



Theoretical round. Sketches for solutions

Note for jury and team leaders. The proposed sketches are not full; the team leaders have to give more detailed explanations for students. But the correct solutions in the students' papers (enough for 8 pts) may be shorter.

α -1. Star rise in Moletai.

1.1. The time elapsed from the rising of the star to culmination is equal to the time elapsed from culmination to set. Therefore, on September 8, this star rose at 02:54 – 02:51 = 0:03. The next time the star will rise after a sidereal day, equal to 23:56, that is, at 23:59 but still on September 8. On September 9 the star will rise after one more sidereal day, that is at 23:55.

1.2. Very roughly one can say that the star is above the horizon $\frac{1}{4}$ days, and under the horizon $-\frac{1}{4}$ days. The star culminates at south. Accordingly, it is above the horizon: $\frac{1}{4}$ of rotation before the culmination and $\frac{1}{4}$ of rotation after, that is, it rises approximately at south-east and sets approximately at south-west. At the picture that should be drawn by the participants, it can be seen even better. Answer: SE. (IOB in Russian).

β -1. RadioAstron. The resolution of the instrument is determined by the working wavelength λ , and the base D. For telescopes the base is just the diameter of the front lens or mirrors, and for interferometers it is the distance between its elements, or rather, the projection of this distance on the plane perpendicular to the direction to the object under research.

$$\delta = \lambda / D.$$

Thus, the maximum resolution is achieved in the case of the minimum wavelength and the maximum base.

$$\delta_{\min} = \lambda_{\min} / D_{\max}.$$

1.1. In our case it is achieved at the wavelength $\lambda_{\min} = 1.2$ cm and the base of the Earth-"Spektr-R", in which "Spektr-R" is at apogee, and the ground-based telescope is at the opposite side of the globe; and while the observations are held in the directions perpendicular to the base of Earth-"Spektr-R", that is perpendicular to the major axis of the orbit:

$$D_{\max} = A + R,$$

where A is the distance at apogee, and R is radius of the Earth. The apogee distance can be found as

$$A = 2a - (R+h),$$

where a is semi-major axis of the orbit, which, in turn, follows Kepler's third law. Comparing the motion of the satellite with the motion of the Moon we get:

$$a^3/a_J^3 = \tau^2/\tau_J^2, \quad a = a_J \cdot (\tau/\tau_J)^{2/3},$$

$$a = 384\,400$$

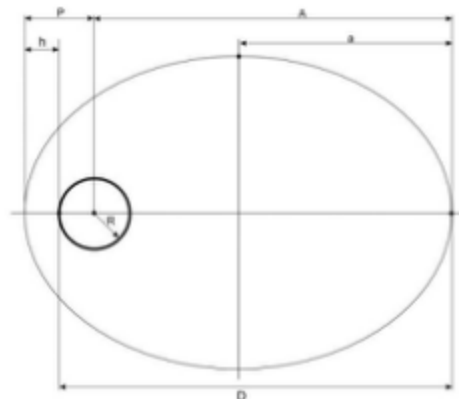
$$\text{km} \times (8.3 \text{ days} / 27.32 \text{ days})^{2/3} = 173\,700 \text{ km}.$$

$$D_{\max} = 2a - (R+h) + R = 2a - h,$$

$$D_{\max} = 2 \times 173\,700 \text{ km} - 600 \text{ km} \approx 347\,000 \text{ km}.$$

$$\delta_{\min} = 1.2 \text{ cm} / 347\,000 \text{ km} \approx 3.5 \cdot 10^{-11} \text{ rad},$$

or in angular seconds:



$$\delta_{\min} = 3.5 \cdot 10^{-11} \text{ rad} \times 206265''/\text{rad} \approx 7 \cdot 10^{-6}'' = 7 \text{ } \mu\text{s}.$$

