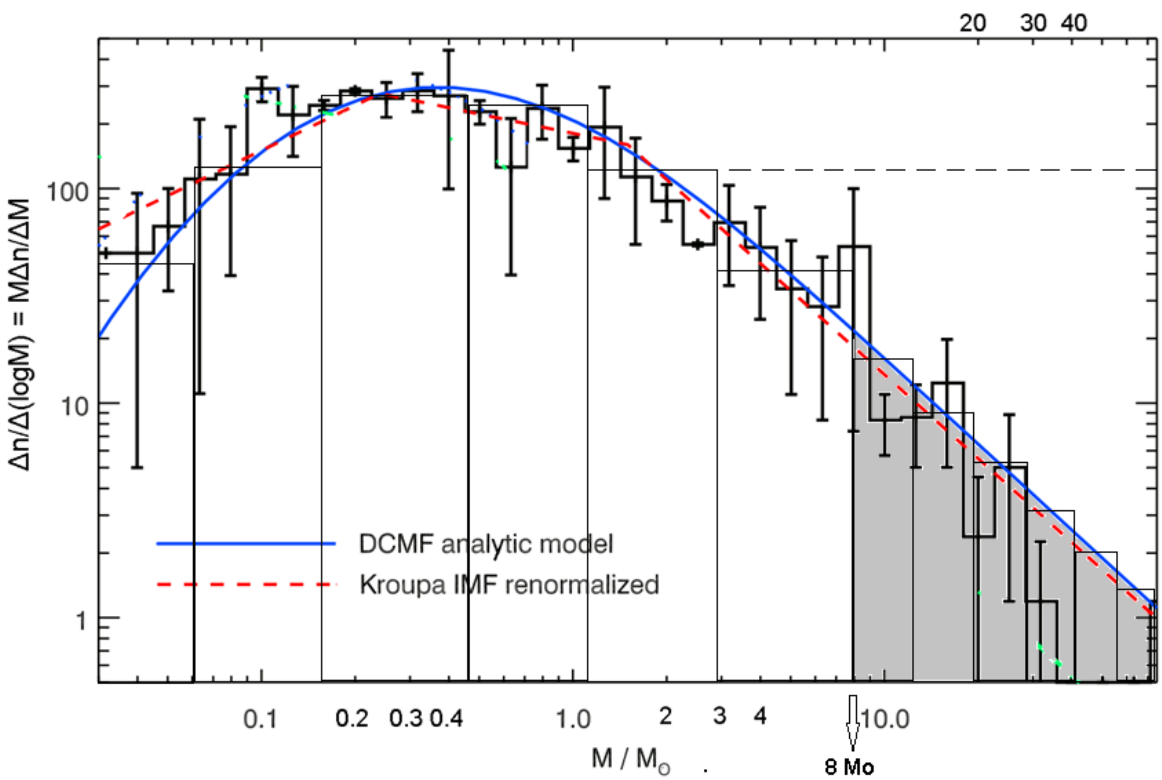
2016 International Astronomy Olympiad

Practical round: solutions

The gravitational constant is γ or G = 6.67×10^{-11} m^3 kg^{-1} s^{-2}

Problem αβ-7. The Initial Mass Function and supernovae.

7.1. *6pts (3pts for reasonably accurate measurements + 3pts for further considerations)*



Stars with an initial mass > 8 $M_\odot$ explode as core-collapse supernovae. Let us mark on the IMF figure the space where these stars reside with grey. Then we divide both the left and the right part of the graph with bins. We calculate the scales on both axes:

X: Δ(lgM) = 1 → ΔX=5.75 cm
Y: Δ(lg(Δn/ΔlogM)) = 1 → ΔY=3.95 cm (defining lgX = $\log_{10}$X)

The base of the Y-axis logarithm is irrelevant – if it is changed, the function is multiplied by a constant and the final results should be the same.

For each bin we calculate the width Δ(lgM) and the height lg(Δn/ΔlogM). We make the rough approximation that Δ(lgM) << 1 so the process is analogous to numerical integration and can be expressed with integrals as well. For further accuracy the function can be extrapolated linearly until it crosses the X-axis at ~ 160 $M_\odot$.

For each bin we then calculate the X-axis center M and the value of Δn that corresponds to the bin width. If the values to the left of the 8 $M_\odot$ vertical line are M_L, Δn_L and to the right are M_H, Δn_H, we can obtain

$$M_{SN} = \sum(M_H \Delta n_H)/\sum(\Delta n_H)$$

$$q = \sum(M_H \Delta n_H)/\sum(M_H \Delta n_H + M_L \Delta n_L)$$

A reasonable result would be $M_{SN} \sim 20\ M_\odot$, $q \sim 30\%$.

Other mathematical methods and bin distributions may be used so these values are correct within an order of magnitude. Participants from group α can avoid the usage of logarithms by considering the Y-axis in the form $M(\Delta n/\Delta M)$ and by doing linear approximations between the X-axis ticks (or by choosing bins using the ticks).

As the galactic star formation rate is said to be $\Delta M/\Delta t = 8\ M_\odot$/yr, and the timescale of the evolution of massive stars is much lower than the change of this rate, the mass rate of supernova explosions is

$q\Delta M/\Delta t \sim 2.4\ M_\odot$/yr

This mass is distributed among objects with an average mass M_{SN} therefore the expected frequency is

$f = q(\Delta M/\Delta t) / M_{SN} \sim 0.12\ yr^{-1}$

or the derived expectations are that there should be a supernova each 8 years on average. Please note that the mass loss of massive stars is not taken into account which leads to significant systematic errors. The mass of supernova progenitors right before the explosion is significantly lower than their initial mass. The actual values are $M_{SN} \sim 12\ M_\odot$, $q \sim 8\%$, and $f = 1/20\ yr^{-1}$.

7.2. *2pts*

The last directly observed galactic supernovae exploded in 1006, 1054, 1572 and 1604 and some of them were type Ia, which are less massive systems with a different mechanism of explosion. So the approximate value of the observed frequency is

$f \sim 1/400\ yr^{-1}$

This value is significantly lower than the one derived in 7.1. A possible explanation is that many explosions are obscured by dust as massive stars are situated in the galactic disk. As we lie inside it, the galactic disk covers a large angular area on the sky which is not easy to monitor. Furthermore, most of the observations historically were conducted from the Northern Hemisphere.