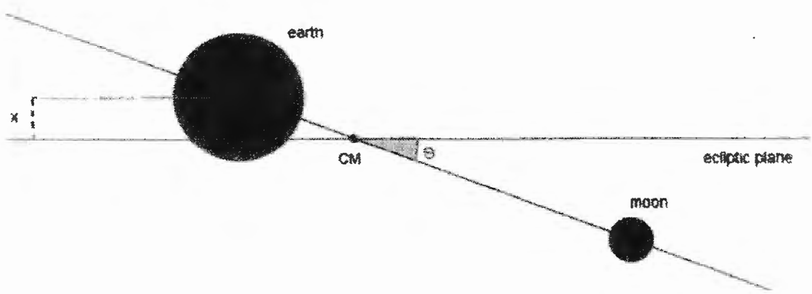
As for the period, it is the period of Jupiter's orbit:

$$\frac{T_{♃}^2}{a_{♃}^3} = \frac{T_{\oplus}^2}{a_{\oplus}^3} = 1\frac{year^2}{AU^3} \quad \therefore \quad T_{jup} = \mathbf{11{,}87\ years}$$

4 pt period of oscillation

Obs: the difference of Jupiter's mass in the Kepler's Third Law represents a difference in less than 0.01 year at the period of Jupiter.

12 The phenomenon is as follow:



The Center of Mass of the Earth-Moon system (CM) lies on the plane of the ecliptic, so as the Lagrangian points of its orbit around the Sun. Let be D the distance between Earth and Moon and $x$the distance between Earth and the CM. We then have:

$$M_{\oplus} \cdot x = M_{☾} \cdot (D - x) \quad \therefore \quad x = \left(\frac{M_{☾}}{M_{\oplus} + M_{☾}}\right) D$$

2 pt equating the CM

The Earth's displacement perpendicular to the ecliptic is $A = 2x \sin\theta$, where $\theta = 5°$ is the inclination of the plane of Moon's orbit relative to the plane of ecliptic. Calculating:

$$A = \frac{2D\ M_{☾}}{M_{\oplus} + M_{☾}} \sin 5° = 810\ km$$

4 pt calculating the displacement

The telescope resolution is given by **3 pt telescope resolution**

$$r'' = 1.22\frac{\lambda}{\delta} = \frac{A}{d}$$

whereδ is the diameter of the objective lens and d is the distance to the telescope to Earth. But, at L4(L5), Earth and Sun forms an equilateral triangle, so d = 1 UA. Using $\lambda = 400\ nm$ (that is, the visible wavelength that allows the smallest telescope), we find $\boldsymbol{\delta = 9.01\ cm}$ **1 pt answer**

13 As seen from an inertial referenceframe outside Earth, the observer must describe a circle parallel to the ecliptic, with constant speed, in a period of one sidereal day. Sincethe ecliptic poles are in zero altitude, the circle will be a great circle.

But the resulting movement should be the actual movement of the observer (in relation of Earth's surface) plus the movement of Earth (in relation to the inertialreferenceframe). After some time t, the observer must describe an arc of length $\phi = 2\pi/T$ on the great circle; at the same time, the Earth will rotate by this same amount.

In other words, the displacement of the observer is the composition of a rotation with an angle φ around the ecliptic axis and a rotation with an angle –φ around the earth's rotation axis. But this is exactly the same construction which describes an analemma of a planet with a circular orbit, inclination of $\epsilon = 23.44°$ between the ecliptic and equator!

So, the important characteristics of the drawing curve are:

a. It is closed
b. Maximum and minimum latitudes are $\theta = \pm 23.44°$
c. It is symmetric around the Equator **1 pt each item**
d. It is symmetric around a meridian
e. It has the shape of an eight (8)

For the calculation: as the observer starts on the southern hemisphere, he will cross the Equator for the first time in the direction south-north. Its velocity relative to an inertial referential will have intensity $v = 2\pi R/T$ and azimuth (the angle with the north-south direction) $\alpha = 90° - \epsilon$. The velocity of the Earth's surface will have the same intensity and azimuth $\alpha = 90°$. Subtracting the vectors, we'll have a resultant vector with modulus

3 pt velocity

$$v' = v\sqrt{2(1 - \cos\epsilon)}$$

And azimuth **2pt direction**

$$\alpha' = -\frac{\epsilon}{2} = -11.72°$$

14. Let be R the radius of the Celestial Sphere and (x, y, z) a coordinate system with origin at the observer, z is the down-up axis, y is the east-west axis and x is the south-north axis.