1.2. In observations of objects located in the direction of the major axis of the orbit of "Spektr-R", the maximum base D will be equal to the sum of the semi-minor axis of the orbit and the radius of Earth:

$$D_{\max} = b + R.$$

Semi-minor axis can be found from the geometric properties of the ellipse:

$$a^2 = b^2 + (a-P)^2, \quad b^2 = [a^2 - (a-P)^2]^{1/2},$$

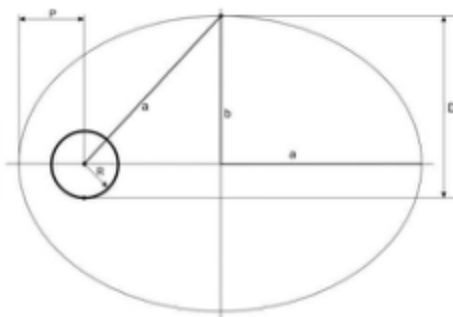
$$b \approx 48\,900 \text{ km}.$$

$$D_{\max} = 48\,900 \text{ km} + 6\,400 \text{ km} \approx 55\,300 \text{ km}.$$

$$\delta_{\min} = \lambda_{\min} / D_{\max} = 1.2 \text{ cm} / 55\,300 \text{ km} \approx 2.2 \cdot 10^{-10} \text{ rad},$$

or in angular seconds:

$$\delta_{\min} = 2.2 \cdot 10^{-11} \text{ rad} \times 206265''/\text{rad} \approx 4.5 \cdot 10^{-5}'' = 45 \text{ } \mu\text{s}.$$



$\alpha\beta$ -2. Gliese 581 g.

2.1. It is obvious that there should be about the same conditions as on the Earth for the existence of albuminous based life on the planet.

(Following in the solution indexes G refer to the system Gliese 581, and indexes \odot and \oplus – to the Sun and Earth).

In the "data of some stars" table one can find the mass of the star Gliese 581, which is equal to $M_G = 0.31 M_\odot$, its parallax $p = 0.16''$, and the apparent bolometric magnitude $m = 8^m.0$.

Knowing the parallax and the apparent bolometric magnitude of the star, one can find its absolute bolometric magnitude:

$$M = m + 5^m + 5^m \lg p = 8^m.0 + 5^m + -3^m.98 \approx 9^m.0.$$

It is $9^m - 4^m.8 = 4^m.2$ greater than the absolute magnitude of the Sun. Thus, the radiation from Gliese 581 is $I_G/I_\odot = 10^{4.2/2.5} \approx 48$ times less than that of the Sun.

Taking into account the Stefan-Boltzmann law, let us compare the temperature regimes on Earth (T_\oplus) and on the planet Gliese 581 g (T_G).

It is known that the balance temperature T on the planet is defined by the balance of incoming and radiated energy. The radiated energy depends only on the surface area and the temperature (the dependence on the temperature is described by the Stefan-Boltzmann $E \sim T^4$, but is not necessary to know the exact relationship for the task). In absence of internal sources of energy (we make this assumption) the radiated energy is equal to the energy from the central luminary. Both the radiated and incoming energies are proportional to the surface area of the planet; hence the balance does not depend on the size of the planet.

The incoming energy is proportional to the radiation of the central star and inversely proportional to the distance to the star:

$$E_\oplus \sim I_\odot/R_\oplus^2, \quad E_G \sim I_G/R_G^2$$

Thus, for the temperatures on Earth and Gliese 581 g to be equal, it is necessary that

$$E_\oplus = E_G \quad \text{or} \quad R_G/R_\oplus = (I_G/I_\odot)^{1/2}.$$

According to the generalized Kepler's third law

$$\tau^2 M/a^3 = \text{const}, \quad \text{that is} \quad \tau \sim a^{3/2}/M^{1/2},$$

or as the orbits are circular,

$$\tau \sim R^{3/2}/M^{1/2}, \quad \tau_G / \tau_\oplus = (R_G/R_\oplus)^{3/2} / (M_G/M_\odot)^{1/2},$$

$$\tau_G = \tau_\oplus (R_G/R_\oplus)^{3/2} / (M_G/M_\odot)^{1/2} = \tau_\oplus (I_G/I_\odot)^{3/4} / (M_G/M_\odot)^{1/2},$$

