Practical round

6. The first gravitational wave detection. In 1916, the year after the final formulation of the equations of general relativity, Albert Einstein predicted the existence of gravitational waves. They are emitted by accelerating masses in a way similar to the emission of electromagnetic waves by accelerating charges. In the case of gravity, like electromagnetism,.when a stationary mass moves suddenly then the gravitational force exerted by it on the test masses changes. However, that change will not happen immediately. Instead, the information that the mass moved will propagate at the speed of light and will take the form of gravitational radiation. When the gravitational wave reaches the test masses, those test masses accelerate, because the gravitational force suddenly changes. For a mass undergoing periodic motion, the gravitational waves it produces will be periodic, and those can accelerate surrounding test masses periodically.

On September 14, 2015, the two detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected the coincident signal GW150914 as shown in Fig. 1. It was the first direct detection of gravitational waves and the first direct observation of a binary black hole system merging to form a single black hole. The distance to the merging black hole binary system was determined to lie somewhere between 250 and 600 Mpc, and the two initial black hole masses as being 36±4 M⊙ and 28±4 M⊙, respectively. When those two black holes merged, they formed a single black hole of mass 62±4M⊙. The missing mass was emitted as energy in gravitational radiation, which was calculated to be 3.0±0.5 M⊙c^2. As usual, 1 M⊙ is one solar mass equal to 2.0×10^{30}kg, and $c = 3.0\times10^{8}$ m/s is the speed of light in vacuum. The peak emitted power of gravitational radiation was calculated at several times of 10^{49} W – more than 10 times greater than the combined power of all light radiated by all the stars in the observable universe.

Due to the tidal forces gravitational waves cause relative displacements (*ΔL*) in the test masses that are proportional to the distance (*L*) between those test masses. Physicists define the *amplitude* of gravitational waves by using the *dimensionless* quantity *h*, defined as

$$h = \Delta L/L$$

which is usually called the dimensionless strain of the gravitational wave. This is the quantity plotted on the vertical axis of Fig.1.

Practical round

6. The first gravitational wave detection. Continue.

Fig. 1. *The gravitational-wave event GW150914 observed by the two LIGO detectors. The vertical axis shows the amplitude of the gravitational wave (the strain, h). The black holes go through the following stages (images placed along the horizontal axis at their roughly corresponding times):*
(1) the inspiral, as the two black holes approach each other;
(2) the merger, as the black holes join together; and
(3) the ringdown, as the single black hole that has newly formed briefly oscillates before settling down.

Using a Newtonian (classical) mechanics approximation do the following tasks:

6.1. Using the masses quoted by the LIGO team, calculate the radii of the horizons of the two black holes before they merged. Include an estimate of the error for the radii..

6.2. From the measurements of the gravitational wave signal presented in the graph above, find the orbital period of the black hole binary at the moment of the merger.

6.3. Estimate the total mass of the initial black hole binary. Use ***only*** information of the observed gravitational radiation from the graph. Do ***not*** use any of the other LIGO results quoted above.

6.4. Find an expression for the energy released in gravitational waves until the time of the black hole merger in terms of the initial black hole masses (M_1 and M_2) only. Using the result from 6.3., calculate the total released gravitational energy if you assume $M_1 = M_2$.

6.5. Find the average power *P* of gravitational wave emission during the last 0.10s before the merger.

6.6. As with most waves, the gravitational wave flux is proportional to the square of the amplitude of the wave. The amplitude (strain) right next to the two merging black holes is approximately $h = (v/c)^2$, where ***v*** is the orbital velocity of the black holes. Estimate the distance to the merging black holes detected by LIGO.

6.7. The LISA experiment aims to measure gravitational waves from colliding supermassive black holes at high cosmological redshift. Estimate the required sensitivity of LISA in terms of the dimensionless strain.