In the superior culmination, the altitude angle is $h = 7.41 + 49.35 = 56.76°$and we have

$$\vec{\mathbf{v}} = v_0\hat{\mathbf{y}} = 2\pi R\,\hat{\mathbf{y}}\ /day$$

$$\vec{\mathbf{r}_0} = R(-\cos h\,\hat{\mathbf{x}} + \sin h\,\hat{\mathbf{z}})$$

$$\mathbf{r(t)} = \mathbf{r_0} + \mathbf{v}t$$

The altitude in function of our coordinate systemis $h = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right)$ .When $t \to \infty$, the coordinates x and z of the star remain constant, while $y \to \infty$, and we have

3 pt **h → 0**

$$h = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right) \xrightarrow[t\to\infty]{} \tan^{-1} 0 = 0°$$

2 pt **A→W**

Similarly, because $y \to \infty$ but x and z remain constant, the final azimuth is the West direction ($A = 90°$ or $A = 270°$, depending on the system used).

2 pt **ratio of distances**

For the magnitude, $m_0 - m_6 = -2.5 \log\frac{F_0}{F_6}$ and $F = \frac{L}{4\pi R^2}$ so $R_6 = 12.88\, R_0$

3 pt **time**

$$|\mathbf{r(t)}| = \sqrt{x^2+y^2+z^2} = R\sqrt{1+4\pi^2 t^2} = 12.88\, R$$

$$t = \mathbf{2.044}\ \boldsymbol{days}$$

15. A known geometrical result is that the projection of a circle in an arbitrary plane is a conic curve (afterwards, projecting a circle is drawing a cone that intersects the given plane). In this particular case, the Sun crosses the horizon at one point, when the tip of the shadow goes to infinity. So the projection must be a parabola.

3 pt **it's a parabola**

The equation of the curve will have the form $y = ax^2 + bx + c$. We just need to find the coefficients of this curve.

1 pt **equation of parabola**

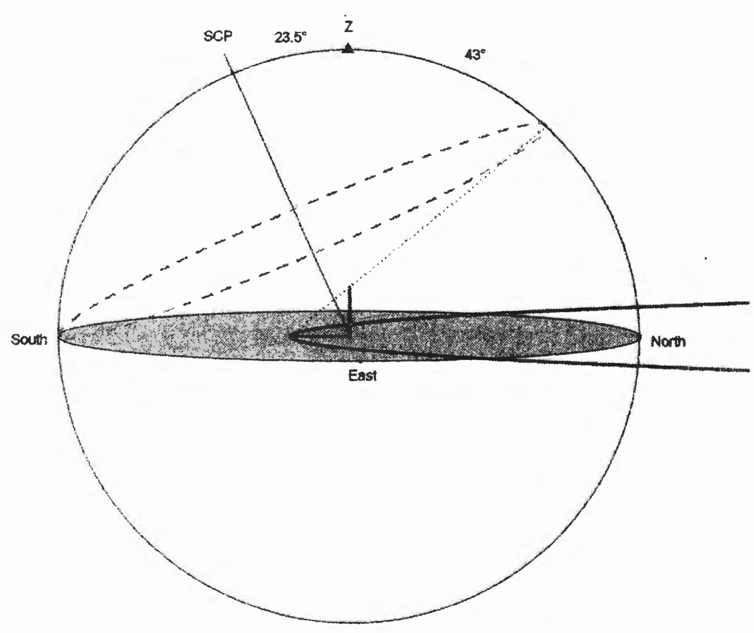
To find c, we can take the Sun in its maximum culmination, lying in the North-South plane. In this case, the altitude angle will be $23°27' + 23°27' = 46°54'$ and, as $x = 0$.

2 pt **finding c**

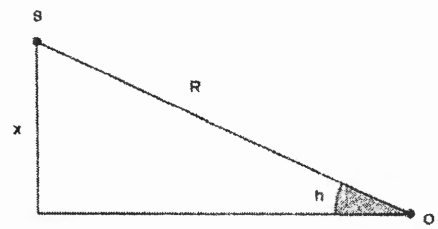
$$y = c = -39.6 \cdot \cot 46.9 = -37.06$$

To find b, we just need to notice that the parabola must be symmetrical around the y axis, so b = 0.

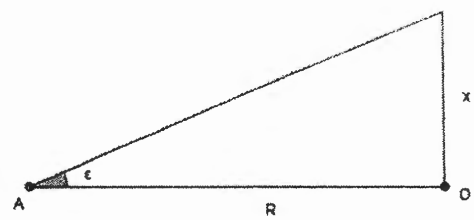
1 pt **finding b**



To find a, we can look at the Sun when it is crossing the First Vertical or the East-West plane. Let's call P this point. Looking at the E-W plane, we have the triangle on the side, where O is the origin of the system of coordinates, h is the altitude angle, R is the radius of the celestial sphere and x is the height of the Sun. We know that the altitude angle is such as $\sin h = x/R$.



On the other hand, looking at the North-South plane, we have another triangle. O is the origin, R is the radius of the celestial sphere (lying in the plane of horizon), A is the point of inferior culmination of the Sun (when it touches the horizon), ε is the obliquity of ecliptic, the hypotenuse lies on the plane containing the Sun trajectory during that day, and x lies in the E-W plane being the height of the Sun as defined above.