$$\tau_G = 365 \text{ days} \times 48^{-3/4} / 0.31^{1/2},$$

$$\tau_G \approx 36 \text{ days}.$$

β -2.2. The size of the planet could be determined by recording the radio-sources from different points of the planet's surface (result of an activity of the civilization). In order to get at least some estimate of the size of the planet, one needs to get a "picture" of radio-sources distribution, and the angular size of the "picture" should exceed the angular resolution at least several times. It should be assumed that as the planet Gliese 581 g is similar to Earth, its diameter is comparable to the diameter of Earth D_E , at least in an order of the value. The angular size of a body with a diameter of Earth D_E in system of Gliese 581 is equal to

$$\alpha = p \times D_E / R_E.$$

$$\alpha = 0,16'' \times 12,8 \text{ thous.km} / 149 \text{ 600 thous.km} \approx 14 \cdot 10^{-6}'' = 14 \mu\text{as}.$$

As we see, this value is comparable with the best angular resolution of RadioAstron, and for the most of positions of the "Spektr-R" it is even less. In order to obtain any reasonable result an "image" have to be taken, that is the angular size of the object have to exceed the angular resolution at least several times. It is not possible to get a "picture", only a spot will appear as evidence of the extended radio-source. Thus only, it is clear that radio-sources are not at one point but at different locations. In the best case it will be able to determine the order of the diameter of the planet (which we had known earlier).

Answer: «нет-но».

Note. The size of the planet Gliese 581 g, determined by other methods, is estimated as 1.4 ± 0.2 of the size of Earth. The ratio of 1.4 does not introduce considerable amendments to the solution.

$\alpha\beta$ -3. Observations from Gliese 581 g.

$\alpha\beta$ -3.1. The magnitude of the Sun, visible from any remote location is

$$m_\odot = m_0 + 5^m \log(206265) - 5^m \lg p.$$

where m_0 is the apparent magnitude of the Sun visible from Earth, and p is the annual parallax of the location.

The magnitude of the Sun as seen from the system of Gliese 581 is

$$m_\odot = -26^m.74 + 26^m.57 - 5^m \log 0.16 = -0^m.17 - 3^m.98 \approx 3^m.8.$$

(And it is not the only way of solution. The value can be found in many other ways, e.g., by formula

$$m_\odot = M - 5^m + 5^m \lg p).$$

$\alpha\beta$ -3.2. While observing from the planet Gliese 581 g, our Sun will be at a point opposite to where the system of Gliese 581 is seen from Earth. Because Gliese 581 is located in the zodiacal constellation of Libra, the sun will be approximately in the opposite zodiacal constellation, that is, in the **constellation of Aries**. (That solution is correct enough.)

Note: Using a sky-chart one can solve the problem more accurately. The point opposite to the location of the system Gliese 581 at the Earth's sky, has the coordinates $03^h 19^m 27^s$ and $07^\circ 43' 20''$. This point is located in the constellation of Taurus (Tau), very close to the borders of the constellations of Cetus (Cet) and Aries (Ari). As the students were not provided by a sky-chart, and question was "what is the approximate constellation", the answer "approximately in the constellation, opposite to the constellation of Libra, that is in the constellation of Aries" is correct.

β -3.3. To keep the heat balance a Gliese 581 g should receive the same amount of energy per unit area as is received at Earth.

(As in the previous solution G indices refer to the system of Gliese 581, and indexes \odot and E – to the Sun and the Earth).

From the given tables one may find that the spectral class of the star Gliese 581 is M3V. According to the Hertzsprung-Russell diagram the temperature of the star, corresponding to this spectral class, is about $T_G = 3300$ K. It is $T_\odot/T_G = 5780/3300 \approx 1.75$ times less than the temperature of the Sun.

According to the Stefan-Boltzmann law the power emitted from the star is $P = \sigma \cdot ST^4$, and in just the same way we may discourse about the apparent angular area from which the radiation is coming.