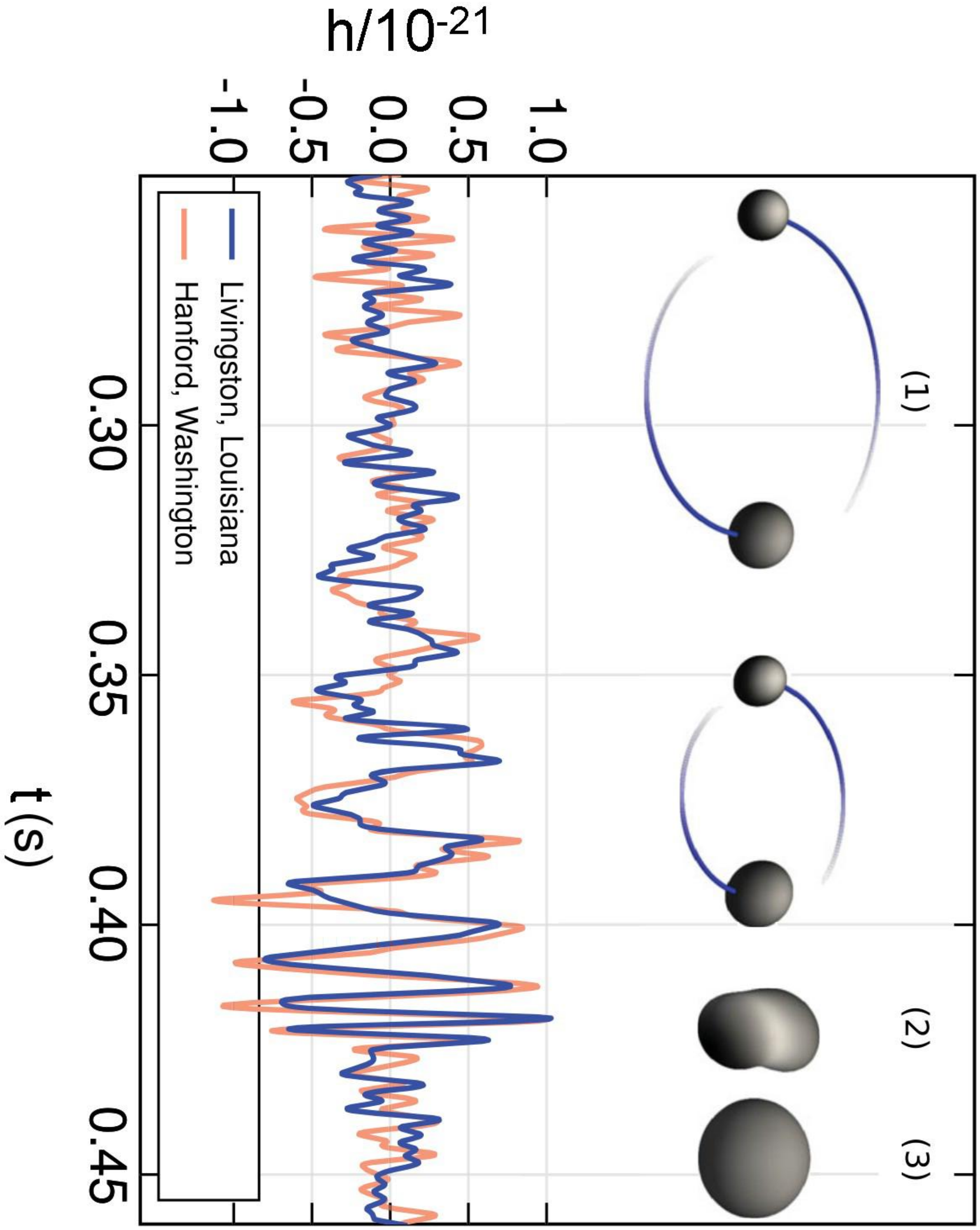
h/10^-21
-1.0
-0.5
0.0
0.5
1.0
Livingston, Louisiana
Hanford, Washington
(1)
(2)
(3)
0.30
0.35
0.40
0.45
t(s)

Fig.1