3 pt
finding a

Then.

$$x = R \tan\epsilon \quad \therefore \quad \sin h = \tan\epsilon \quad \therefore \quad h = 25°42'$$

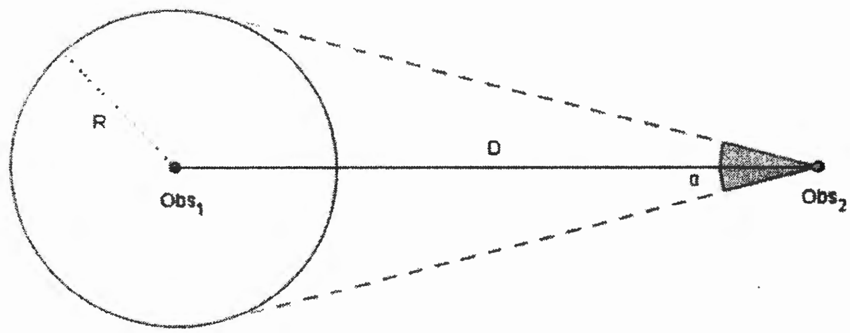
So, when y = 0.$x = 39.6 \cdot \text{cotan}\, 25.7 = 82.28$ and

$$a = -\frac{c}{x^2} = 0.0055$$

And the equation is:

$$\mathbf{y = 0.0055x^2 - 37.1}$$

A. Look at the figure:



$$\tan\alpha \approx \alpha \approx \sin\alpha \approx \frac{2R}{D}$$

A1:

$$\Delta m = m_B - m_A = -2.5 \log\frac{F_B}{F_A}$$

The brightness as seen by the astronomer (F_A) is the brightness of N stars with absolute magnitude

M_0(and luminosity L_0) at a distance D from him:

$$F_A = N\frac{L_0}{4\pi D^2}$$

4pt stating the flux A

We assumethe cluster is homogeneous, sothe density of stars ρ inside the cluster should be constant. The brightness coming from a spherical shell with small thickness ΔR at a distance R' is

$$F_{shell}(R', \Delta R) = \rho V_{shell}\frac{L_0}{4\pi R'^2} = \frac{N}{\frac{4}{3}\pi R^3}4\pi R'^2\Delta R\frac{L_0}{4\pi R'^2} = \frac{3NL_0\Delta R}{4\pi R^3}$$

5pt no-R'

The brightness doesn't depend on the radius of the shell R'. If we divide the cluster in n shells, $R = n\Delta R$, the brightness of the cluster as seen by the astronomer 2 is:

4pt stating the flux B

$$F_B = nF_{shell} = n\frac{3NL_0\Delta R}{4\pi R^3} = \frac{3NL_0}{4\pi R^2}$$

So the difference in the magnitudes Δm is:

2 pt calculate answer

$$\Delta m = -2.5\log\left(\frac{\frac{3NL_0}{4\pi R^2}}{\frac{NL_0}{4\pi D^2}}\right) = -2.5\log\left[3\left(\frac{D}{R}\right)^2\right] = 2.5\log\left(\frac{\alpha^2}{12}\right)$$

A2.

The total energy received is proportional to the objective area:

3 pt x as function of Fs

$$F_A' = F_A\frac{\pi\left(\frac{x}{2}\right)^2}{\pi\left(\frac{d_{pup}}{2}\right)^2} \Rightarrow x = 6\cdot 10^{-3}\sqrt{\frac{F_A'}{F_A}}$$

3 pt calculating the diameter

$$F_A' = F_B \Rightarrow \frac{F_A'}{F_A} = \frac{F_B}{F_A} = \frac{12}{\alpha^2} \Rightarrow x = 6\cdot 10^{-3}\sqrt{\frac{12}{\alpha^2}} = \frac{0.021}{\alpha}\ m$$

A3.

2 pt formulate the fraction of flux

In this case, a fraction $\xi(d) = \frac{A(\alpha)}{4\pi d^2}$ of the light that comes from the distance d is seen by the biologist. $A(\alpha)$is the area of the interior of a cone with aperture α that intersects the sphere with radius d and center in the vertices of the cone (a spherical cap of radius d and height D):

4 pt calculate

$$\cos\left(\frac{\alpha}{2}\right) = \frac{d-D}{d} \text{and } A(\alpha) = 2\pi Dd \Rightarrow A(\alpha) = 2\pi d^2\left[1-\cos\left(\frac{\alpha}{2}\right)\right] \Rightarrow \xi = \sin^2\left(\frac{\alpha}{4}\right)$$

So the fraction is independent of the distance d, andthe fraction of light that the biologist sees is$F_B' = \xi F_B$:

3 pt calculating the new magnitude

$$\Delta m' = m_B' - m_A = -2.5\log\frac{F_B'}{F_A} = -2.5\log\left[\frac{12\sin^2\left(\frac{\alpha}{4}\right)}{\alpha^2}\right]$$

B. **B1.**

1st line: $\{(0.0); (15.210)\}$ $(10^3\text{ly}, \frac{\text{km}}{\text{s}})$

2nd line: $\{(15.210); (60.200)\}(10^3\text{ly}, \frac{\text{km}}{\text{s}})$

4 pt For the lines (2 pt each, considered right with some

$V(D) = \begin{cases} 14D, 0 \le D < 15 \\ 213 - 0.22D, 15 \le D \le 60 \end{cases}$, D in 10^3 l.y. and V in km/s

margin)

B2.

Using the equations from (a):

$$\omega(D) = \frac{V(D)}{D} \Rightarrow \omega(D) = \begin{cases} 14. \quad 0 \le D < 15 \\ \frac{213}{D} - 0.22 \quad ,15 \le D \le 60 \end{cases} \frac{km/s}{10^3 l.y.}$$

1 pt
writing omega

$$\omega_{wave}(D) = 2\omega(D)$$

The time that a spiral arm takes to have one more turn around the galactic center:

4pt
relative velocity

$$\omega_{rel} = \omega_{wave}^{max} - \omega_{wave}^{min}$$

$$\omega_{rel} = \frac{1}{2}\left[\left(\frac{213}{15} - 0.22\right) - \left(\frac{213}{80} - 0.22\right)\right] = 23.9 \frac{\text{km/s}}{10^3 \text{l.y.}}$$

$$P_{sin} = \frac{2\pi}{\omega_{rel}} = \mathbf{8.15 \cdot 10^7 years}$$

1pt
answer

B3.

Tully-Fisher relation estimatesthe luminosity of the galaxy, and considering$\Delta V = 2V_{max}$:

$$L = 16kV_{max}^4$$

2 pt
Use Tully-Fisher with Vmax

We can compare the magnitude of the galaxy with the Sun:

$$m - M = 5\log d - 5 \Rightarrow m - \left(M_{Sun} - 2.5\log\frac{L}{L_{Sun}}\right) = 5\log d - 5$$

$$\log d = 1 + \frac{m - M_{Sun} + 2.5\log\left(\frac{16kV_{max}^4}{L_{Sun}}\right)}{5}$$

2 pt
Compare with the Sun

From the graph: $V_{max} = 230$ km/s:

$$\log d = 1 + \frac{8.5 - 4.8 + 2.5\log(16 \cdot 0.317 \cdot 230^4)}{5} = 6.816 \Rightarrow \quad d = 6.5 \text{ Mpc}$$

2 pt
Answer (use of a reasonable V_max)

B4.

We can estimate the recession velocity of the galaxy using Hubble's law:

$$V_0 = H \cdot d$$

The maximum and minimum wavelengths occurwhere the radial velocity is V_{max}, the radial velocity is:

3pt
Write the velocities correctly

$$V = V_0 \pm V_{max}$$

Using$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$, the wavelengths are:

$$\lambda_{max} = \lambda\left(1 + \frac{Hd + V_{max}}{c}\right)$$

$$\lambda_{min} = \lambda\left(1 + \frac{Hd - V_{max}}{c}\right)$$

2 pt
Doppler law

Calculating the values, using $\lambda = 656.28$ nm:

$$\lambda_{max} = 657.79 \text{ nm}$$
$$\lambda_{min} = 656.78 \text{ nm}$$

B5. Supposing a mass with spherical symmetry distribution and a circular orbit, we have:

5 pt **Estimate the internal mass**

$$\frac{V^2}{D} = \frac{GM_{int}}{D^2} \Rightarrow M_{int} = \frac{V^2 D}{G}$$

From the graph $V(3.0 \cdot 10^4 \text{ l.y.}) = 225 \text{ km/s}$, So the mass is:

$$M_{int} = 2.15 \cdot 10^{41}\text{kg} = 1.08 \cdot 10^{11} M_{sun}$$

B6.

As the mass is derived from the rotation curve, it is necessary to take into account that part of the mass is dark matter, and only part of the mass is in the stars ($\rho_{star} = 1/3$). The fraction of baryonic matter is:

2pt **take in account the dark matter correctly**

$$\rho_{matter} = \frac{f_{baryon}}{f_{matter}} = \frac{4}{22+4} = \frac{4}{26}$$

The number of stars is then:

2 pt **Calculate the number**

$$N = \rho_{matter} \cdot \rho_{star} \frac{M_{galaxy}}{M_{Sun}} = \frac{4}{26}\frac{1}{3} 1.08 \cdot 10^{11} = 5.5 \cdot 10^{9} \text{stars}$$