Thus, since the apparent angular area of a star is proportional to the square of its apparent angular diameter α , the following equations should be correct:

$$\pi/4 \cdot \alpha_G^2 T_G^4 = \pi/4 \cdot \alpha_\odot^2 T_\odot^4 \quad \text{or} \quad \alpha_G = \alpha_\odot (T_\odot/T_G)^2,$$

$$\alpha_G = 32' \cdot (1.75)^2 = 98'.$$

More than three times larger than the Sun!

αβ-4. XVIII century. Midday. Horse-powers per square verst is a power per unit of area, the quantity that dimension is similar to the solar constant. In physics, it is properly called the "power flux".

Let us calculate the solar constant in units, which were coming into operation in the end of the XVIII century.

I method – visual.

The energy-release of the Sun has not changed since the end of the XVIII century and in units of the horse-power is

$$L = 3.86 \cdot 10^{26} \text{ W} / (735.5 \text{ W/hp}) = 5.25 \cdot 10^{23} \text{ horse-powers.}$$

The distance from the Sun to Earth has not changed since the end of the XVIII century as well, and in versts is

$$a = 1 \text{ AU} = 1.496 \cdot 10^{11} \text{ m} / (3 \cdot 500 \cdot 0.7112 \text{ m/vrst}) = 1.402 \cdot 10^8 \text{ vrst.}$$

Thus, the solar constant in units of the end of the XVIII century is

$$A_{XIX} = L / 4\pi a^2 = 5.25 \cdot 10^{23} \text{ hp} / 4\pi(1.402 \cdot 10^8 \text{ vrst})^2 = 2.12 \cdot 10^6 \text{ hp} / \text{vrst}^2.$$

II method – formal.

$$A_{XIX} = A_0 \times k_{(hp/W)} / k_{(vrst/m)}^2 = 1367 \text{ W/m}^2 / (735.5 \text{ W/hp}) \times (1066.78 \text{ m/vrst})^2 = 2.12 \cdot 10^6 \text{ hp} / \text{vrst}^2.$$

4.1. The outskirts of Dubingiai in the XVIII century were located on the same latitude as where they are located in the XXI century, that is, at about latitudes $54.7^\circ - 55.2^\circ$ (latitudes of Vilnius and Moletai respectively, may be found from the map). The value of 23.5° , the inclination of the equatorial plane of the ecliptic, also almost has not changed (strictly speaking, it had declined by about one arc minute). Thus, at the middays of the winter solstice in the XVIII century in the outskirts of the town Dubingiai the sun was at an altitude about

$$h_W = 90^\circ - 55.0^\circ - 23.5^\circ = 11.5^\circ,$$

at the middays of spring and autumn – at an altitude about

$$h_0 = 90^\circ - 55.0^\circ = 35.0^\circ,$$

at the middays of the summer solstice – at an altitude about

$$h_S = 90^\circ - 55.0^\circ + 23.5^\circ = 58.5^\circ.$$

While the altitude of the Sun is h , the power flux to the unit of the territory is

$$W = A \cdot \sin h.$$

Thus, the target values are:

$$\text{in winter} - W_W = A \cdot \sin h_W \approx 420\,000 \text{ hp/vrst}^2,$$

$$\text{in spring and autumn} - W_0 = A \cdot \sin h_0 \approx 1\,220\,000 \text{ hp/vrst}^2,$$

$$\text{in summer} - W_S = A \cdot \sin h_S \approx 1\,810\,000 \text{ hp/vrst}^2.$$

4.2. And what is the capacity of the solar energy incident at those times on a local horse? This value largely depends on the orientation of the horse relative to the Sun than the altitude of the sun above the horizon. Let us assume that the cross-sectional area of a horse perpendicular to the sun-rays is $1-3 \text{ arsh}^2$ (depending on the mentioned above orientation of the horse) and recalculate the value of the solar constant into horse-powers per square arshin.

Distance from the Sun

$$a = 1 \text{ AU} = 1.496 \cdot 10^{11} \text{ m} / 0.7112 \text{ m/arsh} = 2.103 \cdot 10^{14} \text{ arsh.}$$

The solar constant in these units equals to

$$A = L / 4\pi a^2 = 5.25 \cdot 10^{23} \text{ hp} / 4\pi(2.103 \cdot 10^{14} \text{ arsh})^2 = 0.94 \text{ hp} / \text{arsh}^2.$$

Thus, a horse standing in the sun, receives about one to three horse-powers of solar radiation! Just this can be a surprise.

Answer: 1-3 hp.

Note. In solving the problem we emphasize very often that many of the physical parameters are not changed in the last two centuries: the solar constant, the distance from Earth to the Sun, the latitude of location, inclination of the ecliptic to the equatorial plane. Students who also record this constancy should be encouraged to add points.

αβ-5. XXI century. Midday.

5.1. Today (on September 8) the summer time is in operation, so the watches of citizens of Lithuania ahead the universal time by 3 hours. Local mean solar time coincide with this time on the meridian 45° of East longitude, located too far to the east (at the latitude of Lithuania it is in the Nizhny Novgorod region of Russia).

The adding of the equation of time (see figure) is in the range of $-16 - +14$ minutes, and this effect may replace the meridian not more than 4° in longitude. But even with this addition such areas are far beyond the borders of Lithuania.

Answer: «нет-но».

5.2. To answer the second question of the problem you need to consider two periods: summer and winter time. For any of the day during summer time operation the situation is similar to that we discussed in sub-point 1.

The territory of Lithuania is approximately (we can measure it with an accuracy of a quarter degree) between $21^\circ 00'$ and $26^\circ 45'$ of East longitude. At the period when the winter time is in operation, the situation is queerer. Local mean solar time correspond to the meridian 30° East. Thus, in the winter time period mean local solar noon also does not occur before 12:00 of Lithuanian time anywhere in Lithuania.

However, there is a question about the upper culmination of the sun in the problem condition, as a rule it does not coincide with the mean solar culmination at 12:00 of mean solar time. The difference between the apparent (or true) solar time (T_A) and mean solar time (T_M) is defined by the equation of time,

$$T_M = T_A + \eta,$$

where η is parameter of the equation of time, which is hereinafter referred to as simply "equation of time", as it is accepted by astronomers.

The meridian 30° East passes very close to the eastern border of Lithuania (about this meridian St.Petersburg, Nevel, Vitebsk, Orsha, Mogilev and Kiev are located), so one should pay attention to the equation of time. Winter time is in operation from the end of October to the end of March, and within this range the values of the equation of time may be both positive and negative.

Positive values of the equation of time move the apparent solar noon to later time (by the watches of inhabitants), and the negative to earlier time. Using the graph of the equation of time one can determine that the adding is minimal on November 3 and its value in a minimum is -16.4 minutes. The corresponding meridian (for the apparent culmination of the Sun at 12:00 Lithuanian time) is shifted by $4^\circ 06'$ West and by almost a degree enters the territory of Lithuania, till the longitude $\lambda = 25^\circ 54'$.

Thus, the apparent noon will occur at 12:00 of Lithuanian time somewhere at the territory of the country, if the values of the equation of time were below $-3^\circ 45'$ in degrees, and so the values of η were below -15 minutes. And η is below -15 minutes from October 18 to November 17 (approximately, as we can find from the graph).

Answer: «да-yes», October 18 – November 17 (part of this period when winter time is in operation).

Note. There is no universally accepted definition of the sign of the equation of time. This solution is written in accordance with the historically European definition of the sign of the equation of time (positive values of η in January-March, as it is displayed in the supplement page). However, in the British and American (and also translated from English) literature, the definition is inverse (negative values of η in January-March). The members of Jury must consider that solutions using both alternatives of definition are correct (of course, the answer does not depend on the choice of the alternative, turning "+" and "-" at the graph leads to turn the sign of η in the equation (1)).

αβ-6. Supernova remnant.

6.1. Assume that the SNR is a sphere with the linear radius R . Using a ruler we accurately measure the diameter of the circle and the scale length (e.g. in mm) in Fig. SNR (see the figure below, the initial figure was almost 2 times larger). From these measurements we get the radius of the circle $\rho = 48.5$ mm and $100''$ scale length $l = 28.5$ mm. With these data we can calculate the angular radius of the SNR

$$\theta = 100'' \rho / l,$$

$$\theta = 100'' \times 48.5 / 28.5 \approx 170''.$$

If the radius R of the SNR is measured in au, the distance d in pc, and its angular radius θ in arcsec, then

$$R = d \times \theta,$$

$$R = 170'' \times 3400 = 578000 \text{ AU} \approx 8.6 \cdot 10^{16} \text{ m}.$$

The mass of the SNR is confined within the sphere of the radius R .

$$M = \rho \cdot 4/3 \cdot \pi R^3,$$

$$M = 10^{21} \cdot 4/3 \cdot 3,14 \cdot (8.5 \cdot 10^{16})^3 \approx 2.7 \cdot 10^{30} \text{ kg}.$$

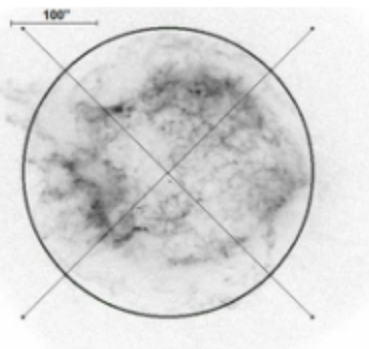
The expansion speed of the SNR is calculated using the expression of the kinetic energy of the SNR

$$E_{kin} = 1/2 M v^2,$$

$$v = (2 \cdot E_{kin} / M)^{1/2} = (2 \cdot 0,01 \cdot E_{SN} / M)^{1/2},$$

$$v = (2 \cdot 0,01 \cdot 10^{46} \text{ J} / 2.7 \cdot 10^{30} \text{ kg})^{1/2} \approx 8.6 \cdot 10^6 \text{ m/s} \approx 10^7 \text{ m/s}.$$

As we see from the image, the darkest parts of it are distributed at distances about 60% of its radius. So we may propose that most mass of the SNR is distributed at these distances. Thus, by the above formulae we have found the average speed of matter at 0.6 R .



Assuming that the expansion velocity was constant we get the age of the SNR:

$$t = 0.6 R / v,$$

$$t = 0.6 \times 8.6 \cdot 10^{16} \text{ m} / 8.6 \cdot 10^6 \text{ m/s} \approx 6 \cdot 10^9 \text{ s} \approx 190 \text{ yr} \approx 200 \text{ yr}.$$

Taking into account that most data in the problem conditions was done very roughly, with maximum 1 significant digit (and even with zero significant digits as only order of magnitude is done: the energy was about 10^{46} J, 1% of which drives the expansion; the average density is 10^{21} kg/m³), the answer should be expressed also with only one significant digit.

So the answer is: ~200 years.

6.2. Using a ruler we measure the distance of the neutron star from the center of circle in Fig. SNR. Say that we get $\sigma = 3$ mm. Then the angular distance δ of the neutron star from the center of the SNR is

$$\delta = 100'' \sigma / l,$$

$$\delta = 100'' \times 3 / 28.5 \approx 10.5''.$$

The linear distance of the neutron star from the center of the SNR is

$$S_N = \sigma \times d,$$

$$S_N = 10.5 \times 3400 \approx 35700 \text{ AU} \approx 5.3 \cdot 10^{12} \text{ km}.$$

The velocity of the motion of the neutron star is

$$u = S_N / t,$$

$$u = 5.3 \cdot 10^{12} \text{ km} / 6 \cdot 10^9 \text{ s} \approx 900 \text{ km/s} \approx 10^3 \text{ km/s}.$$

Also taking into account that most data was done very roughly, the answer should be expressed also as an order of magnitude only or with only one significant digit.

The answer is: $\sim 10^3$ km/s.